COMPLEX FUZZY NEIGHBORHOOD

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According to Paper [3],complex fuzzy number x and its lef end point (p[x]) , right endpoint (Q[x]) and the inferior (infx) on rack set, three orderly real number groups (P[x],Q[x],infx) are one-to-one correspondent.In spatial rectangular coordinate system, orderly real number groups can be indicated by a complex fuzzy point. The distance between two complex fuzzy numbers x_1 and x_2 is

$$d[x_1,x_2]=\max\{|p[x_2]-P[x_1]|,|Q[x_2]-Q[x_1]|,|\inf x_2-\inf x_1|\}$$

which has the three properties of metric space. Thus, complx fuzzy metric space is formed. On the basis of this, we enter complex fuzzy neighborhood and the concept of its boundary contour.

Definition 1. In complex fuzzy metric space, for complex fuzzy point x_0, any real number $\delta>0$, the complex fuzzy points which make $d[x_0, x]<\delta$ hold are called complex fuzzy neighborhood of complex fuzzy point x_0, and is written as $G(x_0, \delta)$. Complex fuzzy point x_0 is called the center of $G(x_0,\delta)$and δ the radius of $G(x_0, \delta)$.

Definition 2. If we talk out the center complex fuzzy point x_0 in complex fuzzy neighborhood $G(x_0, \delta)$, that is

$$\{|x|<d[x_0, x]<\delta\},$$

then we call it the δ deleted complex fuzzy neighborhood of complex fuzzy point x_o, and is written as:

$$\overline{G}(x_o, \delta).$$

Definition 3. Let D be a subset of complex fuzzy number set.

(1). If there exists a certain $\delta>0$ complex fuzzy neighborhood $G(x_o, \delta)$ of complex fuzzy point x_o, so that all the complex fuzzy points in $G(x_o, \delta)$ are points within D, then we call complex fuzzy point x_o is the inner complex fuzzy point of D.

(2). In any complex fuzzy neighborhood $G(x0, \delta)$ of complex fuzzy point x_o, if some complex fuzzy points belong to D and some do not, then we call complex fuzzy point x_o is a bound points of D, and all the bound point of D are the boundary contour of D.

In n-dimensional Euclidean space, neighborhood

$$U(P_o, \gamma)=\{(x_1, x_2, \dots x_n) | \sqrt{(x_1-x_1^0)^2+(x_2-x_2^0)^2+\dots+(x_n-x_n^0)^2}<\gamma\}$$

with $P0(x_1^0, x_2^0, \dots, x_n^0)$ and $\gamma>0$ being its center and radius, respecti vely is a sphere (not including the spherical surface), with P_o and γ being its center of the sphere and radius, respectively. The boundary contour of $U(P_o, \gamma)$ is a spherical surface, with P_o and γ being its center of the sphere and radius, respectively, that is,

$$d(P_o, \gamma)=\gamma.$$

In order to make it clear whether these conclusions are applicable to complex fuzzy neighborhood and its boundary contour in complex fuzzy metric-space, we further study complex fuzzy neighborhood $G(x_o, \delta)$ and its boundary contour.

Let's suppose x0 is an information-type complex fuzzy number i. e. $\inf x_o=0$. we discuss the following two cases:

1) $\delta>1$

According to Definition 1, complex fuzzy number has two cases to make $d[x_o, x]<\delta$ hold

$$\inf x=0$$

and

$$\inf x=1$$

when $\inf x=0$

since $d[x_o, x]=\max\{|P[x]-P[x_o]\},Q[x]-Q[x_o]|, 0\}<\delta$

we have $|P[x]-P[x0]|<\delta$

and $|Q[x]-Q[x0]|<\delta$

that is $P[x_o]-\delta<P[x]<P[x_o]+\delta$

and $Q[x_o]-\delta<Q[x]<Q[x_o]+\delta$

Again since $P[x]<Q[x]$

the figue of complex fuzzy neighborhood is two rectangles.

Obviusly, the figure of $G(x_o, \delta)$ is the following:

When $Q[x_o]-P[x_o]>2\delta$

it is two squares, with its boundary contour being the four sides of these two squares.

When $0<Q[x_o]-P[x_o]<2\delta$

it is two pentagons, with its boundary contour being the four sides of each pentagon (except the one on $P[x]=Q[x]$).

When $Q[x_o]-P[x_o]=0$

it is two isosceles right triangles, with its boundary contour being the two sides of each isoceles right triangle (except the one on $P[x]=Q[x]$).

2) $0<\delta<1$

From Definition 1 and the definition of complex fuzzy number, we know that complex fuzzy numbers which make $d[x_o, x]<\delta$ hold are only imformation-type ones, that is

$$\inf x=0$$

Like 1 we know when $\inf x=0$, the figure of $G(x_o, x)$ are the following:

When $Q[x_o]-P[x_o]>2\delta$,

it is a square,with its boundary contour being the sides of the aquare.

When $0<Q[x_o]-P[x_o]<2\delta$,

it is a pentagon, with its boundary contour being the four sides of the pentagon (except the one on $P[x]=Q[x]$).

When $Q[x_o]-P[x_o]=0$,

it is an isosceles right triangle, with its boundary contour being the two right-angled sides.

In one word, the figure of complex fuzzy neighborhood $G(x_o, \delta)$ can be in either one hald or two halves of complex fuzzy plain. This is determined by $0<\delta<1$ and $\delta>1$. It can be one or two squares, pentagons or isosceles right triangles. This is determined by the relationship among δ, $P[x_o]$ and $Q[x_o]$. The boundary contour of $G(x_o, \delta)$ is not $d[x_o, x]=\delta$. This is also determined by the relationship among δ, $P[x_o]$ and $Q[x_o]$.

Example 1. Try to analyse complex fuzzy neighborhood $G([0, 0], 1)$ and its boundary contour.

Solution: From Definition 1 and the definition of complex fuzzy number, we know

$$d([0, 0], x)=\max\{|P[x]-0|, |Q[x]-0|, |\inf x-0|\}<1.$$

Then we have

$$|P[x]|<1,$$

$$|Q[x]|<1,$$

and $$\inf x=1,$$

that is

$$-1<P[x]<1,$$

$$-1<Q[x]<1,$$

and x is an interval-type complex fuzzy number.

Again since $$P[x]<Q[x],$$

the figure of $G([0, 0], 1)$ is an isosceles right triangle. From Definition 3, we know its boundary is the two right-angled sides.

Example 2. Tye to analyse complex fuzzy neighborhood $G([1, \underset{\sim}{2}], 2)$ and its boundary contour.

Solution: According to Definition 1 and the definition of complex fuzzy nember, the complex fuzzy points of $\{x|\ d[[1, \underset{\sim}{2}], 2]<2\}$ can be divided into two parts.

(1) $\inf x=0$

$$d[[1, \underset{\sim}{2}], x]=\max\{|p[x]-1|, |Q[x]-2|, |\inf x-\inf[1, \underset{\sim}{2}]|\}$$

$$=\max\{|P[x]-1|, |Q[x]-2|\}<2$$

Then we have

$$|P[x]-1|<2,$$

and $$|Q[x]-2|<2,$$

that is

$$-1<P[x]<3,$$

and $$0<Q[x]<4.$$

(2) $\inf x=1$

$$d[[1,\underset{\sim}{2}],x]=\max\{|P[x]-1|, |Q[x]-2|, |1-0|\}$$

$$=\max\{|p[x]-1|, |Q[x]-2|, 1\}<2$$

Then wehave

$$|P[x]-1|<2,$$

and $|Q[x]-2|<2,$

that is

$$-1<P[x]<3,$$

and $$0<Q[x]<4$$

since

$$P[x]<Q[x]$$

from (1) and (2), we know the figure of $G[[1,\underset{\sim}{2}],2]$ is two pentagons. From Definition 3 we know the boundary contour of $G[[1,\underset{\sim}{2}],2]$ is the sides of these two pentagons except the one on $P[x]=Q[x]$.

In form, the concept of complex fuzzy neighborhood and its boundary contour in complex fuzzy metric-space and the concept of the neighborhood and its boundary contour in Euclidean space are very alike. But they are quite different. Complex fuzzy neighborhood and its boundary contour has its own specific characteristics. We must pay much attention to this, and shouldn't apply the properties of neighborhood and its boundary contour in Euclidean space to complex fuzzy neighborhood and its boundary contour on assumptions.

REFERENCES

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