THE ANALYSIS OF LOESS SLOPE INSTABILITY BASED ON FUZZY INFORMATION METHODS

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ABSTRACT
The evaluation of loess slope stability is discussed using method of fuzzy information analysis, first grade and second grade fuzzy approximate inference. The stability of a number of loess slopes is tested using these methods, and it is shown that the results correspond well with field experience. This method of assessing loess slope stability has been applied to a large landslide resear-ch and prevention project in the Lanzhou city area and has been used as a basis for hazard mapping .

KEYWORDS: loess, slope stability, fuzzy information, information distribution,information cncentration, China.

1. INTRODUCTION

A large number of methods may be applied to evaluate slope stability.The methods introduced by non-Chinese researchers (Baker,R.1980)are based on soil mechanical algorithms, with the effective stress methods being the most frequently applied. Chinese researchers put more emphasis on the use of soil mechanics algorithms which are based on engineering geological analyses. However, all these methods have their disadvantages in evaluating the stability conditions of slopes. Many slope stability assessments are carried out using the calculations of safety fac-tors of curved slip surfaces measured along slope profiles. However, these meth-ods do not consider suffiently a number of important slope and soil parameters related to the soil volumen the conditions along the slip surface and other boun-dary conditions. As a result of the greater influence of computers, the develop-ments in related scientific subjects, and continuous development of soil mecha-nics a series of new methods have been developed and are now used instead of the older methods. However, the calculations which need to be performed when using these methods are very complex and are not suitable for the analysis of larger areas. Adex and are not suitable for the analysis of larger areas. Additionally, these methods do not consider the uncertainty involved in assessing the regional variation of slope stability (such as may be expressed using the stochastic, fuzzy or grey character of the analyses). In the following it will be shown that, alth-ough the internal characteristics of a slope remain unclear (grey-fuzzy concept), it is still feasible to analyse the slope stability in a reliable quantitative fashion. To this purpose, the slope stability problem is assessed using an approach which is based on the fuzzy-set and grey theories.

2. BASIC THEORY

2.1 Fuzzy approximate inference.

The fuzzy approximate inference forms a key link between a large system and a system which is based on uncertainty. Mathematically this can be formulated as:

$$B_1 = A_1 \circ R \quad (1)$$

in which (1), and A_i, B_i are the fuzzy subsets from respectively $U=\{u_1,u_2,...,u_n\}$, and $V=\{v_1,v_2,...,v_m\}$ representing the concept of the universe of discourse. R is the fuzzy relation which reflects the obtained experience and knowledge in terms of information. The sign "∘" stands for the calculation rules and the combiniation method.From (1) it is evident that the fuzzy relation is a key link for the establishment of a fuzzy approximate inference. R consists of a certain number of information structures. Zadeh and Mamadani (1977) have suggested that when a condition is assigned to a topic,the information structure looks like 'if A_i, then B_i'. In this paper, the information distribution methods is adopted to evaluate the fuzzy relation factor R.

2.2 Information distribution.
As discussed above, R forms the main component used to establish the fuzzy appro -ximate inference. But how can we evaluate R? In the past,scientists have submit -ted a large number of methods, inducing them as shown in Figure 1.

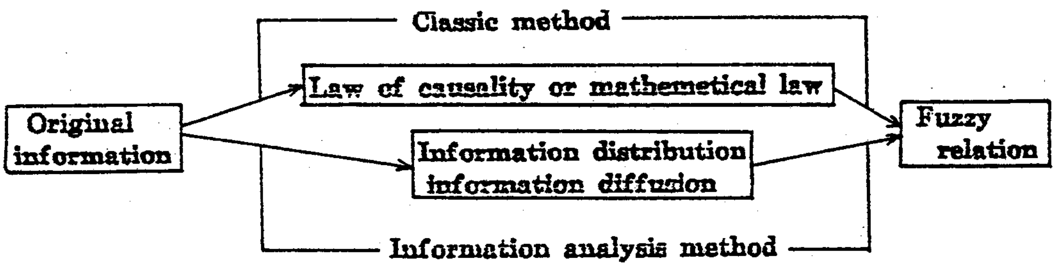

Fig. 1 The way of seeking fuzzy relationship R

Liu (1985) discussed the information distribution in detail. The advantages of his approach are:
(i) it brakes away from subjective mathematical restraints, and does not any disorder of the original data;
(ii) it is simple and direct. It is easy to realize the macro-control of all observation objects.

The method has proved to work well in practical applications. In 1987 Liu publ -ished the mathematical proof of his method. In 1987 Wang,J.D. also published a different way of evaluating R using the 'information diffusion' principle. Its importance for physics is rather explicit,but it is suitable for two dimensional situations. In order to simplify the mathematical explanation, in this paper the information distibution is adapted to evaluate the fuzzy relation matrix in a one-dimensional situation.
Suppose we have two universes of discourse :

$U=\{u_1,u_2,...,u_n\}$ (2)
$V=\{v_1,v_2,...,v_m\}$ (3)

The information matrix Q(n x n) consists of row-element u_i and column element v_j. It records the possibility distribution of U on the V-axis.All original data in the information matrix has provided a unit information, and every unit information is distributed to neighboring controlling points.The formulation can be expressed as follows:

$$Q_{ij}=1-\frac{|u-u_i|}{\Delta}, \quad i=1,2,...,n \quad (4)$$

In this equation, △ is the step distance of different grade basic variables

($\triangle=u_{i+1}-u_i$), Q_{ij} is the elements consisting of the information matrix. The regulated information matrix is the fuzzy relation R.

2.3 Evaluation of A_i on fuzzy approximate inference $B_i=A_i \circ R$.

In order to avoid the subjective effect on the expert information obtained from author's(Wang,J.D.1986) the following formulas have been applied:

I) while $a < a_{min}$, $a_{min} \in A_i$

$$A_i=[1,0,\ldots,0] \qquad (5)$$

II) while $a > a_{max}$, $a_{max} \in A_i$

$$A_i=[0,0,\ldots,1] \qquad (6)$$

III) while $a_{min} < a < a_{max}$

$$A_i=\{Max\{0,1-\frac{|a-a_i|}{\triangle}\}\} \qquad i=1,2,\ldots,n \qquad (7)$$

In formula (7), $\triangle$ is the step distance ($\triangle=a_{i+1}-a_i$). Formulas (5) and (6) show that when the original information element a exceeds the range $A_i(a \in A_i)$, the information on both ends of the element $A_i(a_{min}, a_{max})$ should be stressed.

2.4 Information concentration.

In order not to loose the information obtained by the fuzzy approximate inference, the information concentration principle is described as follows (cf. Wang,J.D.; 1985, 1986, and 1987):

$$u=\sum_{i=1}^{n} b_i^k u_i / \sum_{i=1}^{n} b_i^k \qquad (8)$$

In this equation u is the final result of the evaluation variable,b_i is the subordinate to the number of i elemts evaluated by the fuzzy approximate inference, u_i is the value of grade variable i, and k is a constant which depends on certain conditions.

3. CLASSIFICATION OF SLOPE STABILITY AND THE EVALUATION INDEX.

Based on the material properties and the geomorphology of the loess deposits in the Lanzhou region, the variations in slope stability have been grouped into several classes mentioned in Table 1.

In order to evaluate slope stability, it is supposed that seven universes of discourse exist:

- cohesion C (kg/cm^2);
- internal friction angle Φ (degree);
- slope gradient α (degree);
- slope height H (m);
- annual mean rainfall F (mm);
- maximum accelleration of seismic ground motion A_{max} (m/s^2);
- type of slope structure T.

Similarly it is supposed that the universe of discourse to another slope stability S exists which is expressed as:

$$S=\{s_1,s_2,s_3,s_4\}=\{I,II,III,IV\}$$

in which

I is a stable slope,

II is a relatively stable slope;

III is an unstable slope;
IV is an extremely unstable slope.

Table 1.

Grade	Code	structure of slope	geomorphology	earthquake effects
stable	I	fluvial terrace, loess ridge, or slope consisting of sand stone or metamorphic rocks	undercutting of the slope does not occur. Slope angles do not exceed 30°	earthquakes with intensities of 9 and less will not lead to slpoe failure
relatively stable	II	loess,deposits overlying mudstones (frequently occurring in the mountainous areas)	only slight erosion of the toe of the slopes occurs. Slope angles do not exceed 30°	earthquakes with intensities of 8 and less will not lead to slope failure.Higher intensity earthquakes may lead to small landslides or tanta
unstable	III	loess deposits overlying mudstones and mudstones.The rock strata dip at angles of about 10 to 20° (generally found outside the mountainous zone)	Intensive undercutting of the slopes. Degree of slope lies between 30–45° .Large vertical intervals.	earthquakes with intensities of about 7–8 may lead to small landslides or tanta.Higher intensity earthquakes will result in large scale slope failure
extremely unstable	IV	loess deposits overlying mudstones, the dip of the contact zone being greater than 25° . Slopes consisting of interbedded mudstones and malm stones (frequently occurring outside the mountainous zone with dips of the strata of about 20 to 30°	Intensive undercutting at the foot of the slope. Tension cracks may be found in the upper parts of he slopes.Slopet angles are greater than 45° . Very large vertical intervals	many large landslides will occur as a result of earthquakes. The landslides masses are extremely elongated

* Earthquake intensities are according to the Chinese scale.

4. Evaluation index and fuzzy relation of slope stability.

4.1 Lithological characters, slope gradient and slope height.
Generally, the engineering properties derived from laboratory tests are

expressed in temrs of characteristic cohesion and internal friction angles of the soils. However, also other lithological characters have a great influence on slope stability. Therefore it is necessary to establish the fuzzy relationship between these two factors (cohesion and the internal friction angle) and general slope stability (which is affected by a wide range of factors).

Suppose that there are two universes of discourse: cohesion C (kg/cm^2) and slope stability S.

$C=\{c_1,c_2,\ldots,c_6\}=\{0.05,0.25,0.45,0.65,0.85,1.05\}$
$S=\{s_1,s_2,s_3,s_4\}=\{I,II,III,IV\}$
The choice of the controlling points has an extreme influence on the fuzzy relation. Therefore it is necessary to increase the accuracy by using a large number of tests and calculations. The basic variable which will be used to calculate the controlling points will be divided into six grades, with a step distance $\triangle=0.2$ The information matrix $Q(6\times4)$ consists of c_i as a row variable and s_j as a column variable (see Table 2).

Suppose that the original data (c,s_j) exists, then the information that it distributes Q_{ij} can be calculated using equation (4). E.G., if the original data is (0.40,III), then $c_i=c_2=0.25$, and $c_{i+1}=c_3=0.45$

$$Q_{2,3}=1-\frac{|0.4-0.25|}{0.2}=0.25$$

$$Q_{3,3}=1-\frac{|0.4-0.45|}{0.2}=0.75$$

For example, we can engage in applying this fuzzy logic approach to 40 original data groups which were collected in the loess region. After overlaying the corresponding element of matrix Q, the information distribution table can be constructed . We can obtain the fuzzy relation $R_{c,s}$ (see Table 2) of C and S after regulating the various grades of slope stability in the information table. This is achieved by dividing the maximum value in each column by the value of various elements in the column.

Table 2. The fuzzy relationship $R_{c,s}$ between C and S.

C(kg/cm^2)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
c_1(0.05)	0	0.403	1.000	1.000
c_2(0.25)	0.050	1.000	0.956	0.250
c_3(0.45)	0.410	0.925	0.397	0.046
c_4(0.65)	0.170	0.149	0	0
c_5(0.85)	0.170	0.209	0	0
c_6(1.05)	1.000	0	0	0

Table 3. The fuzzy relationship $R_{\Phi,s}$ between Φ and S

Φ (degree)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
ϕ_1(2.5)	0	0	0	1.000
ϕ_2(7.5)	0	0	0.136	0.053
ϕ_3(12.5)	0	0.038	0.457	0.733
ϕ_4(17.5)	0	0.440	1.000	0.320
ϕ_5(22.5)	0	0.024	0.857	0.368
ϕ_6(27.5)	0.427	0.268	0.300	0.158
ϕ_7(32.5)	0.311	1.000	0.429	0
ϕ_8(37.5)	0.745	0.144	0.030	0
ϕ_9(42.5)	1.000	0	0	0
ϕ_{10}(47.5)	0.553	0	0	0

In a similar way , by using the information distribution methodm the values Φ, α, and H in these 40 groups can be distributed into the matrixes $Q_{\Phi,s}$(10×4), $Q_{\alpha,s}$(7×4), and $Q_{h,s}$(9×4). The fuzzy relationships $R_{\Phi,s}$, $R_{\alpha,s}$ and $R_{h,s}$ can be obtained for the internal friction angle Φ, the slope gradient α, the slope height H, and slope stability S after regularing. (see Tables 3 to 5).

Table 4 The fuzzy relationship $R_{\alpha,s}$ between α and S

α (degree)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
α_1 (5)	0.278	0.136	0	0
α_2 (15)	1.000	0.320	0.452	0
α_3 (25)	0.278	1.000	1.000	0
α_4 (35)	0.556	0.252	0.575	0.474
α_5 (45)	0.111	0.039	0.356	1.000
α_6 (55)	0	0	0	0.737
α_7 (65)	0	0	0	0.474

Table 5 The fuzzy relationship $R_{h,s}$ between H and S

H(meter)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
h_1(5)	1.000	1.000	1.000	0.452
h_2(55)	0.987	0.913	0.495	0.258
h_3(105)	0.484	0.364	0.561	1.000
h_4(155)	0.238	0.526	0.209	0.227
h_5(205)	0.268	0.283	0.359	0.620
h_6(255)	0	0.040	0.060	0.026
h_7(305)	0	0.364	0.260	0.194
h_8(355)	0	0	0	0.784
h_9(405)	0	0	0	0.314

4.2 Annual mean rainfall.
The Lanzhou loess region is situated in the semi-arid zone and thus rainfall is limited. Additionally the spatial variations are small making it difficult to apply the S_r index to the fuzzy approximate inference (see Table 6).

4.3 Seismic ground motion parameter.
The occurrence of high intensity earthquakes, i.e. intensities of 6, 7, 8, or 9, is only considered in general seismic engineering. The maximum accellerations of the corresponding ground movement are respectively 0.04g, 0.125g, 0.25g, and 0.40g, being the gravitational accelleration (m/s²). The fuzzy relationhip $R_{a,s}$ between the maximum acceleration of ground movement in the 40 groups of original materials and the slope stability classes can be evaluated using the information distribution method as described above.

Table 6 Subsets S_r of memership grade

F(mm)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
< 300	1.00	0.60	0.20	0
300–400	0.60	1.00	0.40	0
400–500	0	0.50	1.00	0.5
>500	0	0.20	0.60	1.0

Table 7 The fuzzy relationship $R_{a,s}$ between A max and S

Amax(m/s²) (g)	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
a (0.04)	1.000	1.000	0.411	0.102
a (0.125)	0.479	0.641	1.000	0.855
a (0.250)	0.216	0.470	1.000	1.000
a (0.400)	0	0	0.834	1.000

4.4 The types of slope structure.
The various types of slope structure have an extreme influence on slope stability. However, this index cannot be expressed in a quantitative way. Therefore, the slope stability is graded on the basis of expert knowledge (see Table 8 and Figure 2). Only a limited number of slope types are expressed in Table 8 and Figure 2. In practice S_s is evaluated using expert knowledge based on the assessment of concrete field conditions.

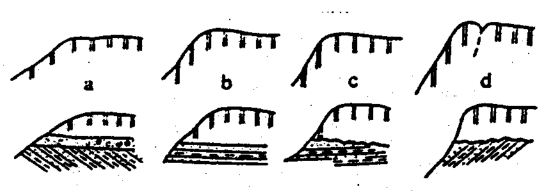

Figure 2. The types of various slope structures

Table 8 Subsets S_t of membership grade

structure type	sign	I(s_1)	II(s_2)	III(s_3)	IV(s_4)
Fig.2(a)	t_1	1.00	0.60	0.10	0
Fig.2(b)	t_2	0.60	1.00	0.30	0
Fig.2(c)	t_3	0.10	0.30	1.00	0.60
Fig.2(d)	t_4	0	0	0.70	1.00

5. CALCULATION EXAMPLE.

The reliability of the methodology discussed above can be tested and verified by using several typical slope stability classes which fall outside the range of the original 40 classes (which were used to establish the method).

The large landslide at Ba Feng Shan, Dong Xiang county, approximately 80km southwest of Lanzhou, is used to test the method. The parameters of this site are mentioned in Table 9.

Table 9 Various parameters of Bafeng Mountain Landslide

c(kg/cm²)	φ(°)	α(°)	h(m)	F(mm)	Amax(m/s²)	type of contact
0	0	44	340	549.8	0.125	t_4

The memebership grade of slope stability is first inferred by the single factor C. From Table 2 it is possible to derive the following values:

$$C_{min}=C_2=0.05(kg/cm^2),$$

and therefore

$$c = 0 < c_{min},$$

From equation (5) we know that C=[1,0,0,0,0,0]. The value of the various memership grade of slope stability (S_c) as inferred by C is obtained by using equation (1):

$$S_c=C \cdot R_{c,s} \qquad (9)$$

in which "$\circ$" is a combination operator. Various types of operators exist, but generally the maximum and minimum operators as mentioned in Zadeh (1975) are being used in papers discussing this methodology. However, many studies have shown that as a result of the use of these operators it is easy to loose information. Therefore, in this paper multiplication operators are adopted, because it is thought that form a smaller risk for loss of data. Applying this type of operator and substituting it in equation (9) has the following result:

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$$S_c=[1,0,0,0,0,0]\cdot\begin{bmatrix}0 & 0.403 & 1.000 & 1.000\\ 0.060 & 1.000 & 0.956 & 0.250\\ 0.410 & 0.925 & 0.397 & 0.046\\ 0.170 & 0.149 & 0 & 0\\ 0.170 & 0.209 & 0 & 0\\ 1.000 & 0 & 0 & 0\end{bmatrix}$$

$$=[0,0.403,1.000,1.000]$$

Other indexes may be obtained as follows:

$S_\varphi=[0,0,0,1.000]$; $S_\alpha=[0.156,0.063,0.378,0.947]$;
$S_h=[0,0.055,0.040,0.896]$; $S_\gamma=[0,0.200,0.600,1.000]$;
$S_w=[0.479,0.641,1.000,0.855]$; $S_t=[0,0,0.700,1.000]$.

Evaluating the relationship between slope stability and each variable seperately, in order to assess the relative importance of each variable, it is necessary to use a second grade fuzzy approximate inference:

$$B^{(2)}=A^{(2)}\circ R^{(2)} \qquad (10)$$

in which $A^{(2)}$ is the weighting factor of every single variable. Wang (1989) discussed several practical examples to evaluate the weighting function of the grey related degree. According to this method it is possible to assess $A^{(2)}$ for each group:

$$A^{(2)} = [0.102,0.102,0.201,0.189,0.080,0.120,0.206]$$

$R^{(2)}$, in equation (10), stands for the combination of S_c, S_φ, $S_\alpha, S_h, S_\gamma, S_w, S_t$:

$$R^{(2)}=\begin{bmatrix}S_c\\ S_\varphi\\ S_\alpha\\ S_h\\ S_\gamma\\ S_w\\ S_t\end{bmatrix}=\begin{bmatrix}0 & 0.403 & 1.000 & 1.000\\ 0 & 0 & 0 & 1.000\\ 0.156 & 0.063 & 0.378 & 0.947\\ 0 & 0.055 & 0.040 & 0.696\\ 0 & 0.200 & 0.600 & 1.000\\ 0.479 & 0.641 & 1.000 & 0.855\\ 0 & 0 & 0.700 & 1.000\end{bmatrix}$$

and therefore

$$B^{(2)} = A^{(2)}\circ R^{(2)}$$

$$= [0.089,0.164,0.498,0.912] \longrightarrow$$

0.089/stable + 0.164/relatively stable + 0.498/unstable + 0.912/extremely unstable.

This example shows that this slope at Ba Feng Shan is extremely unstable which is competely in accordance with the practical situation. A number of simulations have been carried out for a.o. seven slopes around San Tai Ge in Lanzhou city. The results of that study are also in accordance with the expert assessment of the practical situation. Subsequently, the method has been applied to a large landslide research and prevention project in the Lanzhou city area and has been used as a basis for hazard mapping .

6. CONCLUSIONS

In this paper the uncertainty of the change in slope stability has been considered using a quantitative method. On the basis of seven variables which are thought to determine slope stability in the Lanzhou region it has been shown that it is possible to assess the slope stability for individual sites. The results of these analyses are in accordance with the assessment based on independent expert assessment of these sites.

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