NEW VARIANTS OF MODAL OPERATORS IN INTUITIONISTIC FUZZY MODAL LOGIC

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The definition of the intuitionistic fuzzy sets [1] is the basis for defining of the basic elements of the intuituinistic fuzzy logics [2-5], where two variants of an intuitionistic fuzzy propositional calculus (IFPC), a variant of an intuituinistic fuzzy predicative logic (IFPL), two variants of intuitionistic fuzzy modal calculus (IFMC) and a variant of temporal intuitionistic fuzzy logic (TIFL) are constructed.

Following the ideas from [2-5] and using the notation from there, we shall construct a variant of intuitionistic fuzzy modal logics (IFMLs).

To each proposition (in the classical meaning) one can assign its truth value: truth - denoted by 1, or falsum - 0. In the case of fuzzy logics this truth value is a real number in the interval [0, 1] and can be called "truth degree" of a particular proposition. Here we add one more value - "falsum degree" - which will be in the interval [0, 1] as well. Thus one assigns to the proposition p two real numbers $\mu(p)$ and $\gamma(p)$ moreover the constraint is valid:

$$\mu(p) + \gamma(p) \leq 1.$$

Let this be done by a evaluation function V defined such that:

$$V(p) = \langle \mu(p), \gamma(p) \rangle.$$

Hence the function $V: S \dashrightarrow [0, 1] \times [0, 1]$ gives the truth and falsum degrees from the class of all propositions.

The evaluation function V can be defined in different ways.

We assume that the evaluation function V is defined so that it assigns to the logical truth T: $V(T) = \langle 1, 0 \rangle$, and to the logical falsum F: $V(F) = \langle 0, 1 \rangle$.

We shall discuss below the truth and falsum degrees of propositions which result from the application of logical operations (unary and binary) over output propositions which have known values of its evaluation function.

The negation $\neg p$ of the proposition p will be defined through:

$$V(\neg p) = \langle \gamma(p), \mu(p) \rangle.$$

When the values V(p) and V(q) of the propositions p and q are known, the evaluation function V can be extended by its definition also for operations "&", "$\veebar$" and "$\supset$" through:

$$V(p \,\&\, q) = \langle \min(\mu(p), \mu(q)), \max(\gamma(p), \gamma(q)) \rangle,$$

$$V(p \veebar q) = \langle \max(\mu(p), \mu(q)), \min(\gamma(p), \gamma(q)) \rangle,$$

$$V(p \supset q) = \langle \max(\gamma(p), \mu(q)), \min(\mu(p), \gamma(q)) \rangle,$$

and

$$V(p) \wedge V(q) = V(p \,\&\, q),$$

$$V(p) \vee V(q) = V(p \veebar q),$$

$$V(p) \rightarrow V(q) = V(p \supset q);$$

and for operators:

$$V(\Box p) = \langle \mu(p), 1-\mu(p) \rangle,$$

$$V(\Diamond p) = \langle 1-\gamma(p), \gamma(p) \rangle,$$

and

$$\Box V(p) = V(\Box p),$$

$$\Diamond V(p) = V(\Diamond p).$$

Now we shall define two new operators from a modal type, for which:

$$V(!p) = \langle \max(1/2, \mu(p)), \min(1/2, \gamma(p)) \rangle,$$

$$V(?p) = \langle \min(1/2, \mu(p)), \max(1/2, \gamma(p)) \rangle,$$

and

$$!V(p) = V(!p),$$

$$?V(p) = V(?p).$$

The concepts an Intuitionistic Fuzzy Tautology (IFT) and Intuitionistic fuzzy Safety (IS) we shall define through (cf. [6]):

"p is an IFT" iff "if $V(p) = \langle a, b \rangle$, then $a \geq b$",

and

"p is an IS" iff "if $V(p) = \langle a, b \rangle$, then $a \geq 1/2$".

Obviously, if p is an IS, then p is an IFT.

Let $p \equiv q$ iff $(p \supset q) \,\&\, (q \supset p)$.

Let below $V(p) = \langle a, b \rangle$, $V(q) = \langle c, d \rangle$.

THEOREM 1: The following assertions are IFTs (24.0 and 24.1 in [7]):

(a) $\neg !p \equiv ?\neg p$,

(b) $!p \equiv \neg ?\neg p$,

(c) $\neg ?p \equiv !\neg p$,

(d) $?p \equiv \neg !\neg p$.

(e) $p \supset\ !p$,

(f) $?p \supset p$.

Proof: (a) $V(\neg !p \equiv ?\neg p) =$

$= (\langle \min(1/2,\ b),\ \max(1/2,\ a)\rangle \rightarrow \langle \min(1/2,\ b),\ \max(1/2,\ a)\rangle)$

$\wedge\ (\langle \min(1/2,\ b),\ \max(1/2,\ a)\rangle \rightarrow \langle \min(1/2,\ b),\ \max(1/2,\ a)\rangle)$

$= \langle \max(\max(1/2,\ a),\ \min(1/2,\ b)),\ \min(\max(1/2,\ a),\ \min(1/2,\ b))\rangle$

$\wedge\ \langle \max(\max(1/2,\ a),\ \min(1/2,\ b)),\ \min(\max(1/2,\ a),\ \min(1/2,\ b))\rangle$

$= \langle \max(\max(1/2,\ a),\ \min(1/2,\ b)),\ \min(\max(1/2,\ a),\ \min(1/2,\ b))\rangle$

From:

$\max(\max(1/2,\ a),\ \min(1/2,\ b)) - \min(\max(1/2,\ a),\ \min(1/2,\ b))$

$\geq \max(1/2,\ a) - \max(1/2,\ a) = 0$

follows the validity of (a).

(b) - (d) are proved analogically.

(e) $V(p \supset\ !p) =$

$\langle a,\ b\rangle \rightarrow \langle \max(1/2,\ a),\ \min(1/2,\ b)\rangle$

$= \langle \max(1/2,\ a,\ b),\ \min(1/2,\ a,\ b)\rangle$

and obviously:

$\max(1/2,\ a,\ b) - \min(1/2,\ a,\ b) \geq 0$,

i.e. (e) is valid.

(f) is proved analogically.

THEOREM 2: The following assertions are IFTs:

(a) $\Box !p \equiv\ !\Box p$,

(b) $\Box ?p \equiv\ ?\Box p$,

(c) $\Diamond !p \equiv\ !\Diamond p$,

(d) $\Diamond ?p \equiv\ ?\Diamond p$.

Proof: (a) $V(\Box !p \equiv\ !\Box p)$

$= (\langle \max(1/2,\ a),\ 1-\max(1/2,\ a)\rangle \rightarrow \langle \max(1/2,\ a),\ \min(1/2,\ 1-a)\rangle)$

$\wedge\ (\langle \max(1/2,\ a),\ \min(1/2,\ 1-a)\rangle \rightarrow \langle \max(1/2,\ a),\ 1-\max(1/2,\ a)\rangle)$

$= \langle \max(1-\max(1/2,\ a),\ 1/2,\ a),\ \min(\max(1/2,\ a),\ 1/2,\ 1-a)\rangle$

$\wedge\ \langle \max(1/2,\ a,\ \min(1/2,\ 1-a)),\ \min(\max(1/2,\ a),\ 1-\max(1/2,\ a)\rangle$

$= \langle \max(1-\max(1/2,\ a),\ 1/2,\ a),\ \min(\max(1/2,\ a),\ 1/2,\ 1-a)\rangle$

From:

$\max(1-\max(1/2,\ a),\ 1/2,\ a) - \min(\max(1/2,\ a),\ 1/2,\ 1-a)$

$\geq 1/2 - 1/2 = 0$

followis the validity of (a).

(b) - (d) are proved analogically.

It is checked directly, that there are no connections between

XYp and YXp, where $X \in \{D_{\alpha}, F_{\alpha,\beta}, G_{\alpha,\beta}, H_{\alpha,\beta}, H^{*}_{\alpha,\beta}, J_{\alpha,\beta}, J^{*}_{\alpha,\beta}\}$ and $Y \in \{!, ?\}$ (see [9]).

THEOREM 3: For every propositional form A, !A is an IS;

THEOREM 4: For proposition p, if p is an IS, then:

(a) $\Diamond p \supset !p$,

(b) $?p \supset \Box p$.

Proof: (a) $V(\Diamond p \supset !p)$

$= \langle 1-b, b\rangle \rightarrow \langle \max(1/2, a), \min(1/2, b)\rangle$

$= \langle \max(1/2, a, b), \min(1/2, a, b)\rangle$

and obviously:

$\max(1/2, a, b) - \min(1/2, a, b) \geq 0$,

i.e. (a) is valid.

(b) is proved analogically.

THEOREM 5: For every proposition p:

$$V(!?p) = V(?!p) = \langle 1/2, 1/2\rangle.$$

Proof: $V(!?p) = \langle \max(1/2, \min(1/2, a)), \min(1/2, \max(1/2))$

$= \langle 1/2, 1/2\rangle$,

$V(?!p) = \langle \min(1/2, \max(1/2, a)), \max(1/2, \min(1/2))$

$= \langle 1/2, 1/2\rangle$.

The geometrical interpretations of both new modal operators are given in Fig. 1 - 3 (cf. [8]) with respect of the geometrical interpretation of proposition, over which they are applied.

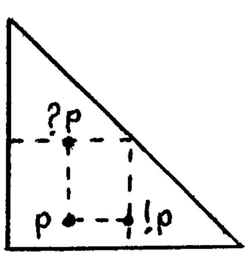

Fig. 1

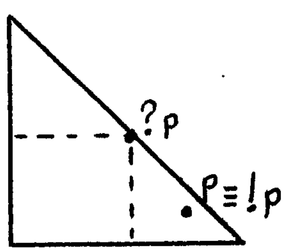

Fig. 2

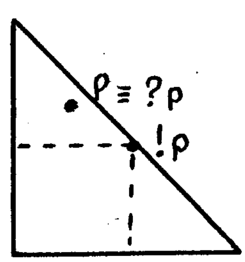

Fig. 3

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