FUZZY WEAKLY IRRESOLUTE MAPPINGS

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ABSTRACT

In this paper, we introduce and study the fuzzy weakly irresolute mapping in fuzzy topological spaces.

Key words: Fuzzy semiopen set; fuzzy strongly semiopen set; fuzzy weakly irresolute mapping.

1. PRELIMINARIES

In this paper, A°, A^{-}, $A_{\circ}$, A_{-} and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A.

Definition 1.1[2]. Let A be a fuzzy set of a fuzzy topological space (X,δ). Then A is called

(1) a fuzzy strongly semiopen set of X iff there is a $B\in\delta$ such that $B\leqslant A\leqslant B^{-\circ}$.

(2) a fuzzy strongly semiclosed set of X iff there is a fuzzy closed set B in X such that $B^{\circ-}\leqslant A\leqslant B$.

Definition 1.2[2]. Let A be a fuzzy set of a fuzzy space (X,δ). Then

$$A^{\Delta}=\bigcup\{B:\ B\leqslant A,\ B \text{ fuzzy strongly semiopen}\}$$

is called the fuzzy strong semi-interior of A and

$$A^{\sim}=\bigcap\{B:\ A\leqslant B,\ B \text{ fuzzy strongly semiclosed}\}$$

is called the fuzzy strong semi-closure of A.

Definition 1.3. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called

(1) a fuzzy strongly semicontinuous mapping if $f^{-1}(B)$ is a fuzzy

strongly semiopen set of X for each $B \in \tau$[2].

(2) a fuzzy irresolute mapping if $f^{-1}(B)$ is a fuzzy semiopen set of X for each fuzzy semiopen set B of Y[4].

(3) a fuzzy S-irresolute mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X for each fuzzy strongly semiopen set B of Y[3].

2. FUZZY WEAKLY IRRESOLUTE MAPPINGS

Definition 2.1. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y, f is called a fuzzy weakly irresolute mapping if $f^{-1}(B)$ is a fuzzy semiopen set of X for each fuzzy strongly semiopen set B of Y.

Definition 2.2. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y , f is said to be fuzzy weakly irresolute at a fuzzy point p in X, if fuzzy strongly semiopen set B of Y and $f(p)\leqslant B$, there exists a fuzzy semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$.

Theorem 2.3. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be a mapping from a fuzzy space X to another fuzzy space Y. Then the following are equivalent:

(1) f is fuzzy weakly irresolute.

(2) $f^{-1}(B)$ is a fuzzy semiclosed set of X for each fuzzy strongly semiclosed set B of Y.

(3) $f(A_{-})\leqslant(f(A))^{\sim}$ for each fuzzy set A of X.

(4) $(f^{-1}(B))_{-}\leqslant f^{-1}(B^{\sim})$ for each fuzzy set B of Y.

(5) $f^{-1}(B^{\Delta})\leqslant(f^{-1}(B))_{\circ}$ for each fuzzy set B of Y.

(6) $f^{-1}(B)\leqslant(f^{-1}(B))^{\circ -}$ for each fuzzy strongly semiopen set B of Y.

(7) $f^{-1}(B)\geqslant(f^{-1}(B))^{-\circ}$ for each fuzzy strongly semiclosed set B of Y.

(8) f is fuzzy weakly irresolute for each fuzzy point p in X.

Proof. We prove only (1)<=>(8).

(1)=>(8): Let f be fuzzy weakly irresolute, p be a fuzzy point in X and B be a fuzzy strongly semiopen set of Y such that $f(P)\leqslant B$. Then $p\leqslant f^{-1}(B)$. Let $A=f^{-1}(B)$, then A is fuzzy semiopen set of X, and so $f(A)=ff^{-1}(B)\leqslant B$. Thus f is fuzzy weakly irresolute for each fuzzy point p in X.

(8)=>(1): Let B be a fuzzy strongly semiopen set of Y and p be a fuzzy point in $f^{-1}(B)$. Then $p\leqslant f^{-1}(B)$, i.e., $f(p)\leqslant B$. From hypothesis there exists a fuzzy semiopen set A of X such that $p\leqslant A$ and $f(A)\leqslant B$, hence

$$p\leqslant A\leqslant f^{-1}f(A)\leqslant f^{-1}(B)$$

and

$$p\leqslant A=A_{\circ}\leqslant (f^{-1}(B))_{\circ}.$$

Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$, $f^{-1}(B)\leqslant(f^{-1}(B))_{\circ}$, i.e., $f^{-1}(B)$ is a fuzzy semiopen set of X. Thus f is fuzzy weakly irresolute.

Theorem 2.4. Let $f:(X,\delta)\rightarrow(Y,\tau)$ be one-to-one and onto, where X and Y are fuzzy spaces. f is a fuzzy weakly irresolute mapping iff $(f(A))^{\Delta}\leqslant f(A_{\circ})$ for each fuzzy set A of X.

Theorem 2.5. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2. Then the product $f_1\times f_2:X_1\times X_2\rightarrow Y_1\times Y_2$ of fuzzy weakly irresolute mappings $f_1:X_1\rightarrow Y_1$ and $f_2:X_2\rightarrow Y_2$ is fuzzy weakly irrselote.

3. MUTUAL RELATIONSHIPS

Remark 3.1. For the mapping $f:X\rightarrow Y$ the following statements are valid:

f fuzzy irresolute ⇘
f fuzzy weakly irresolute=>f fuzzy semicontinuous.
f fuzzy S-irresolute ⇗

None is reversible.

Example 3.2. Let X={a, b, c}, and A, B, C be fuzzy sets of X defined as follows:

$A(a)=0.2$, $A(b)=0$, $A(c)=0.3$;

$B(a)=0.5$, $B(b)=0.4$, $B(c)=0.4$;

$C(a)=0.5$, $C(b)=0.4$, $C(c)=0.6$.

(1) Let $\delta=\{0, A, 1\}$, and $\tau=\{0, C, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy semicontinuous. But f is not fuzzy weakly irresolute.

(2) Let $\delta=\{0, A, B', 1\}$ and $\tau=\{0, B, 1\}$. Consider the identity mapping $f:(X,\delta)\to(X,\tau)$. Then f is fuzzy weakly irresolute. But f is not fuzzy irresolute neither fuzzy S-irresolute.

Proposition 3.3. Let $f:X\to Y$ and $g:Y\to Z$ be mappings. Then the following statements are valid:

(1) If f is fuzzy weakly irresolute and g is fuzzy S-irresolute, then $g\circ f$ is fuzzy weakly irresolute.

(2) If f is fuzzy weakly irresolute and g is fuzzy strongly semicontinuous, then $g\circ f$ is fuzzy semicontinuous.

(3) If f is fuzzy irresolute and g is fuzzy weakly irresolute, then $g\circ f$ is fuzy weakly irressolute.

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