THREE KINDS OF FUZZY DATA MODELS FOR RELATIONAL DATABASES

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Keywords: Fuzzy relational data model, fuzzy valued relational data model, fuzzy valued fuzzy relational data model

1.Introdution

In ordinary database systems, most processing has assumed that the data represented was exact, correct, well-formulated, with no provisions for cosidering otherwise. In order to represent the real world and deal with real world enterprises we must strive to broaden the types of data such systems can process.

In this paper, the concepts and the operations of fuzzy relational data model, fuzzy valued relational data model and fuzzy valued fuzzy relational data model are described.There models are considered as extensions of codd's classical relational data model, they provide necessary basis to establish fuzzy relational database management systems.

Throughout this paper, let Ui be finite set, F(Ui) a class of all fuzzy subsets of Ui, i=1,2,...,n, $\Pi_{i=1}^{n}Ui=U_1*U_2*...*U_n$, $\Pi_{i=1}^{n}F(Ui)=F(U_1)*F(U_2)*...*F(U_n)$.

2.The concepts and operations of fuzzy relational data model

Definition 1. A fuzzy relation $\widetilde{R}$ is a fuzzy subset of $\Pi_{i=1}^{n}Ui$, defined by $\mu_{\widetilde{R}}$, i.e.,

$$\mu_{\widetilde{R}}: \Pi_{i=1}^{n}Ui \longrightarrow [0,1],$$

we denote fuzzy relation R as

$$\widetilde{R}=\{(u, \mu_{\widetilde{R}}(u));\ u=(u_1,u_2,...,u_n)\in \Pi_{i=1}^{n}Ui\}$$

Since R = $\{u;\ \mu_{\widetilde{R}}(u)>0,\ u\in \Pi_{i=1}^{n}Ui\}$ = supp $\widetilde{R}$ = $\Pi_{i=1}^{n}Ui$, R is a classical

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Since R = $\{u;\ \mu_{\widetilde{R}}(u)>0,\ u\in \Pi_{i=1}^{n}Ui\}$ = supp $\widetilde{R}$ = $\Pi_{i=1}^{n}Ui$, R is a classical relation, $\widetilde{R}$ is called a fuzzy relation corresponding to R, denoted by

$$\widetilde{R} = (R, \mu_{\widetilde{R}})$$

where $\mu_{\widetilde{R}}$ is called a grade of membership of u in R. If for every $u \in R$, $\mu_{\widetilde{R}}(u)=1$, then $\widetilde{R} = R$.

For operations fuzzy relational data model, by definitions of operations of fuzzy subset, if $\widetilde{R} = (R , \mu_{\widetilde{R}})$ and $\widetilde{S} = (S , \mu_{\widetilde{S}})$, we have

(1) $\widetilde{R} \cup \widetilde{S} = (R \cup S, \mu_{\widetilde{R}} \vee \mu_{\widetilde{S}})$,
where $(\mu_{\widetilde{R}} \vee \mu_{\widetilde{S}})(u) = \max(\mu_{\widetilde{R}}(u), \mu_{\widetilde{S}}(u))$, $u \in \Pi_{i=1}^{n} Ui$;
(2) $\widetilde{R} \cap \widetilde{S} = (R \cap S, \mu_{\widetilde{R}} \wedge \mu_{\widetilde{S}})$,
where $(\mu_{\widetilde{R}} \wedge \mu_{\widetilde{S}})(u) = \min(\mu_{\widetilde{R}}(u), \mu_{\widetilde{S}}(u))$, $u \in \Pi_{i=1}^{n} Ui$;
(3) $R - S = (T , \mu_{\widetilde{T}})$
where $T = (R - S) \cup \{ u ; \mu_{\widetilde{R}}(u) - \mu_{\widetilde{S}}(u) > 0, u \in R \cap S \}$,
$\mu_{\widetilde{T}}(u) = \max(\mu_{\widetilde{R}}(u) - \mu_{\widetilde{S}}(u), 0)$;
(4) for every $0<\lambda<1$,
$(\widetilde{R})_\lambda = \{ u; \mu_{\widetilde{R}}(u) \geq \lambda, u \in \Pi_{i=1}^{K} Ui\} = R_\lambda \subset \Pi_{i=1}^{n} Ui$, $R = \bigcup_{0<\lambda\leq 1} R$.
(5) projection, for any $A = \Pi_{i=1}^{K} Uij$,
$\widetilde{R}[A] = (R[A], \mu_{\widetilde{R}[A]}) = \{(u[A], \mu_{\widetilde{R}}(u)); u \in R \}$,
where $u[A] = (u_{i_1}, u_{i_2}, \ldots, u_{i_K}) \in A$.
(6) join $\bowtie$,let $\widetilde{R}$ be a fuzzy relation in $\Pi_{j=1}^{K1} U_{ij}$, $\widetilde{S}$ be a fuzzy relation in $\Pi_{m=1}^{K2} U_{jm}$, then
$\widetilde{R} \bowtie_F \widetilde{S} = (R \bowtie_F S, \mu_{\widetilde{R}} \bowtie_F \widetilde{S})$,
where $R \bowtie_F S = \{ rs; r \in R, s \in S, F(r, s) = 1\}$,
$\mu_{\widetilde{R} \bowtie \widetilde{S}}(rs) = \min(\mu_{\widetilde{R}}(r) , \mu_{\widetilde{S}}(s))$,
$r = (u_{i_1}, u_{i_2}, \ldots, u_{i_{K1}})$ and $s = (u_{j_1}, u_{j_2}, \ldots, u_{j_{K2}})$
and F is a Boolean function representing condition of join .
(7) selection $R\{F\} = \{(u , \mu_{\widetilde{R}}(u)) ; u \in R, F(u, \mu_{\widetilde{R}}(u)) = 1\}$,
where F is a Boolean function representing condition of selection and F can contain $\mu_{\widetilde{R}}$.

3.The concepts and operations of fuzzy valued relational data model

Definition 2. A fuzzy valued relation R* is a subset of $\Pi_{i=1}^{n} F(Ui)$.

Obviously, for every $r* = (\widetilde{u}_1, \widetilde{u}_2, \ldots, \widetilde{u}_n) \in R*$, $\widetilde{u}_i$ is a fuzzy subset of Ui, i = 1,2,...,n.

relation, $\widetilde{R}$ is called a fuzzy relation corresponding to R, denoted by

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(2) $\widetilde{R} \cap \widetilde{S} = (R \cap S, \mu_{\widetilde{R}} \wedge \mu_{\widetilde{S}})$,
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Since fuzzy valued relation is a classical relation, operations of fuzzy valued relation and operations of classical relation are same. In the following, we define "<", ">", "≤", "≥", "=" of Boolean function F in fuzzy valued relational data model.

Definition 3. Let $\tilde{u}_1$ and $\tilde{u}_2$ are two fuzzy subsets of U, $\mu_{\tilde{u}_1}$ and $\mu_{\tilde{u}_2}$ are two membership functions of $\tilde{u}_1$ and $\tilde{u}_2$ respectively.

(1) $\tilde{u}_1 < \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) < \mu_{\tilde{u}_2}(u)$;
(2) $\tilde{u}_1 > \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) > \mu_{\tilde{u}_2}(u)$;
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(5) let $U = \{ u_1, u_2, \ldots, u_n \}$, $\tilde{u}_1 = \tilde{u}_2$ if and only if, for given $\varepsilon > 0$, $n(\tilde{u}_1, \tilde{u}_2) > 1-\varepsilon$, where $n(\tilde{u}_1, \tilde{u}_2)$ is a near degree of $\tilde{u}_1$ and $\tilde{u}_2$, i.e.,

$$n(\tilde{u}_1, \tilde{u}_2) = \sum_{i=1}^{n}(\mu_{\tilde{u}_1}(u_i) \wedge \mu_{\tilde{u}_2}(u_i)) / \sum_{i=1}^{n}(\mu_{\tilde{u}_1}(u_i) \vee \mu_{\tilde{u}_2}(u_i)).$$

4. The concepts and operations of fuzzy valued fuzzy relational data model

Definition 4. A fuzzy valued fuzzy relation R# is a fuzzy subset of $\Pi_{i=1}^{n} F(Ui)$ defined by $\mu_{R\#}$, i.e.,

$$\mu_{R\#} : \Pi_{i=1}^{n} F(Ui) \longrightarrow [0,1]$$

we denote fuzzy valued fuzzy relation R# as

$$R\# = \{(u*, \mu_{R\#}(u*)); u* = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n) \in \Pi_{i=1}^{n} F(Ui)\}$$

Since $R* = \{ u*; \mu_{R\#}(u*) > 0, u* \in \Pi_{i=1}^{n} F(Ui)\} = \text{supp}R\#$, R* is a classical relation, R# is called a fuzzy relation corresponding to R*, denote by

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where $\mu_{R\#}$ is called a grade of membership of u* in R*. If for every $u* \in R*$, $\mu_{R\#}(u*) = 1$, then R# = R*.

For operations of fuzzy valued fuzzy relational data model, by operations of fuzzy valued relational data model and fuzzy relational data model, if $R\# = (R*, \mu_{R\#})$ and $S\# = (S*, \mu_{S\#})$, we have

(1) $R\# \cup S\# = (R* \cup S*, \mu_{R\#} \vee \mu_{S\#})$;
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(3) $R\# - S\# = (T* , \mu_{T\#})$

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(4) for every $0< \lambda <1$,

$(R\#)_\lambda = \{ u*; \mu_{R\#}(u*) \geqslant \lambda , u* \in \Pi_{i=1}^{n} F(Ui)\} = R*_\lambda \subset \Pi_{i=1}^{n} F(Ui)$ and

$R* = \bigcup_{0<\lambda\leq 1} R*_\lambda$;

(5) $R\#[A] = \{(u*[A], \mu_{R\#}(u*)); u* \in R*\}$;

(6) $R\# \bowtie_F S\# = (R* \bowtie_F S* , \mu_{R\# \bowtie_F S\#})$;

(7) $R\#\{F\} = \{ (u*, \mu_{R\#}(u*)); u* \in R*$ and $F(u*, \mu_{R\#}(u*)) = 1\}$.

5. Conclusions

The fuzzy valued fuzzy relational data model is an extension of either of the fuzzy relational data model and the fuzzy valued relational data model, it not only facilitates the representation of fuzziness in data item itself in database, but also can easily represents fuzziness in the association between data records.

Fuzzy database systems will find a number of applications in such fields as Artificial Intelligence, Decision Support Systems, Expert Computer Systems where fuzzy data play an inmportant role.

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