ON F-SUSLIN SETS

Deli Zhang
Dept. of Math, Jilin Prov. Inst. of Education, Changchun, Jilin, 130022. P.R.China

Caimei Guo
Dept. of Basic Science, Changchun Univ. Changchun, Jilin, 130022, P.R.China.

Abstract: In the paper, the concept of F-Suslin sets (fuzzy Suslin sets) is given, the results similar to classical Suslin sets summarized in [1] are obtained.

Introduction: It is well known that Suslin sets [1] (or Analytic sets [2]) is an important branch of pure math. and play much important role in modern analysis, theory of measure, etc. [1, 2]. Since Prof. Zadeh built the theory of fuzzy sets [4], it has entered almost all fields of classical math. and lead to many fuzzy branches, such as F-topology, F-measure, and so on. The paper's aim is generalizing the classical Suslin sets to fuzzy circustance. The authors hope it can become the beginning of the investigation of F-Suslin sets theory.

Let X be a set, $\mathcal{F}(X)$ be the fuzzy power set. For $\tilde{X} \subset \mathcal{F}(X)$, let $\tilde{X}_d$, $\tilde{X}_\delta$, $\tilde{X}_s$ $\tilde{X}_\sigma$ denote the classes obtained by applying to the element of $\tilde{X}$ the operations of finite intersections(d), countable intersections(δ), finite unions(s), and countable unions(σ), respectively. the symbols $\tilde{X}_{\sigma\delta}$, $\tilde{X}_{ds}, \cdots$, and so on will denote the classes obtained by succesive application of the indicated operations in the obvious (left to right). $\tilde{X}$ is called a F-paving on X iff ϕ, X belong to $\tilde{X}$, then the pair (X,

$\tilde{X}$) will be called a F-paved set. Let $(X, \tilde{X})$ and $(Y, \tilde{Y})$ be two F-paved sets, we denote by $\tilde{X}\times\tilde{Y}$ the class of all sets of the form $A\times B$ where $A\in\tilde{X}$, $B\in\tilde{Y}$, and $(A\times B)(x, y)=A(x)\wedge B(y)$.

Definitions:

1. A F-Suslin scheme on a set X is a function ε from the set P into $\mathcal{F}(X)$, where P is the set of multi-index $p=(p_1, p_2, \cdots\cdots p_n)$ which is a finite ordered set of natural numbers N. An alternative way thinking of a F-Suslin scheme is as an indexed family $\mathcal{E}=\{E(p_1, p_2, \cdots\cdots, p_n)\in\mathcal{F}(X):\ p_i\in N\}$.

2. The F-Suslin operation on $\mathcal{F}(X)$ is the function α from the collection of all F-Suslin schemes on X to $\mathcal{F}(X)$ defined by

$$\alpha(\varepsilon)=\bigcup_{p\in N^\infty}\bigcap_{k\in N}\varepsilon(p_1, p_2, \cdots, p_n)$$

where N^∞ is the set of all sequences of natural numbers. The value of α at ε is called a F-Suslin set, that is, the collection of F-Suslin sets is the range of α. Corresponding the other view of F-Suslin scheme , we use a different symbol $A(\cdot)$ for the F-Suslin operation, then

$$A(\mathcal{E})=\bigcup_{p\in N^\infty}\bigcap_{k\in N}E(p_1, p_2, \cdots, p_n)=\bigcup_{p\in N^\infty}\bigcap_{k\in N}\varepsilon(p_1, p_2, \cdots, p_n)$$

Example: Let $E\in\mathcal{F}(X)$, consider the constant F-Suslin scheme $\varepsilon(\cdot)\equiv E$ then $\alpha(\varepsilon)=E$, so that every fuzzy set is a F-Suslin set. The elements in $A(\tilde{X})$ is called $\tilde{X}$-F-Suslin sets.

3. Let $(X, \overline{X})$ be a paved set (nonfuzzy), the paving $\overline{X}$ is said to be semicompact if every countable subclass of elements of $\overline{X}$ has the finite intersection property, i. e. if for every sequence $\{A_i: i\in N\}\subset\overline{X}$, $\bigcap_{k=1}^{r}A_k\neq\phi$ for each $r\in N$,

together imply that $\bigcap_{K=1}^{\infty} A_k \neq \phi$ 。

Example: Let $N_p=\{q \in N : q|k=p\}$, where $q|k$ is the initial segment $(q_1, q_2, \cdots, q_k)$, $p=(p_1, p_2, \cdots, p_k) \in P$. Further, let $\overline{N}_p$ denotes the set $\{\phi, N \ N_p, p \in P\}$, obviously, $\overline{N}_p$ is the semicompact paving on N [1].

4. Let $(X, \tilde{X})$ and $(Y, \tilde{Y})$ be two F-paved sets, For an element $C \in \tilde{X} \times \tilde{Y}$ we define $Pr_X C \in \mathcal{F}(X)$ as followed:

$$Pr_X C(x)= \bigvee_{y \in Y} C(x, y) \qquad (x \in X).$$

Main result:

The following results are the extension of the corresponding results in [1], their proofs are not very difficult by comparing with the proofs in [1] and applying the operations of fuzzy sets [4].

Proposition 1. Let $(X, \tilde{X})$ be a F-paved set, then $\tilde{X} \subset A(\tilde{X})$.

Proposition 2. Let $(X, \tilde{X})$ be a F-paved set, s.t. $\tilde{X}=\tilde{X}d$. Then for any $S \in \mathcal{F}(X)$, the following statements are equivalent:

(i). $S \in A(\tilde{X})$;

(i). $S=Pr_X C$. where $C \in (X \times \overline{N}_p)$

(iii) . There exists a set Y with a semicompact paving (nonfuzzy) $\overline{Y}$ and a set C belongs to $(\tilde{X} \times \overline{Y})_{\sigma\delta}$, s.t. $S=Pr_X C$.

Proposition 3. If $(X, \tilde{X})$ is a F-paved set such that $\tilde{X}=\tilde{X}d$, then the class $A(\tilde{X})$ of $\tilde{X}$-F-Suslin sets is closed under the operations of countable intersections and countable unions.

Proposition 4. Let $(X, \tilde{X})$ and $(Y, \tilde{Y})$ be two F-paved sets, then

$$A(\tilde{X}) \times A(\tilde{Y}) \subset A(\tilde{X} \times \tilde{Y})$$

Proposition 5. Let $(X, \tilde{X})$ be a F-paved set, $(Y, \bar{Y})$ be a paved set (nonfuzzy), and $\bar{Y}$ is semicompact, then for all $S \in A(\tilde{X} \times \bar{Y})$, we have $Pr_X S$ is in $A(\tilde{X})$.

Proposition 6. Let $(X, \tilde{X})$ be a F-paved set, then

$$A(A(\tilde{X}))=A(\tilde{X})$$

That is, $\tilde{X}$-F-Suslin sets is closed under the F-Suslin operation.

Proposition 7. Let (X, A, m) be a complete measure space, and $\tilde{A}$ is the family of all A-measurable fuzzy subsets of X i. e.

$\tilde{A}=\{A \in \mathcal{F}(X): \mu_A: X \rightarrow [0, 1]$ is A-measurable$\}$.

Then we have $A(\tilde{A})=\tilde{A}$

That is, the class of A-measurable fuzzy sets is closed under the F-Suslin operation.

Remark: when we use the concepts of F-topology, defining F-Analytic sets as the continuous image of F-Suslin sets, we can discuss the theory similarly, and the authors are working on the problem, some results (such as separation properties [2]) have been obtained.

Ref.

1. E. Klein & A. C. Thompson, Theory of Correspondences, John & wiley NewYork 1984.
2. C, A,. Rogers & etc., Analytic Sets, Academic Press, NewYork 1984.
3. D, Dubois & H. Prade, Fuzzy Sets and Systems, ibid, 1980.
4. L. A. Zadeh, Fuzzy sets, Inform. and control 8(1965), 338-353.