INTUITIONISTIC FUZZY SETS AND EXPERT ESTIMATIONS

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The Intuitionistic Fuzzy Sets (IFSs) [1] can be applied in all places where the ordinary fuzzy sets have applications. Now we shall illustrate one of the possible ways for using the IFSs in expert estimations.

Let both index sets I and J be given. We shall mark the indeces of the different data and observations by their elements, respectively.

Let the data generate the set $D = \{D_i / i \in I\}$, where D_i has the form:

$$D_i = \{<S_{i,a}, \mu_{i,a}, \nu_{i,a}, \alpha_{i,a}> / a \in A_i\},$$

where: $i \in I$, A_i is the set of the indeces of the elements of D_i; $S_{i,a}$ is an element of the data for which $\mu_{i,a}$ and $\nu_{i,a}$ are respectively the smallest true-degree and the highest false-degree which this element can have to be admissible and $\mu_{i,a} + \nu_{i,a} \le 1$; and $\alpha_{i,a}$ is its priority about the other elements of the set $pr_1 D_i$ and $\mu_{i,a}, \nu_{i,a}, \alpha_{i,a} \in [0, 1]$, where $pr_1 Z$ is the set of first projections of m-dimensional set Z ($m \ge 1$; when $m = 1$: $pr_1 Z = Z$).

Let $<S_{i,a}, \mu_{i,a}, \nu_{i,a}, \alpha_{i,a}> \in D_i$ only if $\mu_{i,a} > 0$ and $\alpha_{i,a} > 0$.

Let $A = \bigcup_{i \in I} A_i$.

Let the observations generate the set $N = \{N_j / j \in J\}$, where N_j has the form:

$$N_j = \{<S_{j,b}, \bar{\mu}_{j,b}, \bar{\nu}_{j,b}> / b \in B_j\},$$

where: $j \in J$, B_j is the set of the indeces of the elements of N_j; $S_{j,b}$ is a data element for which $\bar{\mu}_{j,b}$ and $\bar{\nu}_{j,b}$ are respectively the measured (determined, calculated) true-degree and the measured (determined, calculated) false-degree of this element, $\bar{\mu}_{i,a}$, $\bar{\nu}_{i,a} \in [0, 1]$ and $\bar{\mu}_{j,b} + \bar{\nu}_{j,b} \le 1$.

Let $B = \bigcup_{j \in J} B_j$.

Then for every $i \in I$ and for every $j \in J$ we define the number:

$$\sigma_{i,j} = \begin{cases} 0, & \text{if } A_i \cap B_j = \emptyset \\ \left(\sum_{a=b \in A_i \cap B_j} \alpha_{i,a} . sg(\bar{\mu}_{j,b} - \mu_{i,a}).(\mu_{i,a} - \bar{\mu}_{j,b})^2 + \right. \\ \left. sg(\gamma_{i,a} - \bar{\gamma}_{j,b}).\ ((\gamma_{i,a} - \bar{\gamma}_{j,b})^2)^{1/2} \right) / card(A_i \cap B_j), \\ \text{if } A_i \cap B_j \neq \emptyset \end{cases}$$

which determines the degree of accordance between the observation N_j and the data D_i, where

$$sg(x) = \begin{cases} 0, & \text{ако } x < 0 \\ 1, & \text{ако } x \geq 0 \end{cases}.$$

For the decreasing of the number of checks, for every data D_i can be defined a natural number d_i , for which the value of $\sigma_{i,j}$ to be calculated only for these observations, for which

$$\sum_{a=b \in A_i \cap B_j} sg(\bar{\mu}_{j,b} - \mu_{i,a}).sg(\gamma_{i,a} - \bar{\gamma}_{j,b}) \geq d_i .$$

For the calculated values $\sigma_{i,j}$ it can be calculated the numbers

$$\sigma^m_j = \min_{i \in I} \sigma_{i,j} \quad \text{and} \quad \sigma^M_j = \max_{i \in I} \sigma_{i,j},$$

which correspond to the minimal and maximal degrees which are admissiblle for observation N_j in relation to data D_i. It can easily be seen that for every $i \in I$, for which $j \in J$: $\sigma_{i,j} \in [0, 1]$. The validity of the assertion follows from the fact that for $a = b \in A_i \cap B_j$:

$$\alpha_{i,a} . ((\mu_{i,a} - \bar{\mu}_{j,b})^2 + (\gamma_{i,a} - \bar{\gamma}_{j,b})^2)^{1/2} \leq 1$$

and the common number of all addends is $card(A_i \cap B_j)$.

Then, to the observation N_j we can juxtapose the couple $<\sigma^m_j, 1 - \sigma^M_j>$. It can directly be seen that set

$$\{<N_j,\ \sigma^m_j,\ 1 - \sigma^M_j>/\ N_j \in N\}$$

is an IFS.

In some concrete applications, the concepts "data" and "observation" can be interpreted, e.g. as "disease" and "symptom", respectively.

Let there be m experts $E_1, E_2, \ldots, E_m$, p objects (processes) for expert estimation $P_1, P_2, \ldots, P_p$ and q criteria for the estimation $Q_1, Q_2, \ldots, Q_q$. Let every expert have himself (current) rating $\delta_i \in [0, 1]$ and himself (current) number of participations in expert investigations Γ_i (the values of both numbers correspond to his last expert estimation). The expert's rating can be interpreted, e.g., as

$$\delta_i = \left(\sum_{j=1}^{q} \delta_{i,j}\right)/q$$

where $\delta_{i,j}$ are elements of the index matrix (see [2])

$$T = \begin{array}{c|c} & Q_1 \quad Q_2 \quad \ldots \quad Q_q \\ \hline E_1 & \\ E_2 & \delta_{i,j} \\ \vdots & (1 \le i \le m, \\ E_m & 1 \le j \le q) \end{array}$$

and $\delta_{i,j}$ is the rating of i-th expert about j-th criterion (we assume that i-th expert can be specialist with different degrees about different criteria; the case when i-th expert is equally good specialist about the different criteria is a particular case).

Let i-th expert E_i $(1 \le i \le m)$ give the following estimations, which are described by index matrix

$$S_i = \begin{array}{c|c} & P_1 \quad P_2 \quad \ldots \quad P_p \\ \hline Q_1 & \\ Q_2 & \langle \alpha^i_{j,k}, \beta^i_{j,k} \rangle \\ \vdots & (1 \le i \le m, \\ & 1 \le j \le q, \\ Q_q & 1 \le k \le p) \end{array}$$

where: $\alpha^{i}_{j,k}, \beta^{i}_{j,k} \in [0, 1]$ and $\alpha^{i}_{j,k} + \beta^{i}_{j,k} \leq 1$.

Then, we can construct index matrix

$$S = \begin{array}{c|c} & P_1 \quad P_2 \quad \dots \quad P_p \\ \hline Q_1 & \\ Q_2 & \langle \alpha_{j,k}, \beta_{j,k} \rangle \\ \vdots & \\ Q_q & \begin{array}{c} (1 \leq j \leq q, \\ 1 \leq k \leq p) \end{array} \end{array}$$

where

$$\alpha_{j,k} = (\sum_{i=1}^{m} \partial_i . \alpha^{i}_{j,k})/m$$

$$\beta_{j,k} = (\sum_{i=1}^{m} \partial_i . \beta^{i}_{j,k})/m$$

or (more precisely)

$$\alpha_{j,k} = (\sum_{i=1}^{m} \partial_{i,j} . \alpha^{i}_{j,k})/m$$

$$\beta_{j,k} = (\sum_{i=1}^{m} \partial_{i,j} . \beta^{i}_{j,k})/m$$

Obviously, $\alpha_{j,k} + \beta_{j,k} \leq 1$.

This index matrix contains the average experts' estimations in relation to experts' ratings.

Let every one of the criteria Q_j $(1 \leq j \leq q)$ have itself priority $\psi_j \in [0, 1]$.

We can determine for every object (process) P_k the global estimation $\langle \alpha_k, \beta_k \rangle$, where

$$\alpha_k = (\sum_{j=1}^{q} \psi_j . \alpha_{j,k})/q$$

$$\beta_k = (\sum_{j=1}^{q} \psi_j . \beta_{j,k})/q$$

* * *

Let objects (processes) have about the different criteria the following (objective) values after the end of the expert estimations:

$$\begin{array}{c|c} & P_1 \quad P_2 \quad \dots \quad P_p \\ \hline Q_1 & \\ Q_2 & \langle a_{j,k}, b_{j,k} \rangle \\ \vdots & (1 \le j \le q, \\ Q_q & 1 \le k \le k) \end{array}$$

where: $a_{j,k}, b_{j,k} \in [0, 1]$ and $a_{j,k} + b_{j,k} \le 1$.

Then the new rating ∂'_i and the new number of participations in expert investigations Γ'_i of the expert will be:

$$\Gamma'_i = \Gamma_i + 1,$$

and

$$\partial'_i = (\Gamma_i . \partial_i + c_M - c_i + C - \Gamma_i . \partial_i . (c_M - c_i + C))/\Gamma'_i,$$

where

$$c_i = (\sum_{j=1}^{q} \sum_{k=1}^{p} ((\alpha_{j,k} - a_{j,k})^2 + (\beta_{j,k} - b_{j,k})^2)^{1/2})/p.q,$$

$$c_M = \min_{1 \le i \le m} c_i,$$

and C is a coefficient of the admissible mistake-tolerance for the experts.

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The described above ideas will be included in a project for a global generalized net-model of hospital activities (see [3]). Some of the ideas, which are interpreted in the terms of the IFSs are generated from books on mathematical statistics, but the author does not know similar results in fuzzy set-interpretation although it is very likely that such researches exist.

REFERENCES:

[1] Atanassov K. Intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 20 (1986), No. 1, 87-96.

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[3] M. Tetev, J. Sorsich and K. Atanassov, Generalized nets model of hospital activities, AMSE Periodical, Vol. 17, No. 1, 1993, 55-64.