A NOTE ON (FUZZY) CONFLICTS (AND CONTRADICTION SETS).

(Nicola Umberto ANIMOBONO, C.P. 2099, I 00100 Roma AD, Italy)

Basic notions of the conflict theory, introduced by Pawlak [1,2], are shortly presented and followed by a fuzzy approach. A post-scriptum on Shi's [3] contradiction sets is gived.

Keywords: conflicts theory, fuzzy conflicts, contradiction sets.

Notations: $\# A$ cardinality of A set,
Δ_A diagonal (all (x,x)) of A set.

1. CONFLICTS [2].

Let U be a some finite set with $\# U = n \geq 2$ and $L = \{-1, 0, +1\} \subseteq \mathbb{R}$.

Basic definitions.

By giving a relationship function $\varphi : U \times U \rightarrow L$ a couple $(x,y) \in U \times U$ is called:

- alley iff $\varphi(x,y) = +1$,
- neutral iff $\varphi(x,y) = 0$,
- enemy iff $\varphi(x,y) = -1$.

The couple $C = (U, \varphi)$ is called the configuration (of U).

By C are definible 3 (disjoint binary) relations on U:

$R_C^+ = \varphi^{-1}(\{+1\})$, $R_C^- = \varphi^{-1}(\{-1\})$, $R_C^o = \varphi^{-1}(\{0\})$

called, resp., alliance, conflict and neutrality.

If $R_C^- = \emptyset$ C is conflictless else conflict configuration;

$R_C^+ = U^2$ is called total alliance, $R_C^- = U^2 - \Delta$ total conflict;

$A_C = R_C^+ \cup R_C^-$ is called activity.

Regularity.

The configuration $C = (U, \varphi)$ is called <u>regular</u> iff

R1 - $\Delta \subseteq R_C^+$

R2 - $(R_C^+)^{-1} = R_C^+$

R3 - $(R_C^-)^{-1} = R_C^-$

R4 - $R_C^+ \circ R_C^+ \subseteq R_C^+$

R5 - $R_C^- \circ R_C^- \subseteq R_C^+$

R6 - $R_C^- \circ R_C^+ \subseteq R_C^-$

R7 - $R_C^+ \circ R_C^+ \subseteq R_C^-$

(if $n \geq 3$, R1÷R6 are indipendents), else <u>non-regular</u>.

Proposition [4].

$C = (U, \varphi)$ is regular iff

- A_C is equivalence relation,
- $\forall (x,y),(y,z) \in A_C \Rightarrow \varphi(x,y) \cdot \varphi(y,z) = \varphi(x,z)$.

2. EXTENSIONS.

Żakowski [4] extend the class of functions of relationship, by using as range the real interval $I = [-1,+1] = \{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$.

Let $\phi : U^2 \rightarrow I$ be.

The corresponding same, of 1 sect., definitions are analogously extended to configuration $C_\phi = (U, \phi)$.

The couple $(x,y) \in U^2$ is called: <u>alley</u> iff $\phi(x,y) > 0$, <u>neutral</u> iff $\phi(x,y) = 0$ and <u>ennemy</u> iff $\phi(x,y) < 0$.

The value $\phi(x,y)$ is called the <u>alliance degree</u>.

The configuration $C_\phi = (U,\phi)$ is called extensively regular iff

1. $\phi(x,x) > 0$
2. $\phi(x,y) > 0 \Rightarrow \phi(y,x) > 0$
3. $\phi(x,y) < 0 \Rightarrow \phi(y,x) < 0$
4. $\phi(x,y) > 0,\ \phi(y,z) > 0 \Rightarrow \phi(x,z) > 0$
5. $\phi(x,y) < 0,\ \phi(y,z) < 0 \Rightarrow \phi(x,z) > 0$
6. $\phi(x,y) < 0,\ \phi(y,z) > 0 \Rightarrow \phi(x,z) < 0$
7. $\phi(x,y) > 0,\ \phi(y,z) < 0 \Rightarrow \phi(x,z) < 0.$

Let $\sigma : I \rightarrow L$ be such that $\sigma(0) = 0$, $\sigma(r) = \frac{r}{|r|}$ if $r \neq 0$

and $\varphi = \phi \circ \sigma$:

$$\begin{array}{ccc} U & \xrightarrow{\ \varphi\ } & \\ \phi \downarrow & & \searrow \\ [-1,1] & \xrightarrow{\ \sigma\ } & \{-1,0,+1\} \end{array} .$$

<u>Corollary</u>.

C_ϕ is ext.regular iff C is regular.

<u>Corollary</u>.

$\phi(x,y) = 0,\ \phi(y,z) \neq 0 \Rightarrow \phi(x,z) = 0 \quad \forall x,y,z \in U.$

3. <u>FUZZY APPROACH</u>.

Let U be a nonempty universal set, $\Lambda = [0,1] = \{ r \in \mathbb{R} \mid 0 \leq r \leq 1 \}$, $\theta : U^2 \rightarrow \Lambda$ define a <u>fuzzy relation</u> on U. θ is called:

F1- reflexive if $\theta(x,x) = 1 \ \forall x \in U$

F2- ε-reflexive if $\theta(x,x) \geq \varepsilon$ (with $\varepsilon \in]0,1]$)

F3- weakle-reflexive if $\theta(x,y) \leq \theta(x,x) \wedge \theta(y,y) \quad \forall (x,y) \in U^2$

F4- simmetric if $\theta(x,y) = \theta(y,x) \quad \forall (x,y) \in U^2$

F5- max-$*$ transitive if $\theta(x,y) * \theta(y,z) \leq \theta(x,z) \quad \forall (x,y,z) \in U^3$

F6- proximity if θ is F1,F4

F7- similarity if θ is F5,F6.

Let $\vartheta \in]0,1[= \{ r \in R \mid 0 < r < 1 \}$ be a threshold.

A couple $(x,y) \in U^2$ is called: ϑ-fuzzly alley iff $\theta(x,y) > \vartheta$, ϑ-fuzzly neutral iff $\theta(x,y) = \vartheta$ and ϑ-fuzzly enemy iff $\theta(x,y) < \vartheta$; $\theta(x,y)$ is called the fuzzy alliance degree between x and y. Analogously to 2 sect. by changing ϑ to 0, the configuration $C_\theta = (U, \theta)$ is defined f.regular.

Let $\lambda_\vartheta : \Lambda \rightarrow L$ be s.t. $\lambda_\vartheta(r) = +1$ if $r > \vartheta$, $= 0$ if $r = \vartheta$, $= -1$ if $r < \vartheta$ and let $\varphi = \theta \circ \lambda_\vartheta$ be.

<u>Corollary</u>.

(U, θ) is f.regular iff (U, φ) is regular.

<u>Example</u>.

$U = \{x,y,z,t\}$, $\theta \subseteq U \times U$, $\theta(a,b) \in \{1/4, 1/2, 2/3, 3/4, 1\}$;

θ	x	y	z	t
x	1	1/2	1/4	2/3
y	1/2	1	3/4	2/3
z	1/4	3/4	1	2/3
t	2/3	2/3	2/3	1

For $\vartheta = 2/3$ it is:

$$\theta^+_\vartheta = \{ (x,x),(y,y),(y,z),(z,y),(z,z),(t,t) \}$$
$$\theta^-_\vartheta = \{ (x,y),(x,z),(y,x),(z,x) \}$$
$$\theta^0_\vartheta = \{ (x,t),(y,t),(z,t),(t,x),(t,y),(t,z) \}$$

$A_\vartheta = \theta^+_\vartheta \cup \theta^-_\vartheta$ $\quad (U/A_\vartheta = \{ \{x,y,z\}, \{t\} \})$ and (U, θ) is f.regular.

Instead, for $\vartheta = 1/2$ $(A_\vartheta(x,t) = 2/3 > \vartheta$, $A_\vartheta(t,z) = 2/3 > \vartheta$, $A_\vartheta(x,z) = 1/4 < \vartheta)$ or for $\vartheta = 3/4$ $(A_\vartheta(z,x) = 1/4 < \vartheta$, $A_\vartheta(x,y) = 1/2 < \vartheta$, $A_\vartheta(z,y) = 3/4 = \vartheta)$ (U, θ) is not f.regular.

Moreover; for $\vartheta = 2/3$ θ is not max-min transitivity $(\theta(x,y) \wedge \theta(y,z) = 1/2 \wedge 3/4 = 1/2 > 1/4 = \theta(x,z))$ and for bounded difference $\ominus$ $(a \ominus b = 0 \vee (a + b - 1))$ θ is not max-$\ominus$ transitivity $(\theta(x,t) \ominus \theta(t,z) = 2/3 \ominus 2/3 = 1/3 > 1/4 = \theta(x,z))$.

A fuzzy relation $\pi : U^2 \to \Lambda$ s.t. $\pi \geq \theta$ is called <u>pacification</u> of θ (i.e. $\pi(x,y) \geq \theta(x,y) \quad \forall x,y \in U$).
The max-min transitive closure τ of θ, $\tau(x,y) = \bigvee_{u \in U} \{\theta(x,u) \wedge \theta(u,y)\}$, is a fuzzy pacification of θ.

4. POST-SCRIPTUM ON SHI's CONTRADICTION SETS [3].

A <u>philosophycal</u> approach to the sets is presented by Shi [3,5]. The classic logic and set theory use the dicotoumy: true/false, one/zero, yes/no. Shi considere the definition/existence (intension/extension) of a set by a propriety. For Shi a concept (propriety) is (philosophycally) proponible only if we have in mind/experience the its interlinked contrasts (ego/non-ego). Essentially, this is a "gnoseology" philosophycal question.
For Shi, an unclair concept is not a concept (in essence). He distinguishe [5] the <u>imagination</u> for the <u>vague intension</u> and the <u>concept in essence</u> for the <u>clear intension</u>: the first is the <u>intuition-tought</u> and the second the <u>logic-tought</u>.
To make clear the definition of fuzzyness, he try make clear the definition of contradiction. Against to the contradiction as "contrary quality" (opposition) (life/death, correct/wrong, yin/yang), Shi considere the contradiction as "difference quali ty" (fast/slow, fat/thin).
For its (philosophycal) investigations, he propose a "contradiction set" model. Here, I try to shortly give a (my) mathematical traslation of Shi's (mathematically presented) ideas.

Definitions.

A concept, called **elaborate topic** P, determine a **contradiction set** $\overset{\ominus}{N}_t$, represented by a t-uple $\{a_i \mid 1 \leq i \leq t \; (t \in \mathbb{N})\}$, called **t-kind**, and any a_i, called **vision**, is a t_i-uple $\{a_{ij} \mid 1 \leq j \leq t_i \; (j \in \mathbb{N})\}$.

The t-kind is called:

- **standard** type if $t = 3$
- **two-leafe** type if $t = 2$
- **one-leaf** type if $t = 1$
- **extensive** type if $t \geqslant 4$
- **empty** if $t = 0$.

(For Shi, the Cantor's sets are 1-kind and the Zadeh's sets are extensive type).

Moreover, the t-kind of the contradiction set $\overset{\ominus}{N}_t$ is determined by a mental perspective (**vision-angle**: method of making contra diction).

By writing $\bar{a}_i = \alpha(a_i)$, let $\alpha : \overset{\ominus}{N}_t \rightarrow [0,1]$ be s.t. $\alpha(\overset{\ominus}{N}_t) = \{\bar{a}_i\}$ and $\sum_1^t i \;\; \bar{a}_i = 1$: the function α is called **vision-angle**.

Example.

Let $\overset{\ominus}{N}_t = \left\{ \{a_{ij}\}_{1 \leq j \leq t_i} \right\}_{1 \leq i \leq t}$ be and $n = \sum_1^t {}_i t_i$. Let $\alpha : \overset{\ominus}{N}_t \rightarrow [0,1]$ be s.t. $\bar{a}_i = \alpha(a_i) = t_i/n$. Then $\sum_{1i}^t \bar{a}_i = 1$: α is a vision angle for $\overset{\ominus}{N}_t$.

Shi's operators.

$\overset{\ominus}{N}_t = \{a_i\}_t \quad \overset{\ominus}{M}_t = \{b_i\}_t$:

$\overset{\ominus}{N}_t \subseteq \overset{\ominus}{M}_t$ iff $a_i \subseteq b_i \quad \forall i \in [1,2,\ldots,t] \subseteq \mathbb{N}$

$\overset{\ominus}{N}_t \cup \overset{\ominus}{M}_t = \{(a_i \cup b_i)_i\}_t$

$\overset{\ominus}{N}_t \cap \overset{\ominus}{M}_t = \{(a_i \cap b_i)_i\}_t$.

Re-definitions.

A function $a:\Omega\to\Lambda$ is called an attribute and $a(\omega)$ $(\omega\in\Omega)$ the attribute value in ω.

Let $\omega_0\in\Omega$ be a given element, called compare-element, and $a_0 = a(\omega_0)$, called vision-angle.

The triplet (a_U, a_C, a_L), where $a_U = a^{-1}(]a_0 1])$, $a_C = a^{-1}(\{a_0\})$, and $a_L = a^{-1}([0,a_0[)$, is called contradiction set $\overset{\circ\!-}{a}$.

We define:

$\neg\,\overset{\circ\!-}{a} = \overset{\circ\!-}{\bar{a}}$ s.t. $\bar{a}(\omega) = 1 - a(\omega)$ complementation

$\overset{\circ\!-}{a}\cup\overset{\circ\!-}{b} = \overset{\circ\!-}{v}$ s.t.

$$v(\omega) = \begin{cases} a(\omega)\vee b(\omega) & \omega\in a_U\cup b_U \\ a(\omega)\wedge b(\omega) \quad \text{iff} & \omega\in a_L\cap b_L \\ a_0\wedge b_0 & \text{otherwise} \end{cases} \qquad \text{union}$$

$$\overset{\circ\!-}{a}\cap\overset{\circ\!-}{b} = \overset{\circ\!-}{w} \quad \text{s.t.} \quad w(\omega) = \begin{cases} a(\omega)\wedge b(\omega) & \omega\in a_U\cap b_U \\ a(\omega)\vee b(\omega) \quad \text{iff} & \omega\in a_L\cup b_L \\ a_0\vee b_0 & \text{otherwise} \end{cases} \qquad \text{intersection}$$

It results:

$\neg\,\overset{\circ\!-}{a} = (a_L, a_C, a_U)$

$\overset{\circ\!-}{a}\cup\overset{\circ\!-}{b} = (a_U\cup b_U, (a_C\cap b_C)\cup(a_C\cap b_L)\cup(a_L\cap b_C), a_L\cap b_L)$

$\overset{\circ\!-}{a}\cap\overset{\circ\!-}{b} = (a_U\cap b_U, (a_C\cap b_C)\cup(a_C\cap b_U)\cup(a_U\cap b_C), a_L\cup b_L)$

$\neg\,(\overset{\circ\!-}{a}\cup\overset{\circ\!-}{b}) = \neg\,\overset{\circ\!-}{a}\cap\neg\,\overset{\circ\!-}{b}$

$\neg\,(\overset{\circ\!-}{a}\cap\overset{\circ\!-}{b}) = \neg\,\overset{\circ\!-}{a}\cup\neg\,\overset{\circ\!-}{b}$.

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