COMPOSITIONS OF INTUITIONISTIC FUZZY RELATIONS

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Let E be a fixed set. Following the notations in [1] an IFS A in E is an object having the form:

$A = \{<x, \mu_A(x), \gamma_A(x)> / x \in E\}$,

where functions $\mu_A: E \longrightarrow [0, 1]$ and $\gamma_A: E \longrightarrow [0, 1]$ define the degree of membership and the degree of nonmembership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E$ $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

An intuitionistic fuzzy relation (IFR) is an intuitionistic fuzzy set on a cartesian product of universes.

Let E1, E2 and E3 be three universes. Let R and P be two intuitionistic fuzzy binary relations on E1xE2 and E2xE3 respecively. Simularly to the composition of fuzzy relations introdused by Zadeh [2] the first and second composition of the IFRs R and P will be noted with Ro_1P and Ro_2P respectively and defined by:

$$Ro_1P = \{<(x,z), \max_{y \in E2}(\min(\mu_R(x,y), \mu_P(y,z)), \min_{y \in E2}(\max(\gamma_R(x,y), \gamma_P(y,z))> / x \in E1 \;\&\; y \in E2 \;\&\; z \in E3\}$$

$$Ro_2P = \{<(x,z), \min_{y \in E2}(\max(\mu_R(x,y), \mu_P(y,z)), \max_{y \in E2}(\min(\gamma_R(x,y), \gamma_P(y,z))> / x \in E1 \;\&\; y \in E2 \;\&\; z \in E3\}$$

It can be easily checked that Ro_1P and Ro_2P are still IFRs.

Let cR and cP be cylindrical extentions in E1xE2xE3 of IFRs R and P respectively.

Theorem 1. (a) $Ro_1P = (cR \cap cP)^1_{E1xE3}$

(b) $Ro_2P = (cR \cup cP)^2_{E2xE3}$

Proof:

For (a):

from

$$\mu_{Ro_1P}(x,z) = \max_{y \in E2}(\min(\mu_R(x,y), \mu_P(y,z)) =$$

$$= \max_{y \in E2}(\min(\mu_{cR}(x,y,z), \mu_{cP}(x,y,z)) = \mu_{(cR \cap cP)^1_{E1xE3}}(x,z)$$

and

$$\gamma_{Ro_1P}(x,z) = \min_{y\in E2}(\max(\gamma_R(x,y),\ \gamma_P(y,z)) =$$

$$= \min_{y\in E2}(\max(\gamma_{cR}(x,y,z),\ \gamma_{cP}(x,y,z)) = \gamma_{(cR\cap cP)^1_{E1\times E3}}(x,z)$$

follows $Ro_1P = (cR\cap cP)^1_{E1\times E3}$.

(b) is proved analogically.

Let R^q_{E2} and P^q_{E2} be q-projections of IFRs R and P respectively over E2. The IFR $cR\cap cP$ will be called a connection of IFRs R and P when $R^1_{E2} = P^1_{E2}$. The IFS $cR\cup cP$ will be called a connection of IFRs R and P when $R^2_{E2} = P^2_{E2}$.

Theorem 2. If $cR\cap cP$ is a connection of IFRs R and P:

(a) $R = (cR\cap cP)^1_{E1\times E2}$ & $P = (cR\cap cP)^1_{E2\times E3}$

(b) $c(Ro_1P)\cap cR$ is a connection of IFRs Ro_1P and R

(c) $c(Ro_1P)\cap cP$ is a connection of IFRs Ro_1P and P

Proof:

For (a):

from

$$\mu_{(cR\cap cP)^1_{E1\times E3}}(x,y) = \max_{z\in E3}(\min(\mu_{cR}(x,y,z),\ \mu_{cP}(x,y,z))) =$$

$$= \min(\mu_R(x,y),\ \max_{z\in E3}\mu_{cP}(x,y,z)) = \min(\mu_R(x,y),\ \max_{z\in E3}\mu_P(y,z)) =$$

$$= \min(\mu_R(x,y),\ \max_{x\in E1}\mu_R(x,y)) = \mu_R(x,y)$$

and

$$\gamma_{(cR\cap cP)^1_{E1\times E3}}(x,y) = \min_{z\in E3}(\max(\gamma_{cR}(x,y,z),\ \gamma_{cP}(x,y,z))) =$$

$$= \max(\gamma_R(x,y),\ \min_{z\in E3}\gamma_{cP}(x,y,z)) = \max(\gamma_R(x,y),\ \min_{z\in E3}\gamma_P(y,z)) =$$

$$= \max(\gamma_R(x,y),\ \min_{x\in E1}\gamma_R(x,y)) = \gamma_R(x,y)$$

follows $R = (cR\cap cP)^1_{E1\times E2}$

Analogically $P = (cR\cap cP)^1_{E2\times E3}$.

For (b):

from

$$\mu_{(Ro_1P)^1_{E1}}(x) = \max_{z\in E3}\mu_{Ro_1P}(x,z) =$$

$$= \max_{z\in E3}(\max_{y\in E2}(\min(\mu_R(x,y),\ \mu_P(y,z)))) =$$

$$= \max_{y\in E2}(\min(\mu_R(x,y),\ \max_{z\in E3}\mu_P(y,z))) =$$

$$= \max_{y \in E2} (\min (\mu_R(x,y), \max_{x \in E1} \mu_R(x,y))) =$$

$$= \max_{y \in E2} \mu_R(x,y) = \mu_{R^1_{E1}}(x)$$

and

$$\gamma_{(Ro_1P)^1_{E1}}(x) = \min_{z \in E3} \gamma_{Ro_1P}(x,z) =$$

$$= \min_{z \in E3} (\min_{y \in E2} (\max (\gamma_R(x,y), \gamma_P(y,z)))) =$$

$$= \min_{y \in E2} (\max (\gamma_R(x,y), \min_{z \in E3} \gamma_P(y,z))) =$$

$$= \min_{y \in E2} (\max (\gamma_R(x,y), \min_{x \in E1} \gamma_R(x,y))) =$$

$$= \min_{y \in E2} \gamma_R(x,y) = \gamma_{R^1_{E1}}(x)$$

follows $(Ro_1P)^1_{E1} = R^1_{E1}$.

(c) is proved analogically.

Theorem 3. If cRUcP is a connection of IFRs R and P:

(a) $R = (cRUcP)^2_{E1xE2}$ & $P = (cRUcP)^2_{E2xE3}$

(b) $c(Ro_2P)UcR$ is a connection of IFRs Ro_2P and R.

(c) $c(Ro_2P)UcP$ is a connection of IFRs Ro_2P and P.

Proof: Analogically to the theorem 2.

Let E be an universe and IFSs Ai ⊂ E for i = 1,...,n. Let α_i and β_i be such numbers that $\alpha_i \geq 0$ & $\beta_i \geq 0$ for i=1,...n and $\sum_{i=1}^{n} (\alpha_i + \beta_i) = 1$. The convex combination of IFSs A1, A2, ..., An will be defined by:

$$A = \{\langle x, \sum_{i=1}^{n} \alpha_i \cdot \mu_{Ai}(x), \sum_{i=1}^{n} \beta_i \cdot \gamma_{Ai}(x)\rangle \; / \; x \in E\}$$

It can be easily checked that A is still an IFS.

REFERENCES

[1] Atanassov K. Intuitionistic fuzzy sets, Fuzzy sets and systems Vol. 20 (1986) No 1, 87-96

[2] Zadeh L.A. The consept of a linguistic variable and its application to approximate reasoning. American Elsevier Publishing Company, New York 1973