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THE EQUIVALENCE OF COMPLEX FUZZY FUNCTION LIMIT IN DIFFERENT METRIC SPACES

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I. Conditions Of Equivalence Of Function Limit In Different Distances

According to document [1],[2],[3], we can set different distances to the same set and enter different concepts of function limit in different metric spaces.

Let S be a set. Set two distances on it, ρ_A and ρ_B, which satisfy the three axioms of distance. That is, (S, ρ_A) and (S, ρ_B) form two metric spaces. The limit of function f(x) is defined as follows:

Definition 1. Let x_0 be an accumulation point of S. $k \in S$. If for any given $\varepsilon > 0$ there always exists $\delta > 0$ so that when $0 < \rho_A(x, x_0) < \delta$, we have

$$\rho_A(f(x), K) < \varepsilon$$

Then we say the limit of f(x) is K, and is written as:

$$\lim_{\rho_A(x,x_0)\to 0} f(x) = K$$

Definition 2. Let x_0 be an accumulation point of S. $K \in S$. If for any given $\varepsilon > 0$, there always exists $\delta > 0$ so that when $0 < \rho_B(x, x_0) < \delta$, we have

$\rho_B(f(x),K)<\varepsilon$

Then we say the limit of f(x) is K,and is written as:

$$\lim_{\rho_B(x,x_0)\to 0} f(x) = K$$

Definition 3. If function f(x) satisfies:

$$\lim_{\rho_A(x,x_0)\to 0} f(x) = K \Longleftrightarrow \lim_{\rho_B(x,x_0)\to 0} f(x) = K$$

then we say the limit concepts in the metric spaces (S,ρ_A) and (S,ρ_B) of function f(x) are identical,or the distances ρ_A and ρ_B are equivalent on function limit concepts

Theorem 1. If there exists positive real numbers $K_1\leqslant K_2$ so that for any $x\in S$, $x_0\in S$, there will be

$$K_1\rho_B(x,x_0)\leqslant\rho_A(x,x_0)\leqslant K_2\rho_B(x,x_0)$$

then

$$\lim_{\rho_A(x,x_0)\to 0} f(x) = k \Longleftrightarrow \lim_{\rho_B(x,x_0)\to 0} f(x) = k$$

that is, ρ_A and ρ_B are equivalent on limit concepts of function f(x).

Proof: Sufficiency

Let $\lim_{\rho_B(x,x_0)\to 0} f(x) = K$,according to Definition 2,for any given $\varepsilon>0$, $\varepsilon/k_2>0$, there exists $\delta_1>0$.When $0<\rho_B(x,x_0)<\delta_1$, we have

$$\rho_B(f(x),K)<\varepsilon/K_2$$

Take $\delta=K_1\cdot\delta_1$,then when $0<\rho_A(x,x_0)<\delta$,we have

$$0<\rho_B(x,x_0)\leqslant 1/k_1\,\rho_A(x,x_0)<\delta_1$$

Therefore,

$$\rho_A(f(x),K)\leqslant K_2\,\rho_B(f(x),K)<K_2\cdot\varepsilon/K_2=\varepsilon$$

According to Definition 1,

$$\lim_{\rho_A(x,x_0)\to 0} f(x) = k$$

Necessary Condition

Let $\lim_{\rho_A(x,x_0)\to 0} f(x) = k$, according to Definition 1, for any given $\varepsilon > 0$, $\varepsilon \cdot k_1 > 0$, there exists $\delta_2 > 0$. When $0 < \rho_A(x,x_0) < \delta_2$, we have

$$\rho_A(f(x),k) < \varepsilon \cdot k_1$$

Take $\delta = \delta_2 / K_2$, then when $0 < \rho_B(x,x_0) < \delta$, We have

$$0 < \rho_A(x,x_0) \leqslant K_2 \rho_B(x,x_0) < \delta_2$$

Therefore,

$$\rho_B(f(x),K) \leqslant 1/K_1 \, \rho_A(f(x),K) < 1/K_1 \cdot \varepsilon \cdot K = \varepsilon$$

According to Definition 2,

$$\lim_{\rho_B(x,x_0)\to 0} f(x) = K$$

II. The Equivalence Of Complex Fuzzy Function Limit In Different Metric Spaces

Now, based on the above mentioned, we would study the equivalenec of complex fuzzy function limit in different metric spaces.

According to Document [4], any complex fuzzy number x can be indicated as [P[x], Q[x]], or [P[x]$_\sim$ Q[x]]. It has definate left end point P[x], right end point Q[x] and infimum infx. On the contrary, when P[x], Q[x] and infx are given, the complex fuzzy number x is asserted with them .

Here, $P[x]<Q[x]$, $\inf x \in \{0,1\}$.

Definition 4. Let R be a complex fuzzy subset, and $x \in R$, $y \in R$, the distance between x and y is defined as:

$$d[x,y]=\max(|P[x]-P[y]|, |Q[x]-Q[y]|, |\inf x-\inf y|)$$

According to Document[3],[4], R forms a complex fuzzy metric space under $d[x,y]$. From Document[1],[2],[3], we know when $\inf x = \inf y$, $d[x,y]$ becomes the distance of the two fuzzy numbers.

Definition 5. Let R be a complex fuzzy subset, and $x \in R$, $y \in R$. The distance between x and y is defined as:

$$|\overline{xy}| = \sqrt{(P(x)-P(y))^2+(Q(x)-Q(y))^2+(\inf x-\inf y)^2}$$

According to Document[3],[4], R forms a complex fuzzy metric space under $|\overline{xy}|$. From Document[1],[2],[3], we know when $\inf x=\inf y$, $|\overline{xy}|$ becomes the distance of the two fuzzy numbers.

Definition 6. Let $f(x)$ be a complex fuzzy function, with R being its domain of definition. A and x_0 are complex fuzzy numbers. If for any given $\varepsilon>0$, there always exists $\delta>0$. When $0<d[x,x_0]<\delta$, we have

$$d[f(x),A]<\varepsilon$$

Then we say when x approaches x_0, the limit of complex fuzzy function $f(x)$ is A, and is written as:

$$f(x)\to A \quad (d[x,x_0]\to 0)$$

or

$$\lim_{d(x,x_0)\to 0} f(x)=A$$

Definition 7. Let $f(x)$ be a complex fuzzy function, with R being its domain of definition. A and x are complex fuzzy numbers. If for any given $\varepsilon>0$, there always exists $\delta>0$. When $0<|\overline{xx_0}|<\delta$, we have

$$|f(x),A|<\varepsilon$$

Then we say when x approaches x_0, the limit of f(x) is A, and is written as:

$$f(x)\to A \quad (|\overline{xx_0}|\to 0)$$

or

$$\lim_{|\overline{x\,x_0}|\to 0} f(x)=A$$

Now, we would discuss the equivalence of complex fuzzy function limit in the two above mentioned metric spaces

Lemma: Let R be a complex fuzzy number set. If for any taken $x,y\in R$, we have

$$d[x,y]\leqslant|\overline{xy}|\leqslant\sqrt{3}d[x,y]$$

proof:

From Definition 4 and 5, we have

$$d[x,y]=\max\{|P[x]-P[y]|,|Q[x]-Q[y]|,|\inf x-\inf y|\}$$

$$|\overline{xy}|=\sqrt{(P[x]-P[y])^2+(Q[x]-Q[y])^2+(\inf x-\inf y)^2}$$

Let us Suppose

$$\max\{|P[x]-P[y]|,|Q[x]-Q[y]|,|\inf x-\inf y|\}=|P[x]-P[y]|$$

Then

$$d[x,y]=|P[x]-P[y]|$$

and

$$|P[x]-P[y]|<|\overline{xy}|<\sqrt{3}|P[x]-P[y]|$$

Therefore

$$d[x,y]<|\overline{xy}|<\sqrt{3}d[x,y]$$

From Lemma and Theorem 1 we obtain:

Theorem 2. Let f(x) be a complex fuzzy function, and

R be a complex fuzzy number set. $A, x_0 \in R$. Then we have

$$\lim_{d[x,x_0]\to 0} f(x)=A \iff \lim_{|x-x_0|\to 0} f(x)=A$$

From the above mentioned we know that in different metric spaces, the complex fuzzy function limit is equivalent. This is one of the basic theories of complex fuzzy function limit and will be of great theoretical significance in further founding complex fuzzy function limit and complex fuzzy derivation numbers.

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