Fuzzy Pre-semi-open subsets

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Abstract: In this paper, fuzzy pre-semi-open subsets in a fuzzy topological spaces are introduced and studied. In addition, the relation between it and the other fuzzy semi-topological subsets lastly, we descibe fuzzy S^{*}-closed spaces in terms of fuzzy pre-semi-open sets.

Keywords: Fuzzy semi-open sets; fuzzy pre-semi-open sets; fuzzy semi-regularly semi-open sets; S^{*}-closed spaces.

Thoughout the paper (X,T) or simiply X stands for a fuzzy topological space (fts, for short) in Chang's [2] sense. A fuzzy point in X with support $x \in X$ and value α $(0<\alpha \leqslant 1)$ is denoted by x_α. For a fuzzy set A in X, the notations A^-, A^o, and (1-A) are used to respectively stand for the closure, interior and complement of A, whereas the constant fuzzy sets taking on respectively the values 0 and 1 on X are designated by 0_x and 1_x respectively. A fuzzy set A in X is said to be q-coincident with a fuzzy set B written as A q B, iff there is $x \in X$ such that $A(x)+B(x)>1$ [4]. Abbreviating the word "neighbourhood" by nbd,we say that A is a semi-q-nbd of a fuzzy point x_α iff there exists a fuzzy semi-open set V such that $x_\alpha\, q V \leqslant A$ [3]. The fuzzy semi-closure of a fuzzy set A in X, to be denoted by A_-, is the union of all fuzzy points x_α such that every semi-open semi-q-nbd of x_α is q-coincident with A. The union of all fuzzy semi-open sets contained in a fuzzy set A is called the fuzzy semi-interior of A,to be denote by A_o.Clearly, for any fuzzy set A, $A^o \leqslant A_o \leqslant A$, and also, $A_o = 1-(1-A)_-$. For an fts X, we shall denote the set of all fuzzy open , closed, semi-open and semi-closed sets by FO(X), FC(X), FSO(X) and FSC(X). respectively.

Lemma 1 Let A be any fuzzy set in X. Then

$$A^{o} \leqslant A_{o} \leqslant A \leqslant A_{-} \leqslant A^{-}.$$

Lemma 2 Let A be any fuzzy set in X. Then

(a) $(A_{-})^{o} = A^{-o}$,

(b) $A^{o-} = (A_{o})^{-}$,

(c) $(A_{-})^{-} = (A^{-})_{-} = A^{-}$,

(d) $(A_{o})^{o} = (A^{o})_{o} = A^{o}$,

(e) $(A_{-})^{o} \leqslant A^{-o} \leqslant A_{-o} \leqslant (A^{-})_{o}$,

(f) $(A^{o})_{-} \leqslant A_{o-} \leqslant A^{o-} \leqslant (A_{o})^{-}$.

In [1], a fuzzy set A in X is defined to be fuzzy pre-open iff $A \leqslant A^{-o}$, Let us now define as follows.

Definition 3 A fuzzy set A in X is said to be fuzzy pre-semi-open iff $A \leqslant A_{-o}$.

We shall use the notations FPO(X) and FPSO(X) to represent the sets of all fuzzy pre-open sets and fuzzy pre-semi-open sets in X respectively.

Obviously, we now have the following.

Thorem 4 For any fts X,

(a) $FO(X) \subset FPO(X) \subset FPSO(X)$,

(b) $FO(X) \subset FSO(X) \subset FPSO(X)$.

The iverse of Theorem 4 is not valid in general.

Example Let X be any non-empty set, consider the fuzzy set A、B、C、D、 give by $A(x)=0.25$, $B(x)=0.6$, $C(x)=0.2$, $D(x)=0.7$, for all $x \in X$. Then for the fts (X,T), where $T=\{0x, 1x, A, B\}=FO(X)$. We have $C \in FPO(X)$, $C \notin FO(X)$, $C \notin FSO(X)$, $D \in FPSO(X)$, $D \notin FPO(X)$, and $D \in FSO(X)$.

Theorem 5 For any fts X,

(a) A is fuzzy pre-open iff there exists a fuzzy open set V in X such that $A \leqslant V \leqslant A^{-}$.

(b) A is fuzzy pre-semi-open iff there exists a fuzzy semi-open set U in X such that $A \leq U \leq A_-$.

Proof. (a) Let $A \in FPO(X)$ then $A \leq A^{-o}$, we put $V = A^{-o}$ be a fuzzy open set and $A \leq A^{-o} = V \leq A^-$. Conversely, if $V \in FO(X)$, such that $A \leq V \leq A^-$, then $A \leq V = V^o \leq A^{-o}$ and thus $A \in FPO(X)$.

(b) Similar to that of (a) and omitted.

Definition 6 A fuzzy set A in X is said to be fuzzy semi-regularly semi-open (semi-closed) iff $A = A_{-o}$ (resp. $A = A_{o-}$).

For an fts X , we shall denote the set of all fuzzy semi-regularly semi-open sets and semi-regularly semi-closed sets by FSRSO(X) and FSRSC(X).

Theorem 7 Let A be any fuzzy set in X. Then

(a) $A \in FSRSC(X)$ iff $(1-A) \in FSRSO(X)$.

(b) $A \in FSRSO(X)$ iff there exist a fuzzy semi-open set B in X such that $A = B_-$.

(c) $A \in FSRSC(X)$ iff there exists a fuzzy semi-closed set C in X such that $A = C_o$.

(d) $A \in FSRSC(X)$ then $A \in FSO(X)$.

Proof. They are clear from the above definition 6.

Theorem 8 Let A be any fuzzy set in X. Then

(a) If $A \in FPSO(X)$ then $A_- \in FSRSC(X)$.

(b) If $(1-A) \in FPSO(X)$ then $A_o \in FSRSC(X)$.

(c) $A \in FPSO(X) \cap FSC(X)$ iff $A \in FSO(X) \cap FSC(X)$.

Proof. (a) Let $A \in FPSO(X)$ then $A \leq A_{-o}$ and hence $A_- \leq A_{-o-}$. Again, $(A_-)_{o-} \leq A_{--} = A_-$ thus we have proved $A_- = A_{-o-}$. Therefore it follows from Difinition 6 that $A_- \in FSRSC(X)$.

(b) Let $(1-A) \in FPSO(X)$ by (a) we have $(1-A)_- \in FSRSO(X)$, thus $A_o = 1-(1-A)_- = 1-(1-A)_{-o} = [1-(1-A)_-]_- = A_{o-}$. Therefore $A_o = A_{o-o} \in FSRSO(X)$

(c) $A \in FPSO(X) \cap FSC(X)$ iff $A_- = A \leq A_{-o} = A_o$ iff $A \in FSO(X) \cap FSC(X)$.

Theorem 9 Let A be any fuzzy set in X, then $A \in FPSO(X)$ iff $A^- = A_{-o}$.

Proof. Let $A \in FPSO(X)$, to prove $A^- = A_{-o}$, by Lemma 2 it is sufficient to prove that $A^{-} \leqslant A_{-o}$. Since $A \in FPSO(X)$ then $A^{-} \leqslant (A_{-o})^{-}$, but $A_{-o} \in FC(X)$, thus $A^{-} \leqslant A_{-o}$.

Conversely, is obvious.

Definition 10 A fuzzy point x_{α} is said to be a fuzzy semi-δ-cluster point (semi-θ-clustor point) of a fuzzy set A in X iff $U_{-o}qA$ (resp, $U_{-}qA$) for every fuzzy semi-open semi-q-nbd U of x_{α}. Then union of all fuzzy semi-δ-cluster points (resp. semi-θ-cluster points) of a fuzzy set A in X is denoted by $[A]_{s\delta}$ (resp, $[A]_{s\theta}$). A is fuzzy semi-δ-closed (resp, semi-θ-closed) iff $A=[A]_{s\delta}$ (resp, $A=[A]_{s\theta}$).

Theorem 11. Let A be any fuzzy set in X, then

$$A_{-} \leqslant [A]_{s\delta} \leqslant [A]_{s\theta}$$

Proof. By Definition 10 is obvious.

Lemma 12 If A is any fuzzy set and B a fuzzy semi-open set in an fts X, such that $A\bar{q}B$, then $A_{-}\bar{q}B$.

Proof. If there exists an $x \in X$ such that $A_{-}(x)+B(x)>1$, then putting $A_{-}(x)=\alpha$, we see that B is a fuzzy semi-open semi-q-nbd of x_{α} with $A\bar{q}B$, whereas $x_{\alpha} \in A_{-}$. Thus we arrive at a contradiction.

Thorem 13 Let $A \in FPSO(X)$, then $A_{-}=[A]_{s\delta}=[A]_{s\theta}$.

Proof. To prove this it is sufficient to prove that $[A]_{s\theta} \leqslant A_{-}$. Now if $x_{\alpha} \bar{\in} A_{-}$ then there exist a fuzzy semi-open semi-q-nbd U of x_{α} such that $U\bar{q}A$ and thus $U\bar{q}A_{-}$. But then $U_{-}\bar{q}A$ (since $A \in FPSO(X)$). Hence $x_{\alpha} \bar{\in} [A]_{s\theta}$. Thus we have proved $[A]_{s\theta} \leqslant A_{-}$.

Definition 14 [5] An fts X is said to be fuzzy S^{*}-closed iff for every fuzzy semi-open cover $\{U_{\alpha}; \alpha \in \Lambda\}$ of X, there exists a finite subset Λ_{o} of Λ such that $\{(U_{\alpha})_{-}; \alpha \in \Lambda_{o}\}$ is a fuzzy cover of X.

Theorem 15 An fts X is fuzzy S^{*}-closed iff for every fuzzy pre-semi-open cover $P=\{P_{\alpha}; \alpha \in \Lambda\}$ of X, there exists a finite subset Λ_{o} of Λ such that $P_{-}=\{(P_{\alpha})_{-}; \alpha \in \Lambda_{o}\}$ is a fuzzy cover of X.

Proof. Let $P=\{P_\alpha ; \alpha \in \wedge\}$ be a fuzzy pre-semi-open cover of X, then for each $\alpha \in \wedge$, $P_\alpha \leqslant (P_\alpha)_{-o}$, thus $P_{-o}=\{(P_\alpha)_{-o}; \alpha \in \wedge\}$ is fuzzy semi-open cover of X, By fuzzy S^*-closedness of X, for P_{-o} there exists a finite subset $\wedge_o$ of $\wedge$ such that $\{(P_\alpha)_{-o}; \alpha \in \wedge_o\}$ is a fuzzy cover of X, and since for any $\alpha \in \wedge$. $(P_\alpha)_{-o-}=(P_\alpha)_-$, hence P_- there exist a finite subcover.

Coversely, Since each fuzzy semi-open set is fuzzy pre-semi -open sets. Thus the conclusion is clear.

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