FUZZY VALUED FUZZY RELATIONAL DATA MODEL

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Abstract

In this paper, the data structures and the operations on fuzzy relational data model, fuzzy valued relational data model and fuzzy valued fuzzy relational data model are described. These models are considered as extensions of Codd's classical relational data model, they provide necessary basis to establish fuzzy relational database management systems.
Keywords: Fuzzy relational data model, fuzzy valued relational data model, fuzzy valued fuzzy relational data model

1. Introduction

As the techniques and methods of representing and processing ideal data have now progressed into well developed information systems, less than ideal or perfect data has become of interest. In ordinary database systems, most processing has assumed that the data represented was exact, correct, well-formulated, with no provisions for cosidering otherwise. In order to represent the real world and deal with real world enterprises we must strive to broaden the types of data such systems can process.

We have proposed a method of represent fuzzy data in database based on the fuzzy set and possibility theory [1]. Here we will discuss some concepts and operations of three kinds of fuzzy relational data models.

In Section 2, we will introduce the concept and operations of fuzzy relational data model that can represent fuzziness in an association between values; In Section 3, we will describe the concepts and operations of fuzzy attribute valued relational data model that can represent fuzziness in a attribute value itself; In Section 4, we will present fuzzy valued fuzzy relational data model, it can represents two types of fuzziness above mentioned, ordinary relational data model, fuzzy relational data model and fuzzy valued relational data model are all some special cases of this model.

Throughout this paper, let U_i be finite set, $F(U_i)$ a class of all fuzzy subsets of U_i, $i = 1,2,\dots,n$, $\prod_{i=1}^{n} U_i = U_1 \times U_2 \times \cdots \times U_n$, $\prod_{i=1}^{n} F(U_i) = F(U_1) \times F(U_2) \times F(U_3) \times \cdots \times F(U_n)$.

2. The concepts and operations of fuzzy relational data model

Definition 1. A fuzzy relation $\tilde{R}$ is a fuzzy subset of $\prod_{i=1}^{n} U_i$, defined by $\mu_{\tilde{R}}$, i.e.,

$$\mu_{\tilde{R}}: \prod_{i=1}^{n} U_i \longrightarrow [0\ ,\ 1],$$

we denote fuzzy relation $\tilde{R}$ as

$$\tilde{R} = \{(u\ ,\ \mu_{\tilde{R}}(u));\ u = (u_1, u_2, \dots, u_n) \in \prod_{i=1}^{n} U_i\}\ .$$

Since $R = \{u;\ \mu_{\tilde{R}}(u) > 0,\ u \in \prod_{i=1}^{n} U_i\} = \operatorname{supp} \tilde{R} \subset \prod_{i=1}^{n} U_i$, R is a classical relation, $\tilde{R}$ is called a fuzzy relation corresponding to R, denoted by

$$\tilde{R} = (R \ , \ \mu_{\tilde{R}}),$$

where $\mu_{\tilde{R}}$ is called a grade of membership of **u** in R. If for every $u \in R$, $\mu_{\tilde{R}}(u) = 1$, then $\tilde{R} = R$.

For operations fuzzy relational data model, by definitions of operations of fuzzy subset, if $\tilde{R} = (R \ , \ \mu_{\tilde{R}})$ and $\tilde{S} = (S \ , \ \mu_{\tilde{S}})$, we have

(1) $\tilde{R} \cup \tilde{S} = (R \cup S \ , \ \mu_{\tilde{R}} \vee \mu_{\tilde{S}})$,

where $(\mu_{\tilde{R}} \vee \mu_{\tilde{S}})(u) = \max\ (\mu_{\tilde{R}}(u) \ , \ \mu_{\tilde{S}}(u))$, $u \in \prod_{i=1}^{n} U_i$;

(2) $\tilde{R} \cap \tilde{S} = (R \cap S \ , \ \mu_{\tilde{R}} \wedge \mu_{\tilde{S}})$,

where $(\mu_{\tilde{R}} \wedge \mu_{\tilde{S}})(u) = \min\ (\mu_{\tilde{R}}(u) \ , \ \mu_{\tilde{S}}(u))$, $u \in \prod_{i=1}^{n} U_i$;

(3) $\tilde{R} - \tilde{S} = (T \ , \ \mu_{\tilde{T}})$,

where $T = (R - S) \cup \{ u;\ \mu_{\tilde{R}}(u) - \mu_{\tilde{S}}(u) > 0,\ u \in R \cap S \}$,

$$\mu_{\tilde{T}}(u) = \max\ (\mu_{\tilde{R}}(u) - \mu_{\tilde{S}}(u) \ , \ 0);$$

(4) for every $0 < \lambda \leq 1$,

$$(\tilde{R})_\lambda = \{u;\ \mu_{\tilde{R}}(u) \geq \lambda,\ u \in \prod_{i=1}^{n} U_i \} = R_\lambda \subset \prod_{i=1}^{n} U_i,\ R = \bigcup_{0<\lambda\leq 1} R_\lambda.$$

Similar to the projection and the join and the selection of classical relational data model, we can define a projection and a join and a selection of fuzzy relational data model:

(5) projection, for any $A = \prod_{j=1}^{k} U_{i_j}$,

$$\tilde{R}[A] = (R[A] \ , \ \mu_{\tilde{R}[A]})$$
$$= \{(u[A] \ , \ \mu_{\tilde{R}[A]}(u));\ u \in R \}$$

$$= \{(u[A], \mu_{\widetilde{R}}(u));\ u \in R\},$$

where $u[A] = (u_{i_1}, u_{i_2}, \ldots, u_{i_k}) \in A$.

(6) join $\bowtie$, let $\widetilde{R}$ be a fuzzy relation in $\prod_{j=1}^{k_1} U_{i_j}$, $\widetilde{S}$ be a fuzzy relation in $\prod_{m=1}^{k_2} U_{j_m}$, then

$$\widetilde{R} \underset{F}{\bowtie} \widetilde{S} = (R \underset{F}{\bowtie} S,\ \mu_{\widetilde{R} \underset{F}{\bowtie} \widetilde{S}}),$$

where $R \underset{F}{\bowtie} S$ is the join of between classical relations R and S, i.e.,

$$R \underset{F}{\bowtie} S = \{rs;\ r \in R,\ s \in S,\ F(r, s) = 1\},$$

$$\mu_{\widetilde{R} \underset{F}{\bowtie} \widetilde{S}}(rs) = \min(\mu_{\widetilde{R}}(r), \mu_{\widetilde{S}}(s)),$$

$$r = (u_{i_1}, u_{i_2}, \ldots, u_{i_{k_1}}) \text{ and } s = (u_{j_1}, u_{j_2}, \ldots, u_{j_{k_2}}),$$

and F is a Boolean function representing conditions of join.

(7) selection

$$\widetilde{R}_{\{F\}} = \{(u, \mu_{\widetilde{R}}(u));\ u \in R,\ F(u, \mu_{\widetilde{R}}(u)) = 1\}$$

where F is a Boolean function representing conditions of selection and F can contain $\mu_{\widetilde{R}}$.

3. The concepts and operations of fuzzy valued relational data model

Definition 2. A fuzzy valued relation R_* is a subset of $\prod_{i=1}^{n} F(U_i)$.

Obviously, for every $r_* = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n) \in R_*$, $\tilde{u}_i$ is a fuzzy subset of U_i, $i = 1, 2, \ldots, n$.

Since fuzzy valued relation is a classical relation, operations of fuzzy valued relation and operations of classical relation are same. In the following, we define " $<$ ", " $>$ ", " $\leq$ ", " $\geq$ ", " $=$ " and " $\neq$ " of Boolean function F in fuzzy valued relational data model.

Definition 3. Let $\tilde{u}_1$ and $\tilde{u}_2$ are two fuzzy subsets of U, $\mu_{\tilde{u}_1}$ and $\mu_{\tilde{u}_2}$ are two membership functions of $\tilde{u}_1$ and $\tilde{u}_2$ respectively.

(1) $\tilde{u}_1 < \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) < \mu_{\tilde{u}_2}(u)$;

(2) $\tilde{u}_1 > \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) > \mu_{\tilde{u}_2}(u)$;

(3) $\tilde{\mu}_1 \leq \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) \leq \mu_{\tilde{u}_2}(u)$;

(4) $\tilde{u}_1 \geq \tilde{u}_2$ if and only if for every $u \in U$, $\mu_{\tilde{u}_1}(u) \geq \mu_{\tilde{u}_2}(u)$;

(5) let $U = \{u_1, u_2, \ldots u_n\}$, $\tilde{u}_1 = \tilde{u}_2$ if and only if, for given $\varepsilon > 0$, $n(\tilde{u}_1, \tilde{u}_2) > 1 - \varepsilon$, where $n(\tilde{u}_1, \tilde{u}_2)$ is a near degree of $\tilde{u}_1$ and $\tilde{u}_2$, i.e.,

$$n(\tilde{u}_1 , \tilde{u}_2) = \sum_{i=1}^{n}(\mu_{\tilde{u}_1}(u_i) \wedge \mu_{\tilde{u}_2}(u_i)) / \sum_{i=1}^{n}(\mu_{\tilde{u}_1}(u_i) \vee \mu_{\tilde{u}_2}(u_i)).$$

4. The concepts and operations of fuzzy valued fuzzy relational data model

Definition 4. A fuzzy valued fuzzy relation R^* is a fuzzy subset of $\prod_{i=1}^{n} F(U_i)$ defined by μ_{R^*}, i.e.,

$$\mu_{R^*}: \prod_{i=1}^{n} F(U_i) \longrightarrow [0 , 1],$$

we denote fuzzy valued fuzzy relation R^* as

$$R^* = \{(u^* , \mu_{R^*}(u^*)); \ u^* = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n) \in \prod_{i=1}^{n} F(U_i)\}.$$

Since $R_* = \{ u^*;\ \mu_{R^*}(u^*) > 0,\ u^* \in \prod_{i=1}^{n} F(U_i) \} = \text{supp} R^*$, R_* is a classical relation, R^* is called a fuzzy relation corresponding to R_*, denote by

$$R^* = (R_* \ , \ \mu_{R^*}),$$

where μ_{R^*} is called a grade of membership of u^* in R^*. If for every $u^* \in R_*$, $\mu_{R^*}(u^*) = 1$, then $R^* = R_*$.

For operations of fuzzy valued fuzzy relational data model, by operations of fuzzy valued relational data model and fuzzy relational data model, if $R^* = (R_* \ , \ \mu_{R^*})$ and $S^* = (S_* \ , \ \mu_{S^*})$, we have

(1) $R^* \cup S^* = (R_* \cup S_* \ , \ \mu_{R^*} \vee \mu_{S^*})$;

(2) $R^* \cap S^* = (R_* \cap S_* \ , \ \mu_{R^*} \wedge \mu_{S^*})$;

(3) $R^* - S^* = (T_* \ , \ \mu_{T^*})$,

where $T_* = (R_* - S_*) \cup \{ u^*;\ \mu_{R^*}(u^*) - \mu_{S^*}(u^*) > 0,\ u^* \in R_* \cap S_* \}$,

$$\mu_{T^*}(u^*) = \max\ (\mu_{R^*}(u^*) - \mu_{S^*}(u^*),\ 0);$$

(4) for every $0 < \lambda \leq 1$,

$$(R^*)_\lambda = \{ u^*;\ \mu_{R^*}(u^*) \geq \lambda,\ u^* \in \prod_{i=1}^{n} F(U_i) \} = R_{*_\lambda} \subset \prod_{i=1}^{n} F(U_i)$$

and

$$R_* = \bigcup_{0<\lambda\leq 1} R_{*_\lambda};$$

(5) $R^*[A] = \{ (u^*[A] \ , \ \mu_{R^*}(u^*));\ u^* \in R^* \}$;

(6) $R^* \underset{F}{\bowtie} S^* = (R_* \underset{F}{\bowtie} S_* \ , \ \mu_{R^* \underset{F}{\bowtie} S^*})$;

(7) $R^*\{F\} = \{ (u^*,\ \mu_{R^*}(u^*));\ u^* \in R_*,\ F(u^*,\ \mu_{R^*}(u^*)=1 \}$

5. Conclusions

We have defined three kinds of non-classical relational data models-- the fuzzy relational data model and the fuzzy valued relational data model and the fuzzy valued fuzzy relational data model. The fuzzy valued fuzzy relational data model is an extension of either of the fuzzy relational data model and the fuzzy valued relational data model, it not only facilitates the representation of fuzziness in data item itself in database, but also can easily representsfuzziness in the association between data records.

Fuzzy database systems will find a number of applications in such fields as Artificial Intelligence, Decision Support Systems, Expert Computer Systems where fuzzy data play an important role.

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