INTUITIONISTIC FUZZY MODEL OF A NEURAL NETWORK

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A new tool for description of the neuron networks is discussed, which is based on the intuitionistic fuzzy set theory (cf. [1]). The neuro-physiological processes, which occur in neurons, are described with intuitionistic fuzzy models (IFMs) in a way similar to the descriptions with fuzzy models (e.g. [2.3]). The difference between both types of models consists in the more powerful description possibilities of the IFMs. Besides, the IFMs allow a new approach, which gives more detailed description of the interaction between the different neurons.

Initially, we shall introduce one operation of IF-type. Here the operation "@" will be defined by analogy to [4]. If $\langle a, b\rangle$ and $\langle c, d\rangle$ are two ordered tuples, where $a, b, c, d \in [0, 1]$ and $a + b \leq 1$, $c + d \leq 1$, then:

$$\langle a, b\rangle @ \langle c, d\rangle = \langle (a + c)/2, (b + d)/2\rangle,$$

$$\langle a, b\rangle @ \langle *, *\rangle = \langle a, b\rangle,$$

$$\langle *, *\rangle @ \langle *, *\rangle = \langle *, *\rangle,$$

for the special symbol "*". Obviously:

$$\langle a, b\rangle @ \langle c, d\rangle = \langle c, d\rangle @ \langle a, b\rangle,$$

i.e. the operation is commutative, but it is not associative - for three tuples A, B and C, (A @ B) @ C is not equal to A @ (B @ C) in the general case. Thus we define for the tuples $A_1, \ldots, A_n$ ($A_i = \langle a_i, b_i\rangle$, $1 \leq i \leq n$):

$$A_1 @ A_2 = \mathop{@}_{i=1}^{2} A_i;$$

$$\mathop{@}_{i=1}^{n} A_i = \langle (\sum_{i=1}^{n} a_i)/n, (\sum_{i=1}^{n} b_i)/n\rangle.$$

Thus "@" is defined over the elements of a given universe, where as in [4] it is defined over IFSs, but its properties are similar to these from [4].

Let a neural network be given (see e.g. [5,6]). A part on this network is shown on Fig. 1. Each neuron N_i, ($1 \leq i \leq n+1$) of the network possesses the structure and parameters, as follows (for

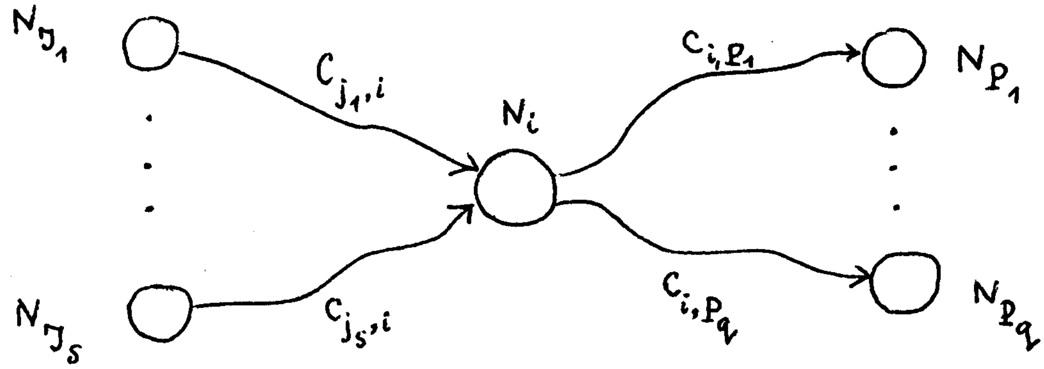

Fig. 1

the notation used see [1,5,6]): somma, dendrits, axon and synapses.

The neuron inputs are the dendrits (here - these are the input connections $C_{j,i}$ ($1 \leq j \leq s$) to the neuron N_i; the neuron outputs are an axon (which in not given on Fig. 1) and synapses - output connections $C_{i,p}$ ($1 \leq p \leq q$); the basic body of the neuron is the somma, in which sommatic potential is accumulated. If there are not connections between neurons N_i and N_j, then the value of $C_{i,j}$ = <*, *>. The sommatic potential of the neuron N_i at the time-moment k is $U_i(k)$. Every neuron can fall into three states: axitatory, inhibitory and normal.

Let $C_{j,i} = \langle \mu^C_{j,i}, \gamma^C_{j,i} \rangle$, where $\mu^C_{j,i}$ and $\gamma^C_{j,i}$ are degrees of exitatory and inhibitory, respectively, and $\mu^C_{j,i} + \gamma^C_{j,i} \leq 1$.

If $\mu^C_{j,i} > \gamma^C_{j,i}$, then the connection is exitatory;

if $\mu^C_{j,i} < \gamma^C_{j,i}$, then the connection is inhibitory

if $\mu^C_{j,i} = \gamma^C_{j,i}$, then the connection is indifferent.

Let $X_j = \langle \mu^X_j, \gamma^X_j \rangle$, where μ^X_j and γ^X_j are constants, related to the determination of $U_i(k)$ and $\mu^X_j + \gamma^X_j \leq 1$, and they are characteristics of the neuron N_j.

These constructions justify the use of the apparatus of the

IFSs.

The model can be presented by the next equations.

$$U_i(k) = \mathop{@}_{j=1}^{n} f(g(X_j), C_{j,i}),$$

for $1 \le i \le n$, where $U_i(k)$ is the sommatic potential of neuron N_i and:

$$g(X_j) = \begin{cases} J_{\alpha',\beta'}(X_j), & \text{if } \mu_j^{Y(k)} > \max(\gamma_j^{Y(k)}, \alpha_j) \ \&\ (\gamma_j^{Y(k)} < \beta_j) \\ \langle *, * \rangle, & \text{otherwise} \end{cases},$$

is the function determining the participation of the neuron N_j at the sommatic potential $U_i(k)$;

$$f(Z, C_{j,i}) = \begin{cases} Z_i \mathbin{@} C_{i,j}, & \text{if } \mu_{j,i}^{C} \ge \gamma_{j,i}^{C} \\ H_{\alpha'',\beta''}(Z_i \mathbin{@} C_{i,j}), & \text{if } \mu_{j,i}^{C} < \gamma_{j,i}^{C} \\ *, & \text{if } Z_i = \langle *, * \rangle \text{ or } C_{j,i} = \langle *, * \rangle \end{cases}$$

is the function determining the participation of the neuron N_j at the sommatic potential $U(k)$. From the above definition it can be seen that $U_i(k) = \langle \mu_i^U, \gamma_i^U \rangle$, where μ_i^U and γ_i^U determine the degrees of positivity and negativity of the sommatic potential of the neuron N_i at time-moment k.

When:

$$\mu_j^{Y(k)} > \max(\gamma_j^{Y(k)}, \alpha_j) \text{ and } \gamma_j^{Y(k)} < \beta_j,$$

then:

$$\mu_i^{Y(k+1)} = \mu_i^X \text{ and } \gamma_i^{Y(k+1)} = \gamma_i^X.$$

Thus the exited neurons at the time-moment k recover their initial state values at the time-moment k+1.

The neuron N_i exitatory and inhibitory degrees values at the time-moment k+1 are given by:

$$Y_i(k+1) = \begin{cases} Y_i(k), & \text{if } U_i(k) = \langle *, * \rangle \\ Y_i(k) \mathbin{@} U_i(k), & \text{otherwise} \end{cases}.$$

Therefore:

$$Y_i(k+1) = \langle \mu_i^{Y(k+1)}, \gamma_i^{Y(k+1)} \rangle,$$

where $\mu_i^{Y(k+1)}$ and $\gamma_i^{Y(k+1)}$ determine the neuron N_i exitatory and inhibitory degrees at time-moment k.

The state of the neuron N_i is:

- exitatory, if $\mu_i^{Y(k+1)} > \gamma_i^{Y(k+1)}$,
- inhibitory, if $\mu_i^{Y(k+1)} < \gamma_i^{Y(k+1)}$,
- normal, if $\mu_i^{Y(k+1)} = \gamma_i^{Y(k+1)}$.

The neurons' IFM described above allows to apply elements of the apparatus of the intuitionistic fuzzy sets with the aim to research the basic model properties. Using this model, we can obtain estimations of the neuron network behaviour. This will be an object of another author's communication.

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