APPLICATION OF FUZZY INFORMATION PROCESSING METHOD IN MULTIVARIATED ANALYSIS FOR BUILDING EARTHQUAKE DAMAGE PREDICTION①

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ABSTRACT

A multivariated analysis model for building earthquake damage prediction is presented in this paper using the principle of information distribution and the formula of fuzzy deduction, in which lattice similarity is used to recognize the fuzzy deduction result. The application of this method is illustrated in an example for single—storey industrial masonry building.

Key words: earthquake damage prediction, information distribution, lattice similarity

1. INTRODUCTION

Recently, a great progress has been made in the area of earthquake damage prediction and many methods have been presented in China[1]. These methods can be classified basically into two categories: one is that the relationship between earthquake intensity and the degree of earthquake damage is found by analysizing historical earthquake damage materials; and the other is the theoretical calculation method.

The earthquake damage prediction method presented in this paper belongs in the first one. As compared with general fuzzy deduction, in this method the grade of membership or membership functions are not determined directly but the fuzzy relationship matrix for one—variated analysis is formed using information distribution method presented in references [2][3][4]. The first step of presented method is to carry out one—variated fuzzy deduction. Then the fuzzy relationship matrix for multivariated analysis is formed by collecting the fuzzy deduction results of one — variated analysis, and multivariated fuzzy deduction is made. Not only theoretical calculation of earthquake respone of structure but also various factors and fuzzy character of historical earthquake damages of structure may

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be considered in this method.

2. THE GRADES OF EARTHQUAKE DAMAGE IN BUILDINGS AND THEIR PREDICTION

At present, the earthquake damage in buildings are classified as five grades in most references [4][5][6], they may be regarded as fuzzy subset A_t in earthquake damage indices universe of discourse V, where V is expressed as

$$V = \{v_1, v_2, \cdots, v_j, \cdots, v_{11}\} = \{0.0\ 0.1\ 0.2 \cdots 1.0\} \qquad (1)$$

and A_t are defined as

$$\left.\begin{array}{l} A_1 = \text{undamaged} = 1/0 + 0.7/0.1 + 0.2/0.2 \\ A_2 = \text{slightly damaged} = 0.2/0 + 0.7/0.1 + 1/0.2 + 0.7/0.3 + 0.2/0.4 \\ A_3 = \text{moderately damaged} = 0.2/0.2 + 0.7/0.3 + 1/0.4 + 0.7/0.5 + 0.2/0.6 \\ A_4 = \text{severely damaged} = 0.2/0.4 + 0.7/0.5 + 1/0.6 + 0.7/0.7 + 0.2/0.8 \\ A_5 = \text{collapse} = 0.2/0.6 + 0.7/0.7 + 1/0.8 + 0.7/0.9 + 0.2/1.0 \end{array}\right\} \qquad (2)$$

These formulas can be expressed in general form as .

$$A_t = a_1/v_1 + a_2/v_2 + \cdots + a_j/v_j + \cdots + a_{11}/v_{11} \qquad (3)$$

where v_j is the j—th grade of earthquake damage index; a_j is the grade of membership of v_j in A_t.

When we carry out earthquake damage prediction using fuzzy deduction, its result is also a fuzzy vector which is called "earthquake damage fuzzy vector" in this paper, it has the same form with Eq. (3) and for simplicity can be expressed in terms of its grades of membership as

$$A = \{a_1, a_2, \cdots, a_j, \cdots, a_{11}\} \qquad (4)$$

Suppose there are n earthquake damage factors, they form earthquake damage factors universe of discourse

$$U = \{u_1, u_2, \cdots, u_i, \cdots, u_n\} \qquad (5)$$

Then earthquake damage factors fuzzy subset is defined as

$$W = w_1/u_1 + w_2/u_2 + \cdots + w_i/u_i + \cdots w_n/u_n \qquad (6)$$

where w_i denotes the grade of membership of u_i in W. Eq. (6) can be expressed also as "earthquake damage factors fuzzy vector" in terms of its grade of membership, that is

$$W = [w_1, w_2, \cdots, w_i, \cdots, w_n] \qquad (7)$$

The earthquake damage prediction is to deduce the earthquake damage fuzzy vector A from the materials of earthquake damage factors by means of the fuzzy relationship between the earthquake damage factors universe of discourse U and the earthquake damage indices V, and to compare the deduction result with Eq. (2) for determining the grade of earthquake damage in building in future earthquake.

3. ONE—VARIATED FUZZY DEDUCTION AND ITS FUZZY RELATIONSHIP MATRIX

One—variated fuzzy deduction is to find the grade of membership which relates factor u_t to the grades of different earthquake damage indices in case the factor u_t is given. Suppose factor u_t changes in the interval [a, b], according to the prescribed analysis precision, we take a discrete universe of discourse as

$$u_t = \{x_1, x_2, \cdots, x_i, \cdots, x_n\} \quad (8)$$

where $x_i \in [a,b]$, $i = 1, 2, \cdots, m$.

Suppose there are N historical earthquake damage information (we call also them as knowledge sample) with n groups of digital data in each information, and there are two components x and A in each group of data, where x denotes the characteristic value of the factor u_t , A is the grade of earthquake damage. We mark a group of digital data in k—th earthquake damage information by $D_k = (x_k, A_k)$, where x_k and A_k denote the information components.

Let us now distribute data component x_k over discrete universe of discourse u_t (*Eq.* (8)). *If* $x_i < x_k < x_{i+1}, x_i, x_{i+1} \in u_t$, then the distributed data proportion can be defined as

$$\left.\begin{aligned} q_k(x_i) &= 1 - \frac{x_k - x_i}{x_{i+1} - x_i} \\ q_k(x_{i+1}) &= 1 - \frac{x_{i+1} - x_k}{x_{i+1} - x_i} \end{aligned}\right\} \quad (9)$$

If $A_k = A_j$, where A_j is a fuzzy subset in Eq. (2), then the data proportion distributed to the j—th earthquake damage index v_j is defined as

$$q_k(v_j) = a_j / \sum a_j \quad (10)$$

where $\sum a_j$ denotes the sum of all grades of membership in the fuzzy subset A_j.

By setting

where

$$\left.\begin{aligned} R_1 &= (r^1_{ij}), R_2 = (r^2_{ij}) \\ r^1_{ij} &= q_{ij}/max\{q_{i1}, q_{i2}, \cdots, q_{i11}\} \\ r^2_{ij} &= q_{ij}/max\{q_{1j}, q_{2j}, \cdots, q_{mj}\} \end{aligned}\right\} \quad (12)$$

we obtain

that is

$$\left.\begin{aligned} R^t &= R_1 \wedge R_2 \\ r_{ij} &= min\{r^1_{ij}, r^2_{ij}\} \end{aligned}\right\} \quad (13)$$

where R^t is the fuzzy relationship matrix for one—variated analysis.

Applying the principle of fuzzy deduction, the earthquake damage fuzzy vector from the single—factor u_t analysis can be obtained as

$$A^t_1 = X \ o \ R^t \quad (14)$$

where X is a fuzzy subset in u_t(*i. e.* , *Eq*(8)), it can be obtained by use of information distribution method (i. e. , Eq(9)); "o" is an operation sign, which will be explained in the next section.

4. MULTIVARIATED FUZZY DEDUCTION AND ITS RECOGNITION

Multivariated fuzzy deduction gives the final improved result on the basis of the deduction result of one—variated analysis. This operation can be expressed as

$$A = W \ o \ R \tag{15}$$

where the fuzzy relationship matrix

$$R = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n^t \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1,11} \\ r_{21} & r_{22} & \cdots & r_{2,11} \\ \vdots & \vdots & & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{n,11} \end{bmatrix} \tag{16}$$

is a collection of all one—variated deduction results A_i^t; A_i^t denotes the fuzzy relationship between the factor u_i and earthquake damage indices universe of discourse V; W denotes earthquake damage factors fuzzy vector.

Different mathematical models may be adopted for the right part of Eq. (14) and (15). In this paper, "weighted mean mathematical model" is used. So "o" denotes general matrix multiplication, and W can be regarded as the weighting vector expressing the importance of various factors.

For a certain building, the grade A of earthquake damage found by Eq. (15) will be recognized by lattice similarity defined by Professor Wang Peizhuang [7] in the following way.

Suppose A, A_t be two fuzzy subsets in universe of discourse V, designating

$$\left.\begin{aligned} A \cdot A_t &= \bigvee_{v \in V} (\mu_A(v) \wedge \mu_{At}(v)) \\ A \odot A_t &= \bigwedge_{v \in V} (\mu_A(v) \vee \mu_{At}(v)) \end{aligned}\right\} \tag{17}$$

the lattice similarity of fuzzy subsets A and A_t is defined as

$$(A, A_t) = \frac{1}{2}[A \cdot A_t + (1 - A \odot A_t)] \tag{18}$$

Eq. (18) is called also as neartude on certain conditions. When the deduction result is recognized, the lattice similarity of A and each A_j in Eq. (2) is calculated respectively, if (A, A_j) is maximum, then the grade of earthquake damage in the building belongs to A_j.

5. APPLICATION OF THE METHOD IN EARTHQUAKE DAMAGE PREDICTION OF SINGLESTOREY INDUSTRIAL MASONRY BUILDING

For illustrating the use of this method, we take earthquake damage information presented in reference [6] as an example. Information are given for 37 masonry buildings constructed at zone of 8 degree of earthquake intensity. The earthquake damage factors universe of discouse is considered as

$$U = \{u_1, u_2, u_3, u_4, u_5\}$$

where u_1 denotes the characteristic value of the major bearing structure of building; u_2 denotes the ratio of the length to the span of building; u_3 denotes the characteristic value of roof truss bracing; u_4 denotes the characteristic value of external nonbearing wall; u_5 denotes the characteristic value of site soil.

Values u_i and the grades of damage A_i for 37 buildings are listed in Table 1, where the definition of A_i are different to those in Eq(2), that is

A_1 = undamaged and slightly damaged = 0.45/0+0.86/0.1+1/0.2+0.65/0.3+0.2/0.4

A_2 = moderately damaged = 0.2/0.2+0.7/0.3+1/0.4+0.7/0.5+0.2/0.6

A_3 = severely damaged = 0.2/0.4+0.7/0.5+1/0.6+0.7/0.7+0.2/0.8

A_4 = collapse = 0.2/0.6+0.65/0.7+1/0.8+0.86/0.9+0.45/1.0 (19)

5.1 FORMATION OF THE FUZZY RELATIONSHIP MATRIX FOR ONE-VARIATED ANALYSIS

Taking the factor u_1 for example, according to the character of data in Table 1, we take discrete universe of discourse as

$$u_1 = (x_1, x_2, \cdots, x_6) = (0.081, 0.187, 0.293, 0.399, 0.505, 0.611)$$

For the building No. 4, the knowledge sample point is $D_4 = (0.419, A_4)$, where 0.419 is between 0.399 and 0.505, it means $x_4 < x < x_5$. From Eq. (9), we get

$$q_4(x_4) = 1 - \frac{0.419 - 0.399}{0.505 - 0.399} = 0.811, \qquad q_4(x_5) = 0.189$$

Similar operations are made for other buildings, then the initinal information distribution matrix is obtained using Eq. (11).

The fuzzy relationship matrix (Table 2) of the factor u_1 is formed by making unitized operation in row nd column direction of matrix according to Eqs. (12) and (13). Similar operations are made for other factors (u_2 to u_5), corresponding fuzzy relationship matrices are obtained and summarzed in Table 3 to 6.

5.2 ONE-VARIATED FUZZY DEDUCTION

Again we take the building No. 4 as an example. For the factor u_1, the elements of matrix R' in Eq. (14) are listed in Table 2, and X is given by

$$X = [0 \quad 0 \quad 0 \quad 0.811 \quad 0.189 \quad 0 \quad 0]$$

Substituting values of R' and X into Eq. (14), we get

Table 1. Earthquake Damage Information and Prediction Results

Number of Building	Considered u_1	u_2	u_3	Factors u_4	u_5	Damage Grades	Prediction Results in This Paper
1	0.598	3.50	0.50	0.75	0.45		A_4
2	0.598	3.50	0.50	0.75	0.45		A_4
3	0.300	1.00	0.65	0.40	0.46		A_4
4	0.419	7.15	0.50	0.68	0.46		A_4
5	0.419	7.15	0.50	0.72	0.46	A_4	A_4
6	0.250	5.25	0.50	0.60	0.40		A_4
7	1.203	2.85	0.50	0.50	0.40		A_4
8	0.574	2.50	0.30	0.70	0.38		A_3-A_4
9	0.375	5.30	0.30	0.35	0.40		A_4
10	0.300	2.52	0.30	0.35	0.30		A_3
11	0.340	2.40	0.50	0.35	0.40		A_3
12	0.301	2.29	0.40	0.60	0.25		A_3
13	0.500	2.10	0.45	0.50	0.35		A_3
14	0.880	5.30	0.10	0.40	0.25		A_3
15	0.590	3.60	0.10	0.25	0.40		A_3
16	0.230	4.20	0.30	0.70	0.25	A_3	A_3
17	0.291	2.50	0.50	0.40	0.30		A_3
18	0.149	3.20	0.35	0.61	0.30		A_3
19	0.540	2.13	0.40	0.62	0.30		A_3
20	0.238	2.40	0.30	0.52	0.30		A_3
21	0.359	2.18	0.25	0.55	0.30		A_3
22	0.621	2.35	0.50	0.30	0.30		A_3
23	0.151	1.70	0.30	0.50	0.35		A_3
24	0.410	1.60	0.55	0.60	0.35		A_3
25	0.350	2.00	0.50	0.30	0.25		A_3
26	0.668	3.20	0.10	0.60	0.25		A_2-A_3
27	0.211	3.50	0.10	0.20	0.25		A_2
28	0.332	2.00	0.50	0.60	0.30	A_2	A_3
29	0.162	3.50	0.50	0.60	0.25		A_2
30	0.181	1.70	0.50	0.70	0.25		A_2
31	0.269	2.30	0.10	0.70	0.15		A_1
32	0.210	3.10	0.10	0.20	0.25		A_1-A_2
33	0.101	1.20	0.10	0.20	0.20		A_1
34	0.121	1.50	0.50	0.20	0.15	A_1	A_1
35	0.141	2.00	0.50	0.70	0.15		A_1
36	0.362	2.50	0.50	0.30	0.15		A_3
37	0.102	3.10	0.50	0.30	0.25		A_1-A_2

A_1^1 = [0. 071 0. 138 0. 210 0. 266 0. 366 0. 565 0. 581 0. 655 0. 763 0. 602 0. 313]
Similar operation are made for other factors (u_2 to u_5), we get
A_2^1=[0. 000 0. 000 0. 040 0. 142 0. 332 0. 400 0. 485 0. 745 0. 869 0. 618 0. 323]
A_3^1=[0. 221 0. 424 0. 603 0. 711 0. 779 0. 828 0. 908 1. 000 0. 986 0. 742 0. 387]
A_4^1=[0. 093 0. 176 0. 272 0. 368 0. 458 0. 551 0. 626 0. 671 0. 731 0. 540 0. 282]
A_5^1=[0. 000 0. 000 0. 000 0. 000 0. 028 0. 087 0. 290 0. 716 1. 000 0. 833 0. 435]

5. 3 MULTIVARIATED FUZZY DEDUCTION AND ITS RECOGNITION

Gathering the results A_1^1 to A_5^1 of one-variated deduction, we obtain the multivariated fuzzy relationship matrix R defined in Eq. (16). Then multivariated fuzzy deduction are conducted using Eq. (15), where the weighting vector W is taken by experience of expert as

$$W = [0.23 \quad 0.22 \quad 0.20 \quad 0.16 \quad 0.19]$$

hence we have

A = [0. 075 0. 145 0. 221 0. 294 0. 392 0. 488 0. 577 0. 758 0. 871 0. 667 0. 348]

This is the deduction result of the building No. 4. The lattice similarity of this result with each A_t in Eq. (19) are calculated, among them $(A, A_4)=0.898$ is the maximum. Therefore, the final earthquake damage prediction result is that the building No. 4 will collapse in the case of suffering 8 degree of earthquake intensity. Similar operations are conducted for other buildings and their prediction results are listed in the last column of the Table 1.

Table 2. Fuzzy Relationship Matrix for the factor u_1

u \ V	0. 0	0. 1	0. 2	0. 3	0. 4	0. 5	0. 6	0. 7	0. 8	0. 9	1. 0
0. 081	0. 438	0. 839	0. 945	0. 565	0. 267	0. 197	0. 169	0. 118	0. 045	0. 000	0. 000
0. 187	0. 270	0. 517	0. 745	0. 897	0. 981	0. 974	0. 672	0. 466	0. 245	0. 090	0. 047
0. 293	0. 119	0. 228	0. 324	0. 380	0. 514	0. 792	1. 000	0. 932	0. 699	0. 459	0. 240
0. 399	0. 088	0. 170	0. 259	0. 328	0. 425	0. 609	0. 608	0. 702	0. 850	0. 660	0. 344
0. 505	0. 000	0. 000	0. 000	0. 000	0. 112	0. 378	0. 464	0. 452	0. 390	0. 352	0. 181
0. 611	0. 000	0. 000	0. 054	0. 191	0. 437	0. 771	0. 823	0. 851	0. 860	0. 505	0. 264

Table 3. Fuzzy Relationship Matrix for the factor u_2

u \ V	0. 0	0. 1	0. 2	0. 3	0. 4	0. 5	0. 6	0. 7	0. 8	0. 9	1. 0
1. 50	0. 407	0. 779	1. 000	0. 795	0. 480	0. 316	0. 293	0. 369	0. 415	0. 360	0. 188
2. 10	0. 114	0. 219	0. 316	0. 384	0. 547	0. 867	1. 000	0. 690	0. 248	0. 056	0. 029
2. 70	0. 214	0. 410	0. 486	0. 348	0. 288	0. 369	0. 484	0. 670	0. 686	0. 439	0. 229
3. 30	0. 199	0. 381	0. 604	0. 858	0. 762	0. 442	0. 333	0. 484	0. 738	0. 596	0. 311
3. 90	0. 000	0. 000	0. 040	0. 142	0. 332	0. 400	0. 485	0. 745	0. 869	0. 618	0. 323

Table 4. Fuzzy Relationship Matrix for the factor u_3

u \ V	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.10	0.381	0.730	0.704	0.613	0.523	0.471	0.367	0.195	0.056	0.000	0.000
0.20	0.000	0.000	0.000	0.000	0.018	0.059	0.077	0.049	0.014	0.000	0.000
0.30	0.000	0.000	0.000	0.000	0.142	0.471	0.693	0.630	0.487	0.429	0.263
0.40	0.000	0.000	0.000	0.000	0.107	0.353	0.459	0.292	0.084	0.0000	0.000
0.50	0.221	0.424	0.603	0.711	0.779	0.828	0.908	1.000	0.986	0.742	0.387

Table 5. Fuzzy Relationship Matrix for the factor u_4

u \ V	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2000	0.416	0.797	1.000	0.866	0.485	0.259	0.183	0.127	0.051	0.000	0.000
0.3375	0.158	0.303	0.392	0.369	0.373	0.452	0.576	0.705	0.741	0.493	0.258
0.4750	0.000	0.000	0.012	0.044	0.247	0.503	0.676	0.733	0.503	0.275	0.144
0.6125	0.046	0.087	0.199	0.407	0.675	0.931	1.000	0.789	0.472	0.262	0.137
0.7500	0.141	0.269	0.348	0.328	0.233	0.157	0.238	0.548	1.000	0.828	0.432

Table 6. Fuzzy Relationship Matrix for the factor u_5

u \ V	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.15	0.449	0.861	1.000	0.490	0.112	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.123	0.237	0.411	0.657	0.886	1.000	0.892	0.558	0.203	0.047	0.025
0.35	0.000	0.000	0.013	0.046	0.250	0.652	1.000	0.852	0.499	0.271	0.142
0.45	0.000	0.000	0.000	0.000	0.028	0.087	0.290	0.716	1.000	0.833	0.435

6. CONCLUSIONS

The multivariated analysis model for building earthquake damage prediction presented in this paper is of the following characters. The fuzzy relationship matrix of one—variated analysis formed by information distribution method does not contain any mathematical assumptions and can more objectively reflect the historical earthquake damage information. It can consider not only theoretical value of earthquake response of structure (as one factor) but also the comprehensive effect of other factors on prediction result. When the deduction result is recognized by lattice similarity, we can more explicitly obtain the grade of earthquake damage in building.

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