Theory of R-convergence of Nets in Fuzzy Lattices and Its Applications

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Abstract: In this paper, we first introduce the concepts of R-convergence of nets and R-closures in fuzzy lattices, and systematically discuss their properties. We obtain more interesting characterizations with respect to almost continuous and R-irresolute order-homomorphisms by means of the theory.

Keywords: fuzzy lattice; R-convergence; R-closure; order-homomorphism; almost continuity; R-irresoluteness

1. Preliminaries

Throughout this paper, L, L_1 and L_2 will always denote fuzzy lattices, i.e. completely distributive lattices with order-reversing involutions"′ ". M, M_1 and M_2 will denote the set of all nonzero $\vee$-irreducible elements i.e. so-called moleculae, or points for short, in L, L_1 and L_2 respectively. $(L(M),\delta)$, $(L_1(M_1),\delta_1)$ and $(L_2(M_2), \delta_2)$ will be topological molecular lattices (briefly, TML) with the topology δ, δ_1 and δ_2 respectively. Put $\eta(e)=\{P \in\delta';\ e\not\leq P\}$ and call the elements in $\eta(e)$ R-neighborhoods of a point $e\in M$[5]. Write $R\eta(e)=\{P\in\eta(e): P=P^{o-}\}$.

A mapping $f:L_1\to L_2$ is said to be an order-homomorphism if the following conditions hold: (H_1) $f(0)=0$; (H_2) $f(\vee A_i)=\vee f(A_i)$; (H_3) $f^{-1}(B')=(f^{-1}(B))'$[6]. An order-homomorphism (briefly, OH) $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ is called almost continuous (R-irresolute) if the inverse image of every regular open element in L_2 is open (regular open) in L_1.

2. Theory of R-convergence of nets in fuzzy lattices

Definition 2.1 Let $(L(M),\delta)$ be a TML, $A\in L$ and $e\in M$. e is in the R-closure of A ($e\leq \bar{A}_R$) if for each $P\in R\eta(e)$, $A\not\leq P$. If $e\leq\bar{A}_R$, then we call e a R-adherence point of A. A is called R-closed if $A=\bar{A}_R$. A is called R-open if A' is R-closed.

The follwing theorem follows immediately from Definition 2.1.

Theorem 2.1 In any TML $(L(M),\delta)$ we have:

(1) The least element 0 and the greatest element 1 of L are R-closed.

(2) $A \leq A^{-} \leq A_R^{-} \leq A_\theta^{-}$[4] for each $A \in L$.

(3) If $A \leq B$, then $A_R^{-} \leq B_R^{-}$ for $A, B \in L$.

(4) $A_R^{-} = \vee \{e \in M: e$ is a R-adherence point of $A\}$ for $A \in L$.

(5) Arbitrary intersections and finite unions of R-closed elements are R-closed.

(6) Every R-closed element is closed.

(7) Every θ-closed element is R-closed.

(8) Every regular closed element is R-closed.

(9) If $A \in \delta$, then $A^{-} = A_R^{-} = A_\theta^{-}$.

Definition2.2 A point e in $(L(M), \delta)$ is said to be a R-cluster point of an element A in $(LM), \delta)$ if (1) $e \leq A_R^{-}$; (2) $e \not\leq A$, or $e \leq A$ and $A \not\leq P \vee b$ for each $P \in \eta(e)$ and each b in M with $e \leq b \leq A$. The union of all R-cluster points of A will be denoted by A_R^d and called the R-derived element of A.

Theorem 2.2 Let $(L(M), \delta)$ be a TML, $A \in L$. Then

(1) $A_R^{-} = A \vee A_R^d$

(2) $(A^d)_R^{-} \leq A_R^{-}$.

(3) A is R-closed iff for each point $e \not\leq A$, there exists $P \in R\eta(e)$ such that $A \leq P$.

Proof. The proofs of (1) and (2) are easy and are omitted. We only check (3). In case A is R-closed and $e \not\leq A$, then $e \not\leq A_R^{-}$, and then there exists $P \in R\eta(e)$ such that $A \leq P$ by Definition2.1. Conversely, if A is not R-closed, then we have a point $e \in M$ such that $e \leq A_R^{-}$ and $e \not\leq A$ by Proposition 2.17 in [5]. However, being $e \leq A_R^{-}$, we know that there is not $P \in R\eta(e)$ such that $A \leq P$. Hence the sufficiency is proved.

Definition 2.3 Let S be a molecular net in $(L(M), \delta)$ and $e \in M$. If for each $P \in R\eta(e)$, S is eventually not in P, then e is called a R-limit point of S (or S R-converges to e), in symbols $S \xrightarrow[R]{} e$. If for each $P \in R\eta(e)$, S is frequently not in P, then e is called a R-cluster point of S (or S R-accumulates to e), in symbols $S \underset{R}{\infty} e$. The union of all R-limit points and all R-cluster points of S will be denoted by R-limS and R-adS respectively.

From the definition, Definition4.17 in [5] and Definition3.3 in [3] one can readily verify the following theorem:

Theorem 2.3 Let S be a molecular net in $(L(M), \delta)$ and $e \in M$. Then we have:

(1) $S \xrightarrow[R]{} e$ iff $e \leq$ R-lim S.

(2) $S \underset{R}{\infty} e$ iff $e \leq$ R-ad S.

(3) lim $S \leq$ R-lim$S \leq \theta$-lim S.

(4) ad $S \leq$ R-ad$S \leq \theta$-ad S.

(5) R-lim $S \leq$ R-ad S.

(6) R-lim S and R-ad S are R-closed.

Where θ-limS and θ-adS denote the union of all θ-limit points[3] and all θ-cluster points [3] of S respectively.

Proof. We only investigate Statement (6). Let $e < (\text{R-lim}S)^{-}_{R}$. Then for each $P \in R\eta(e)$ we have $\text{R-lim}S \nless P$ by Definition 2.1, and hence there is a point $b \in M$ such that $b < \text{R-lim } S$ and $b \nless P$ according to Proposition 2.7 in [5]. Since $P \in R\eta(b)$ and $b < \text{R-lim } S$, by Statement(1), S is eventually not P. So $e < \text{R-lim } S$. This implies that R-lim S is R-closed. Similarly, $\text{R-ad}S = (\text{R-ad}S)^{-}_{R}$.

Theorem 2.4 In a TML $(L(M),\delta)$, $e < A^{-}_{R}$ iff there exists in A a molecular net which R-converges to e.

Proof. Let $e < A^{-}_{R}$; then for each $P \in R\eta(e)$ we have $A \nless P$. In the light of Proposition 2.17 in [5], there is a molecula $S(P)$ in A with $S(P) \nless P$. Take $S = \{S(P) \colon P \in R\eta(e)\}$. Obviously, S is a molecular net in A by virtue of the fact that $R\eta(e)$ is an ideal base and $S \underset{R}{\to} e$. Conversely, if $S = \{S(n) \colon n \in D\}$ is a molecular net in A and $S \underset{R}{\to} e$, then for each $P \in R\eta(e)$, there is $n_0 \in D$ such that $S(n) \nless P$ whenever $n > n_0$ $(n \in D)$. Hence $A \nless P$, and hence $e < A^{-}_{R}$ by Definition 2.1.

Theorem 2.5 Let S be a molecular net in $(L(M),\delta)$ and $e \in M$. Then $S \underset{R}{\infty} e$ iff S has a subnet T satisfying $T \underset{R}{\to} e$.

Proof. Suppose that $S = \{S(n) \colon n \in D\}$ is a molecular net in $(L(M),\delta)$ and $S \underset{R}{\infty} e$. Then for each $P \in R\eta(e)$ and each $n \in D$, there exists $N(P,n) \in D$ such that $N(P,n) > n$ and $S(N(P,n)) \nless P$. Let $E = R\eta(e) \times D$ and define

$$(P_1, N(P_1,n_1)) < (P_2, N(P_2,n_2)) \text{ iff } P_1 < P_2 \text{ and } n_1 < n_2.$$

Then E is a directed set. Take $T(P,N(P,n)) = S(N(P,n))$. Then we obtain a subnet $T = \{T(P, N(P,n)) \colon (P,N(P,n)) \in E\}$ of S. For each $Q \in R\eta(e)$, choose $(Q,N(Q,n)) \in E$, we have $T(P,N(P,n)) \nless Q$ whenever $(P,N(P,n)) > (Q,N(Q,n))$ because of the fact that $T(P,N(P,n)) = S(N(P,n)) \nless P$ and $Q < P$. This shows that T is eventually not in Q, and so T R-converges to e. Conversely, provided that $T = \{T(m) \colon m \in E\}$ is a subnet of S and $T \underset{R}{\to} e$. For each $n_0 \in D$, we have a mapping $N \colon E \to D$ and $m_0 \in E$ such that $N(m) > n_0$ as $m > m_0$ $(m \in E)$. Since T R-converges to e, there is $m_1 \in E$ with $T(m) \nless P$ as long as $m > m_1$ $(m \in E)$ for each $P \in R\eta(e)$. Because E is a directed set, we have $m_2 \in E$ such that $m_2 > m_0$ and $m_2 > m_1$. Hence $T(m_2) \nless P$ and $N(m_2) > n_0$. Let $n = N(m_2)$. Then $S(n) = S(N(m_2)) = T(m_2) \nless P$ and $n > n_0$. This means that S is frequently not in P. Hence $S \underset{R}{\infty} e$.

Theorem 2.6 Assume that S is a molecular net in $(L(M),\delta)$. If S R-converges to $e \in M$, then every subnet of S also R-converges to e.

Proof. The proof is straightforward and is omitted.

3. Applications with Respect to Theory of R-convergence of Nets

In [1], N.Ajmal and S.K. Azad introduced the notion of fuzzy almost continuity at a fuzzy point and obtained a pointwise charactrization of fuzzy almost continuous functions by dual points and fuzzy nets. In this section, we shall present the concepts of almost continuous and R-irresolute order-homomorphisms at a point, which are a proper generalization of that in [1], and get more characters of almost continuity and R-irresoluteness by theory of R-convergence of nets.

Definition 3.1 Let $f:(L_1(M_1),\delta_1)\rightarrow(L_2(M_2),\delta_2)$ be an OH and $e\in M$. f is said to be almost continuous at e if for each $P\in R\eta(f(e))$ we have $(f^{-1}(P))^{-}\in\eta(e)$.

Theorem 3.1 An OH $f:(L_1(M_1)\delta_1)\rightarrow(L_2(M_2),\delta_2)$ is almost continuous iff for each point $e\in M_1$, f is almost continuous at e.

Proof. Suppose that f is almost continuous, $e\in M_1$ and $P\in R\eta(f(e))$. Then $f^{-1}(P)=(f^{-1}(P))^{-}$ by Theorem 2.2 in [2]. Since $f(e)\nleqslant P$ iff $e\nleqslant f^{-1}(P)$, $(f^{-1}(P))^{-}\in\eta(e)$. Hence f is almost continuous at e. Conversely, if f is not almost continuous, then there exists a regular closed element B in $(\delta_2)'$ such that $f^{-1}(B)<(f^{-1}(B))^{-}$. In accordance with Proposition 2.17 in [5] we have a point $e\in M_1$ satisfying $e\leqslant(f^{-1}(B))^{-}$ and $e\nleqslant f^{-1}(B)$. Because $e\nleqslant f^{-1}(B)$ implies $f(e)\nleqslant B$, $B\in R\eta(f(e))$. However, $(f^{-1}(B))^{-}\notin\eta(e)$. Therefore the sufficiency holds.

Theorem 3.2 An OH $f:(L_1(M_1),\delta_1)\rightarrow(L_2(M_2),\delta_2)$ is almost continuous iff for each $A\in L_1$, $f(A^{-})\leqslant(f(A))^{-}_{R}$.

Proof. In case f is almost continuous and $A\in L_1$, then for each point $e\leqslant A^{-}$ and each $P\in R\eta(f(e))$ we have $(f^{-1}(P))^{-}\in\eta(e)$, and then $A\nleqslant(f^{-1}(P))^{-}=f^{-1}(P)$, i.e. $f(A)\nleqslant P$. Therefore $f(e)\leqslant(f(A))^{-}_{R}$ by Definition 2.1. This implies $f(A^{-})\leqslant(f(A))^{-}_{R}$. Conversely, suppose that the condition is satisfied and that B is a regular closed element in L_2. Then we have $f((f^{-1}(B))^{-})\leqslant(ff^{-1}(B))^{-}_{R}\leqslant B^{-}_{R}=B$ by Theorem 2.1 (6), equivalently, $(f^{-1}(B))^{-}\leqslant f^{-1}(B)$. This shows that f is almost continuous.

Theorem 3.3 An OH $f:(L_1(M_1),\delta_1)\rightarrow(L_2(M_2),\delta_2)$ is almost continuous iff for each point $e\in M_1$ and each molecular net S in L_1 which converges to e, f(S) R-converges to f(e).

Proof. Assume that f is almost continuous, $e\in M_1$ and $P\in R\eta(f(e))$. Then $(f^{-1}(P))^{-}\in\eta(e)$ by Theorem 3.1. Let $S=\{S(n): n\in D\}$ be a molecular net in L_1 which converges to e. Then there is $n_0\in D$ such that $S(n)\nleqslant(f^{-1}(P))^{-}=f^{-1}(P)$ whenever $n\geqslant n_0$ $(n\in D)$. Since $S(n)\nleqslant f^{-1}(P)$ implies $f(S(n))\nleqslant P$, $f(S)=\{f(S(n)): n\in D\}$ R-converges to f(e). Conversely, grant that B is a regular closed element in L_2. We shall prove that $(f^{-1}(B))^{-}\leqslant f^{-1}(B)$. For this aim, let $e\leqslant(f^{-1}(B))^{-}$. According to Corollary 4.23 in [5], there exists in $f^{-1}(B)$ a molecular net S which converges to e. Obviously f(S) is a molecular net in B. Hence f(S) R-converges to f(e) by the condition of the theorem, and hence $f(e) < \bar{B}_R = B$, that is, $e < f^{-1}(B)$. This means that $(f^{-1}(B))^- < f^{-1}(B)$. Thus the almost continuity of f follows immediately.

Theorem 3.4 An OH $f:(L_1(M_1),\delta_1) \to (L_2(M_2),\delta_2)$ is almost cotinuous iff for each molecular net S in L_1, $f(\lim S) < R\text{-}\lim f(S)$.

Proof. Presume that S is a molecular net in L_1. By Theorem 2.3 and Theorem 4.21 in [5], we know easily that $f(\lim S) < R\text{-}\lim f(S)$ iff for each point $e \in M_1$ and $e < \lim S$, $f(e) < R\text{-}\lim f(S)$, i.e. $f(\lim S) < R\text{-}\lim f(S)$ iff for each point $e \in M_1$, $S \to e$ implies $f(S) \underset{R}{\to} f(e)$.
Hence the theorem follows from Theorem3.3

Theorem 3.5 An OH $f:(L_1(M_1),\delta_1) \to (L_2(M_2),\delta_2)$ is almost continuous iff for each $B \in L_2$, $(f^{-1}(B))^- < f^{-1}(\bar{B}_R)$.

Proof. Since for each $B \in L_2$, $f^{-1}(B) \in L_1$, from Theorem 3.2 we obtain that if f is almost continuous, then $f((f^{-1}(B))^-) < \overline{(ff^{-1}(B))}_R < \bar{B}_R$. Hence $(f^{-1}(B))^- < f^{-1}(\bar{B}_R)$. Conversely, assume that the condition is true and $A \in L_1$. Then $f(A) \in L_2$ and $A^- < (f^{-1}f(A))^- < f^{-1}(\overline{(f(A))}_R)$. So $f(A^-) < \overline{(f(A))}_R$ and so f is almost continuous by Theorem 3.2.

Theorem 3.6 Let $(L_1(M_1),\delta_1)$ be a C_I TML [5]; then an OH $f:(L_1(M_1),\delta_1) \to (L_2(M_2),\delta_2)$ is almost continuous iff for each point $e \in M_1$ and each molecular sequence S in L_1 which converges to e, f(S) R-converges to f(e).

Proof. The necessity follows from Theorem 3.3. Now we only prove the sufficiency. If f is not almost continuous, then there exists a point $e \in M_1$ such that f is not almost continuous at e. This is the same as there is $Q \in R\eta(f(e))$ with $(f^{-1}(Q))^- \notin \eta(e)$. Let $\{P_n: n \in N\}$ be an increasing R-neighborhood base of e. Then for each $n \in N$ we have $f^{-1}(Q) \nless P_n$, and then there is a molecula S(n) satisfying $S(n) < f^{-1}(Q)$ and $S(n) \nless P_n$. Take $S = \{S(n): n \in D\}$, one easily sees that S is molecular sequence in L_1 which converges to e. However, f(S) does not R-converges to f(e) because for each $n \in N$, $S(n) < f^{-1}(Q)$, i.e. $f(S(n)) < Q$.

Definition 3.2 An OH $f:(L_1(M_1),\delta_1) \to (L_2(M_2),\delta_2)$ is called R-irresolute at a point $e \in M_1$ if for each $P \in R\eta(f(e))$, $\overline{(f^{-1}(P))}_R \in \eta(e)$.

Theorem 3.7 An OH $f:(L_1(M_1),\delta_1) \to (L_2(M_2),\delta_2)$ is R-irresolute iff for each point $e \in M_1$, f is R-irresolute at e.

Proof. Let f is R-irresolute, $e \in M_1$ and $P \in R\eta(f(e))$. Then $f^{-1}(P)$ is regular closed in L_1 by Theorem 4.5 in [2], and then $f^{-1}(P) = \overline{(f^{-1}(P))}_R$ in the light of Theorem 2.1. Since $f(e) \nless P$ implies that $e \nless f^{-1}(P)$, $\overline{(f^{-1}(P))}_R \in \eta(e)$. This shows that f is R-irresolute at e. Conversely, in case f is not R-irresolute, then there is a regular closed element Q in L_2 such that $f^{-1}(Q)$ is not regular closed in L_1. Hence $f^{-1}(Q) < \overline{(f^{-1}(Q))}_R$. From Proposition 2.17 in [5] we have a point $e\in M_1$ such that $e\not< f^{-1}(Q)$ and $e<(f^{-1}(Q))_R^-$. But $e\not< f^{-1}(Q)$ implies $Q\in R\eta(f(e))$. Therefore f is not R-irresolut at e.

Theorem3.8 An OH $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ is R-irresolute iff for each $A\in L_1$, $f(A_R^-)<(f(A))_R^-$.

Proof. Provided that f is R-irresolute and $A\in L_1$. In order to investigate $f(A_R^-)<(f(A))_R^-$, we only need to verify that for each point $e\in M_1$ and $e<A_R^-$, $f(e)<(f(A))_R^-$. For this purpose, in case $P\in R\eta(f(e))$, then $f^{-1}(P)=(f^{-1}(P))_R^-\in\eta(e)$ by Theorem 3.7. Being $e<A_R^-$, we have $A\not< f^{-1}(P)$, i.e. $f(A)\not< P$. Hence $f(e)<(f(A))_R^-$ by Definition 2.1. Conversely, suppose that the condition is satisfied, $e\in M_1$ and $Q\in R\eta(f(e))$. Since $f(e)\not< Q$ iff $e\not< f^{-1}(Q)$, we have $f(((f^{-1}(Q))_R^-)<(ff^{-1}(Q))_R^-<Q_R^-=Q$, that is, $(f^{-1}(Q))_R^-<f^{-1}(Q)$. Therefore $(f^{-1}(Q))_R^-\in\eta(e)$. This shows that f is R-irresolute at e. Hence the sufficiency follows from Theorem 3.7.

Theorem 3.9 An OH $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ is R-irresolute iff for each point $e\in M_1$ and each molecular net S in L_1 which R-converges to e, f(S) R-converges to f(e) in L_2.

Proof. Assume that f is R-irresolute, $e\in M_1$ and $P\in R\eta(f(e))$. Then $(f^{-1}(P))_R^-\in\eta(e)$ by Theorem 3.7. Let $S=\{S(n): n\in D\}$ be a molecular net which R-converges to e in L_1. Then there exists $n_0\in D$ such that $S(n)\not<(f^{-1}(P))_R^-$, specially, $S(n)\not< f^{-1}(P)$ as long as $n\geq n_0$ $(n\in D)$. Hence $f(S)=\{f(S(n): n\in D\}$ R-converges to f(e) by virtue of the fact that $S(n)\not< f^{-1}(P)$ implies that $f(S(n))\not< P$. Conversely, let $A\in L_1$ and $e<A_R^-$ $(e\in M_1)$. By Theorem 2.4, there exists in A a molecular net S which R-converges to e. Obviously, f(S) is a molecular net in f(A). Hence f(S) R-converges to f(e) using the condition of the theorem, and hence $f(e)<(f(A))_R^-$. This means that $f(A_R^-)<(f(A))_R^-$. So f is R-irresolute by Theorem 3.8.

Theorem 3.10. An OH $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ is R-irresolute iff for each molecular net in L_1, $f(\text{R-lim}S)<\text{R-lim}f(S)$.

Proof. The proof follows from Theorem 2.3 and Theorem 3.9 and is omitted.

Theorem 3.11 An OH $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ is R-irresolute iff for each $B\in L_2$, $(f^{-1}(B))_R^-<(f^{-1}(B_R^-))$.

Proof. If f is R-irresolute and $B\in L_2$, then $f^{-1}(B)\in L_1$ and $f((f^{-1}(B))_R^-)<(ff^{-1}(B))_R^-<B_R^-$, equivalently, $(f^{-1}(B))_R^-<f^{-1}(B_R^-)$ by Theorem 3.8. Conversely, if the condition is true, then for each $A\in L_1$ we have $A_R^-<(f^{-1}(f(A)))_R^-<f^{-1}((f(A))_R^-)$, i.e. $f(A_R^-)<(f(A))_R^-$. Hence the R-irresoluteness of f follows from Theorem 3.8.

Theorem 3.12 Let $f:(L_1(M_1),\delta_1)\to(L_2(M_2),\delta_2)$ be an OH. Then the following conditions are equivalent:

(1) f is R-irresolute.

(2) For each R-closed element P in L_2, $f^{-1}(P)$ is a R-closed element in L_1.

(3) For each R-open element G in L_2, $f^{-1}(G)$ is a R-open element in L_1.

Proof. The equivalence between (2) and (3) is clear. Now we prove that (1) is equivalent to (2). Suppose that f is R-irresolute and that P is R-closed in L_2. Then $f^{-1}(P)=f^{-1}(P^{-}_{R})\geqslant(f^{-1}(P))^{-}_{R}$ by Theorem 3.11. On the other hand, $f^{-1}(P)\leqslant(f^{-1}(P))^{-}_{R}$ follows from Theorem 2.1. Hence $f^{-1}(P)$ is R-closed in L_1. Conversely, in case (2) holds, then for each $e\in M_1$ and each $Q\in R\eta(e)$ we have $f^{-1}(P)=(f^{-1}(P))^{-}_{R}$. Since $f(e)\not\leq P$ implies that $e\not\leq f^{-1}(P)$, $(f^{-1}(P))^{-}_{R}\in\eta(e)$ by Theorem 2.1(6). This means that f is R-irresolute at e. Hence f is R-irresolute in the light of Theorem 3.7.

Analogous to proof of Theorem 3.6 we have:

Theorem 3.13 If $(L_1(M_1),\delta_1)$ is a C_1 TML, then an OH $f:(L_1(M_1),\delta_1)\rightarrow(L_2(M_2),\delta_2)$ is R-irresolute iff for each point $e\in M_1$ and each molecular sequence S in L_1 which R-converges to e, f(S) R-converges to f(e).

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