TOPOLOGICAL APPROACH TO ROUGH SETS.

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The rough sets, introduced by Pawlak [1] ten years ago, are defined [2] by using a family of equivalence relations (its intersection is called "indiscernibility" relation). Equivalentely here the rough sets are defined by means of an α-discrete topology.

Keywords: rough set, α-discrete topology, generalized rough set, rough topology.

1. **Introduction.** [2]

Let $U \neq \emptyset$ be an universal set and R an equivalence relation on U.

$X \subseteq U$ is "R-definable" or "R-exact" set if X is the union of some R-equivalence classes ($X = \bigcup_{x \in X} [x]_R$); else: X is "R-indefinable" or "R-rough" set.

If $K = \{R_i\}_{i \in I}$ is a family of equivalence relations on U, $R = \bigcap_{i \in I} R_i$ is also an equivalence relation, denoted by IND(K) and called "indiscernibility" relation on K (and it is $[x]_{IND(K)} = \bigcap_{i \in I} [x]_{R_i}$).

$X \subseteq U$ is "exact" set in K when there is a $R_i \in K$ such that X is R_i-exact set and X is "rough" set in K when for any $R_i \in K$ X is R_i-rough set.

2. α-**discrete topologies.**

(cardinality of A set denoted by $\#$ A)

Let X be a set and $\mathcal{P}(X)$ the totality of its subsets (power set of X). The couple $(X, \mathcal{P}(X))$ is called "discrete" topological space and $\mathcal{P}(X)$ the "discrete" topology of X.

Proposition 1.

Let $(X, \mathcal{T})$ be a topological space and σ the its closed set family.

If $\mathcal{T} = \sigma$ (i.e. any open set is closed also, and versavice), then $(X, \mathcal{T})$ is homeomorfic to $(Y, \mathcal{P}(Y))$ for some Y set, and $\mathcal{T}$ is called "α-discrete" topology where it's $\alpha = \# Y$.

Proof. $\forall\, x \in X$, let $\mathcal{A}_x = \{ A \in \mathcal{T} \mid A \ni x \}$ be and $A_x = \bigcap_{A \in \mathcal{A}_x} A \in \sigma = \mathcal{T}$.

It results: $\forall\, y \in A_x$ it's $A_y = A_x$. Let $x \approx y \iff A_x = A_y$ be: this is an equivalence relation; $\{ [x]_{\approx} \}_{x \in X}$ is a base for $\mathcal{T}$ and a partition of X. $Y = X/_{\approx}$ complete the proof. ∎

Corollary 1.

Any partition π of X is a base for a $\# \pi$-discrete topology called "associated" to π.

Corollary 2.

The α-discrete topological spaces $(X, \mathcal{T})$, with $\alpha < \# X$, are not T_0; they have one only base and their base-open are the connected components and quasicomponents [3] .

Examples.

Let $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ be the integer, rational, real number set resp. and $[y, z[\; = \{ x \in \mathbb{R} \mid y \leq x < z \}$ $\;]y, z[\; = \{ x \in \mathbb{R} \mid y < x < z \}$.

a) $\{[j, j+1[\}_{j\in \mathbb{Z}}$

b) $\{[2j, 2j+2[\}_{j\in \mathbb{Z}}$

c) $\{[2j+1, 2j+3[\}_{j\in \mathbb{Z}}$

d) $\{]p, q[\}_{p,q\in \mathbb{Q}}$

a), b), c) are bases for $\aleph_0$-discrete topologies (all homeomorfic) and d) is a base for euclidean real topology.

3. Rough sets.

It's well-know that partitions and equivalence relations are mutually interchangeables.

Let $\mathcal{R}$ be a family of equivalence relations on U. Let $\{R_i\}_{i\in I} \subseteq \mathcal{R}$ be a subfamily of $\mathcal{R}$. Let π_R be the partition associated to $R = \bigcap_{i\in I} R_i$ and τ the $\#\pi_R$-discrete topology associated to π_R or R.

Let $S \subseteq U$. It's straightforward:

Proposition 2.

S is exact set iff $S \in \tau$; otherwise S is rough set.

We can assume this propriety as

Definition 1.

$S \subseteq U$ is exact set iff $S \in \tau$;

$S \subseteq U$ is rough set iff $S \notin \tau$.

4. Generalized rough sets.

Let $\varrho = \{S \subseteq U \mid S \notin \tau\}$ be the family of rough sets on U.

If $\cup$, $\cap$ and $\complement$ are the union, intersection and complementation (crisp) set-theoretic operators resp., ϱ is closed for $\complement$, but ϱ is not closed for $\cup$ and $\cap$.

Definition 2.

Let (U, τ) be an α-discrete topological space and $S \subseteq U$.

The triplet $(S, \mathring{S}, \overline{S})$ is called "generalised rough set" S.

($\mathring{S}$ is the interior and $\overline{S}$ the closure of S for τ).

Corollary 3.

$S \subseteq U$ is exact set iff $S = \mathring{S} = \overline{S}$; else S is rough set iff $\mathring{S} \subset S \subset \overline{S}$.

5. Rough topologies.

Definition 3.

Let $\varphi \subset \varrho$ be such that ~~φ is~~ $\{\varphi, \emptyset, U\}$ closed for $\cup$ and finite $\cap$. The family $\{\varphi, \emptyset, U\}$ is called __rough topology__ on U.

The examples of sect.2 say that this is a good definition.

6. References.

[1] PAWLAK, Z. - Rough sets - __I.J.Inf.Comp.Sc.__ 11 (1982), 341-356

[2] PAWLAK, Z. - __ROUGH SETS__. Theoretical Aspects of Reasoning about Data - (1991), Kluwer A.P.

[3] WILLARD, S. - __General Topology__ - (1970), Addison-Wesley P.C.