An Approach to Controllable of Fuzzy Algorithms Systems

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Abstract: The fuzzy system which be expressed as the following discrete time equation

$$X_{t+1}=F(U_t,X_t)$$
$$U_t=H(X_t)$$

is called a fuzzy algorithms system. In this paper, we presented a weaker defintion of the difference measure of fuzzy states. On the basis of this difference measure, the concept of ε-convergent was defined, and the control problems of the homogeneous finite states fuzzy algorithms systems was discussed.

Keywords: Fuzzy set, Fuzzy algorithms system, Fuzzy relation, Difference measure, Fuzzy control.

1. Introduction

The approach of the fuzzy systems being with Zadeh and Chang's work[1],[2]. After this, there are lot of further works by many scholars in the fuzzy systems field. Howerver, in the dynamical analysis of fuzzy systems, the description of convergent concept of a fuzzy system's states sequence $\{X_t \mid t=0,1,2,\ldots\ldots\}$ (X_t is a fuzzy set on $\mathfrak{X}$) is very important. but a integrated definition is not still given, at present. Let X be a fuzzy state of the system, $X\in \mathfrak{F}(\mathfrak{X})$, Negoita and Ralescu[3] (1975) suggested the definition which a fuzzy states sequence $\{X_t\}$ converge to X as follows: there is a t', when t>t' have $X_t=X$. Obviously, this definition is too strong. Tong[4] (1978) proposed to replace $X_t=X$ by the equation of the peak pattern of fuzzy states $hgt(X_t)=hgt(X)$; But Chang and Zadeh used the inclusion relation $X_t\subseteq X$ or $hgt(X_t)\subseteq hgt(X)$ for expressing this convergence; Dubois and prade[5] suggested to depict it using the weak inclusion relation or the inequality $hgt(X_t\cap X) > \varepsilon$, where $\varepsilon\in[0,1]$ is a given small number; When the set $\mathfrak{X}$ is a metric space with distance d, for a definite nonfuzzy set X, $d(X,\varepsilon)$ is the ε-neighborhood of X, then Glas[6] suggested that using $[X_t]_\alpha\subseteq d(X,\varepsilon)$ express the convergence of a system states sequence $\{X_t\}$, where $[A]_\alpha$ denotes an α-cut of the fuzzy set A. In consideration of the specific property of applicability of fuzzy systems, if there is not a definition of the distance on $\mathfrak{X}$, then Glas's way will be useless.

In this paper, we presented a weaker definition of difference measure of fuzzy states on fuzzy power set $\mathfrak{F}(\mathfrak{X})$ of the set $\mathfrak{X}$. If all states of a system are the fuzzy subsets on $\mathfrak{X}$, and a difference measure has been defined at $\mathfrak{F}(\mathfrak{X})$, then $\mathfrak{F}(\mathfrak{X})$ is called a fuzzy state space. In this paper, we shall use the concept of difference measure for expressing the convergence of a fuzzy states sequence $\{X_t\}$, and discuss the control problems of the fuzzy algorithms systems.

2. Mathematical expression of fuzzy algorithms systems

A fuzzy algorithms system $\mathfrak{A}$ may be described as the following equations

$$X_{t+1}=F(U_t, X_t), \tag{1}$$

$$U_t=H(X_t). \tag{2}$$

Where X_t and U_t are respectively fuzzy state and input of the system $\mathfrak{A}$ at time t, X_{t+1} is a fuzzy state of the system $\mathfrak{A}$ at time t+1. Here t is an integer time index. The state X_t is a fuzzy set defined on a space (universe) $\mathfrak{X}$ and the input U_t is a fuzzy set defined in a space $\mathfrak{U}$, we denote $X_t \in \mathfrak{F}(\mathfrak{X})$, $U_t \in \mathfrak{F}(\mathfrak{U})$. Their membership functions are denoted as $X_t(x)$ and $U_t(u)$, respectively, $x \in \mathfrak{X}$, $u \in \mathfrak{U}$.

The mapping F describes that state of this system at time t+1 depend on its state and input at time t. H describes that input of the system depend on interior state of the system at same time, i.e.,

$$F: \mathfrak{F}(\mathfrak{U}) \times \mathfrak{F}(\mathfrak{X}) \to \mathfrak{F}(\mathfrak{X}), \tag{3}$$

$$H: \mathfrak{F}(\mathfrak{X}) \to \mathfrak{F}(\mathfrak{U}). \tag{4}$$

The fuzzy system $\mathfrak{A}$ may be seen as a closed loop fuzzy system, and assume that the system is observable. We define U_t as control of the system $\mathfrak{A}$, it only depend on states of the system at same time. The fuzzy algorithms systems described above and its control problems has a lot of practical case. Let $\mathfrak{X}$ and $\mathfrak{U}$ be all finite set, denoted as $\mathfrak{X}=\{x_1, x_2, \ldots, x_n\}$, $\mathfrak{U}=\{u_1, u_2, \ldots, u_m\}$, then $\mathfrak{A}$ is called a finite states fuzzy algorithms system (FSFAS). Let δ be a fuzzy relation on $\mathfrak{U}\times\mathfrak{X}\times\mathfrak{X}$, i.e.,

$$\delta: \mathfrak{U}\times\mathfrak{X}\times\mathfrak{X} \to [0, 1] \tag{5}$$

When $\mathfrak{X}$ and $\mathfrak{U}$ are finite set, δ may be written a three-dimensional fuzzy relation matrix $\delta=[r_{ijk}]_{m\times n\times n}$. We call δ as a fuzzy transition matrix of states of the system $\mathfrak{A}$,

$r_{ijk} \in [0, 1]$ show a grade of possibility of transfering state X_j to state X_k for the system under condition of that system has a fixer input u_i.

Definition 1. If the fuzzy transition matrix δ of a system $\mathfrak{A}$ is independent of time t, i.e., for any $i=1, 2, \ldots, m$; $j, k=1, 2, \ldots, n$, r_{ijk} is a constant, then the system $\mathfrak{A}$ is called a homogeneous system.

This paper will main discuss the homogeneous finite states fuzzy algorithms systems (HFSFAS). In order to convenient for after discusion, we introduce some useful symbols first.

For any given $U_i \in \mathscr{U}$, $X_j \in \mathscr{X}$, we denoted that

$$\delta_{ui,\cdot,\cdot} = \begin{vmatrix} r_{i11} & r_{i12} & \cdots & r_{i1n} \\ r_{i21} & r_{i22} & \cdots & r_{i2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{in1} & r_{in2} & \cdots & r_{inn} \end{vmatrix}, \quad \delta_{ui,xj,\cdot} = (r_{ij1} \; r_{ij2} \; \cdots \; r_{ijn}) \tag{6}$$

$\delta_{ui,\cdot,\cdot} \in \mathscr{F}(\mathscr{X} \times \mathscr{X})$ is a fuzzy transition matrix of states under condition of that system input is u_i. $\delta_{ui,xj,\cdot} \in \mathscr{F}(\mathscr{X})$ is called a fuzzy transition distribution of state x_j under condition of that system input is u_i.

For given fuzzy state $X \in \mathscr{F}(\mathscr{X})$, under input u_i, define the fuzzy transition distribution of fuzzy state X as the following

$$\delta_{ui,X,\cdot} = X \circ \delta_{ui,\cdot,\cdot} \tag{7}$$

$$\delta_{ui,X,\cdot}(x) = \bigvee_{j=1}^{n} [X(x_j) \wedge \delta_{ui,xj,\cdot}(x)], \quad \forall x \in \mathscr{X}$$

where $\wedge, \vee$ stand for sup (max) and inf (min) operatiors respectively. We denote

$$\delta_{\cdot,X,\cdot} = \begin{vmatrix} \delta_{u1,X,\cdot} \\ \delta_{u2,X,\cdot} \\ \vdots \\ \delta_{um,X,\cdot} \end{vmatrix} \triangleq X \circ \delta, \tag{8}$$

then the fuzzy transition distribution of fuzzy state X under condition of that system has a fuzzy input $U \in \mathscr{F}(\mathscr{U})$ is obtained as

$$\delta_{U,X,\cdot} = U \circ \delta_{\cdot,X,\cdot} \tag{9}$$

$$\delta_{U,X,\cdot}(x) = \bigvee_{j=1}^{m} [U(ui) \wedge \delta_{ui,X,\cdot}(x)]$$

Denote that $X=X_t, U=U_t, \quad \delta_{U_t, X_t, \cdot}=X_{t+1}$, Equation (1) may be written the following

$$X_{t+1}=U_t \circ \delta_{\cdot, X_t, \cdot}=U_t \circ (X_t \circ \delta)$$

$$X_{t+1}(x)=\bigvee_{i=1}^{m} \bigvee_{j=1}^{n} [U_t(u_i) \wedge X_t(x_j) \wedge \delta(u_i, x_j, x)] \tag{10}$$

Then a HFSFAS may be described as the following equations

$$X_{t+1}=U_t \circ (X_t \circ \delta) \tag{11}$$

$$U_t=H(X_t) \tag{12}$$

3. Difference measure of fuzzy states and fuzzy state space

In order to show dynamical properties of the fuzzy algorithms systems , it is necessary that we set up a definition of measure representing the difference between fuzzy states. This measure is not unique, it accordingly as distinct aim of fuzzy systems analysis . Thus we give a weaker definition as the following.

Definition 2. Let A, B be two fuzzy set on X. $\| A-B \|$ is called a difference measure of A and B if it satisfies the following conditions:

1°. $\| A-B \| \geqslant 0$, if A is a normal fuzzy set $(\exists\, x \in X, A(x)=1)$, then $\| A-A \|=0$; (13)

2°. $\| A-B \| = \| B-A \|$ (14)

3°. $\| A_1 \cup A_2 - B \| = \| A_1 - B \| \wedge \| A_2 - B \|$ (15)

Definition 3. Let $\mathfrak{X}$ be the states space of system $\mathfrak{A}$ If the definition of difference measure between any two sets on $\mathfrak{F}(\mathfrak{X})$ is given, then call $(\mathfrak{F}(\mathfrak{X}), \| \cdot \|)$ as a fuzzy state space of the system $\mathfrak{A}$.

By the conditions (13)-(15) , generally speaking, when $\| A-B \|=0$, we cannot deduce that A=B. The difference measure on $\mathfrak{F}(\mathfrak{X})$ satisfies the following properties .

Property 1. Let A_1, A_2 and B be fuzzy sets on $\mathfrak{X}$, and $A_1 \supseteq A_2$, then have $\| A_1-B \| \leqslant \| A_2-B \|$

Proof : Because $A_1 \supseteq A_2$ implies $A_1=A_1 \cup A_2$, by (15) we have

$$\| A_1-B \| = \| A_1 \cup A_2 - B \| = \| A_1-B \| \wedge \| A_2-B \| \leqslant \| A_2-B \|$$

Property 2. $\forall A_1, A_2, B \in \mathfrak{F}(\mathfrak{X})$, $\| A_1 \cap A_2 - B \| \geqslant \| A_1-B \| \vee \| A_2-B \|$.

Proof: Because $A_1 \supseteq A_1 \cap A_2$, $A_2 \supseteq A_1 \cap A_2$. by Propery 1 we have $\| A_1-B \| \leqslant \| A_1 \cap A_2 - B \|$ and $\| A_2-B \| \leqslant \| A_1 \cap A_2 - B \|$, then $\| A_1 \cap A_2 - B \| \geqslant \| A_1-B \| \vee \| A_2-B \|$.

Property 3. Suppose that A and B are all normal fuzzy sets on $\mathscr{X}$ and $A \supseteq B$, then $\| A-B \| = 0$.

Proof: From Definition 2 we may establish the following $\| A-B \| = \| A \cup B-B \| = \| A-B \| \wedge \| B-B \| = \| A-B \| \wedge 0$. And because $\| A-B \|$ is not a negative number, then $\| A-B \| = 0$.

Definition 4. Let $\{X_t | X_t \in \mathscr{F}(\mathscr{X}),\ t=1,2,\ldots\}$ be a fuzzy sets sequence, $X' \in \mathscr{F}(\mathscr{X})$ a given fuzzy set and $\varepsilon \in [0,1]$. We say that the fuzzy sets seqnence $\{X_t\}$ is the ε - convergent for the fuzzy set X' if there exists a index T, for all $t > T$ such that $\| X_t - X' \| < \varepsilon$.

4 . Control problems of HFSTAS

In the fuzzy system which can be experssed as (11) and (12), we suppose that δ is a known fuzzy relation matrix. At this time, if the mapping $H: \mathscr{F}(\mathscr{X}) \to \mathscr{F}(\mathscr{U})$ is once determinated, then a fuzzy input sequence $\tilde{U} = \{U_0, U_1, \ldots, U_T, \ldots\}$ may be obtainned by the control H and a initial state $X_0 \in \mathscr{F}(\mathscr{X})$ of the system . The fuzzy states sequence $\tilde{X} = \{X_1, X_2, \ldots, X_{T+1}, \ldots\}$ of the system is also obtained at same time . The sequence $\tilde{X}$ is called the system dynamic response, it may be seen as "a motional locus" on the system fuzzy states space. The dynamical characters of HFSFAS can be shown by this "locus".

From above we easy to see that the change process of $\tilde{X}$ depend on the input sequence $\tilde{U}$ of system $\mathbb{A}$ when the fuzzy transition matrix δ and the initial state X_0 are given. But $\tilde{U}$ depend on the mapping H, then we say that the system dynamic response $\tilde{X}$ depend on the mapping H. According to above discussion we denote the system dynamic response $\tilde{X}$ as $\tilde{X}(H, X_0)$.

We denote $\overline{\mathscr{F}(\mathscr{X})}$ as a collective of all normal fuzzy sets on $\mathscr{X}$, have $\overline{\mathscr{F}(\mathscr{X})} \subset \mathscr{F}(\mathscr{X})$.

Defintion 5. Let $X_0, X' \in \mathscr{F}(\mathscr{X})$ be respectively given inital state and final state of HFSFAS $\mathbb{A}$. The fuzzy system $\mathbb{A}$ is the ε-controllable from X_0 to X' if there exists mapping $H: \mathscr{F}(\mathscr{X}) \to \mathscr{F}(\mathscr{U})$ such that $\tilde{X}(H, X_0)$ is the ε-convergent about the final state X', i.e., there exists a time index T, for all $t > T$ has that

$$\| X_t - X' \| < \varepsilon, \quad X_t \in \tilde{X}(H, X_0) \tag{16}$$

The minimal T satesfing Formla (16) is called the control step length of H from X_0 to X', denote $T = sl(H)$; The fuzzy system $\mathbb{A}$

is complete ε-controllable if $\mathbf{A}$ is the ε-controllable from any initial state $X_0 \in \overline{\mathfrak{F}(\mathcal{X})}$ to any final state $X' \in \overline{\mathfrak{F}(\mathcal{X})}$.

Definition 6. Let $\mathcal{H}$ be a family of all mapping from $\mathfrak{F}(\mathcal{X})$ to $\mathfrak{F}(\mathcal{U})$. Suppose that a HFSFAS is the ε - controllable from an initial state X_0 to a final state X' $(X_0, X' \in \overline{\mathfrak{F}(\mathcal{X})})$. $H \in \mathcal{H}$ is called a ε-optimal control (for the time) from X_0 to X' if the control step length T of H as the following

$$T=\min\{T' \mid T'=sl(H'), \forall H' \in \mathcal{H}\}.$$

Theorem 1. A HFSFAS is the ε-controllable from X_0 to X' $(X_0, X' \in \overline{\mathfrak{F}(\mathcal{X})})$ if and only if there exists a $k \leqslant n$, such that

$$\| X_0 \, \triangle^k - X' \| \leqslant \varepsilon \tag{17}$$

where $\triangle = \bigcup_{i=1}^{m} \delta_{u_i}, \ldots,$ $\triangle^k = \underbrace{\triangle \circ \triangle \circ \ldots \circ \triangle}_{k}$.

Proof: By (6) we know that $\triangle$ is a binary fuzzy relation on $\mathcal{X}$, denote $\triangle = [\overline{r_{ij}}]_{n\times n}$, where $\overline{r_{ij}}$ is a grade of possibility of transfering the system's state x_i to x_j by one step under all possible exact inputs. Denote $\triangle^p = [\overline{r_{ij}}^p]_{n\times n}$, $\overline{r_{ij}}^p$ shows a grade of possibility of transfering the system's state x_i to x_j under all possible p-step exact input $u^{(1)} u^{(2)} \ldots u^{(p)}$ $(u^{(i)} \in \mathcal{U},\ i=1, 2, \ldots, p)$. Thus the fuzzy matrix series $\bigcup_{p=1}^{\infty} \triangle^p$ shows the grade of possibility of the states transition under all possible input forms (including every step lengths).

For $X_0 \in \overline{\mathfrak{F}(\mathcal{X})}$, by (7) we know that $X_0 \circ (\bigcup_{p=1}^{\infty} \triangle^p) \in \mathfrak{F}(\mathcal{X})$ shows the fuzzy transition distribution of fuzzy state X_0 under all possible input forms.

According to the exposition of $\triangle^K$, the sufficiency of Theorem 1 is obvious. We only proof the necessity.

Because the system is the ε-controllable from X_0 to X', then must has that

$$\| X_0 \circ (\bigcup_{p=1}^{\infty} \triangle^p) - X' \| \leqslant \varepsilon \tag{18}$$

For n order fuzzy square matrix $\triangle$, the following formula

$$\bigcup_{p=1}^{\infty} \triangle^p = \bigcup_{p=1}^{n} \triangle^p$$

is hold according as theory of fuzzy matrix series. Thus that

$$\| X_0 \circ (\bigcup_{p=1}^{\infty} \triangle^p) - X' \| = \| X_0 \circ (\bigcup_{p=1}^{n} \triangle^p) - X' \| = \| \bigcup_{p=1}^{n} X_0 \circ \triangle^p - X' \|$$

$$= \bigwedge_{p=1}^{n} \| X_0 \circ \triangle^p - X' \| \tag{19}$$

By (19), there exists a $k \leqslant n$, has

$$\| X_0 \circ (\bigcup_{p=1}^{n} \triangle^p) - X' \| = \| X_0 \circ \triangle^k - X' \| \leqslant \varepsilon$$

This completes our proof.

We denote binary fuzzy relation matrix on $\mathcal{X}$ as

$$R^* = \bigcup_{p=1}^{n} \triangle^p = [r_{ij}^*]_{n \times n}$$

Let $R_i \in \mathcal{F}(\mathcal{X})$ be ith row of R^*, it may be seen as a fuzzy set on $\mathcal{X}$, i. e., $R_i \in \mathcal{F}(\mathcal{X})$,

$$R_i = r_{i1}^*/x_1 + r_{i2}^*/x_2 + \ldots + r_{in}^*/x_n \tag{20}$$

And let B_{ik} and I_k be two fuzzy states on $\mathcal{X}$ and I_k a normal fuzzy set, $I_k \in \overline{\mathcal{F}(\mathcal{X})}$ with membership functions

$$B_{ik}(x) = \begin{cases} r_{ik}^*, & x = x_k, \\ 0, & \text{Othrerwise.} \end{cases} \qquad I_k(x) = \begin{cases} 1, & x = x_k, \\ 0, & \text{Otherwise.} \end{cases} \tag{21}$$

$i, k = 1, 2, \ldots, n.$ Obviously, has $R_i = \bigcup_{k=1}^{n} B_{ik}$.

Theorem 2. A HFSFAS is complete ε-controllable if

$$\max \| B_{ik} - I_k \| \leqslant \varepsilon,$$
$$i, k = 1, 2, \ldots, n$$

where B_{ik} and I_k are definde as (21).

Proof: For given arbitrary inital state $X_0 \in \overline{\mathcal{F}(\mathcal{X})}$ and final state $X' \in \overline{\mathcal{F}(\mathcal{X})}$, we might suppose that $x_1 \in \mathcal{X}$ is the kernel of fuzzy set X_0, $x_k \in \mathcal{X}$ the kernel of fuzzy set X', then $X_0 \supseteq I_1$, $X' \supseteq I_k$.

By Property 2 and the following equality

$$I_1 \circ (\bigcup_{p=1}^{n} \triangle^p) = R_1 = \bigcup_{k=1}^{n} B_{1k}$$

$$\| X_0 \circ (\bigcup_{p=1}^{n} \triangle^p) - X' \| \leqslant \| I_i \circ (\bigcup_{p=1}^{n} \triangle^p) - X' \| \leqslant \| B_{ik} - X' \| \leqslant \| B_{ik} - I_k \| .$$

Because $\| B_{ik} - I_k \| \leqslant \varepsilon$, so we get

$$\| X_0 \circ (\bigcup_{p=1}^{n} \triangle^p) - X' \| \leqslant \varepsilon .$$

It shown that the system is the ε-controllable from X_0 to X' based on Theorem 1, and X_0, X' are arbitrary normal fuzzy sets. For reasons given above, we shown that system is complete ε-controllable.

Corollary: Denote that $R^* = \bigcup_{p=1}^{n} \triangle^p = [r_{ij}^*]_{n \times n}$. if $r_{ij}^* = 1$, $\forall i, j = 1, 2, \ldots n$, then the system is complete ε-controllable.

Theorem 3. (Existentially of ε – optimal control) If a HFSFAS A is the ε-controllable from X_0 to X', then the optimal control H exists in the system $\mathbb{A}$, and the ε - optimal control step length equal to the minimum exponent k satisfying the inequality $\| X_0 \circ \triangle^k - X' \| \leqslant \varepsilon$.

Proof: If a HFSFAS is the ε-controllable, then by Theorem 1 there must exists a $k \leqslant n$, such that when $T<K$ has $\| X_0 \circ \triangle^T - X' \| > \varepsilon$ and $\| X_0 \circ \triangle^k - X' \| \leqslant \varepsilon$. It shown that there exists an input string $u^{(1)}, u^{(2)}, \ldots u^{(k)}$ such that transform the system's states from inital state X_0 to final state X' and has $\| X_k - X' \| \leqslant \varepsilon$. We take the system's fuzzy input as $U = \{U_0, U_1, \ldots, U_{K-1}\}$, where

$$U_T(u) = \begin{cases} 1, & u = u^{(T+1)}, \\ 0, & \text{Otherwise.} \end{cases} \qquad U_T = H(X_T). \quad T = 0, 1, 2, . k-1.$$

It is easy to see that H is an optimal control and the step length equal to k.

Although we can show that a fuzzy system is the ε-controllable from a given fuzzy state X_0 to X' $(X_0, X' \in \mathscr{F}(\mathscr{X}))$, but a key problem of transfering system's state X_0 to X' such that $\| X - X' \| \leqslant \varepsilon$ under a finite input step lengths depend on whether right control H is sought or not. When selected the control H is not suitable, the control sept lengths will increase or the system will out of control . From above analysis we know that to set up control H depend on the system's dynamical character δ. In some practical applications, δ can be obtained by experiment and statistical methods.

In present some practical applications, the control $H: \mathfrak{F}(\mathfrak{X}) \rightarrow \mathfrak{F}(\mathfrak{U})$ is often given using artificial experiences. First a group control rules 'if $\mathfrak{X}$ is X_i, then $\mathfrak{U}$ is U_i' $(i=1, 2, \ldots, p)$ is sumed up according to practical experiences, By Mamdani's method [8], the system's control H may be expressed as a fuzzy relation on $\mathfrak{X} \times \mathfrak{U}$

$$H = \bigcup_{i=1}^{p} (X_i \times U_i) \tag{22}$$

And a HFSFAS will be written the following

$$X_{t+1} = U_t \circ (X_t \circ \delta) \tag{23}$$
$$U_t = X_t \circ H \tag{24}$$

From (23) and (24), a HFSFAS is also written as $X_{t+1} = (X_t \circ H) \circ (X_t \circ \delta)$.

R.M. Tong had discussed some properities of the fuzzy system with Form (24) in paper [9].

When H is the conclusion of artificial experiences, recured system dynamic response $X(H, X_0)$ of by (22) is not usual an optimal sequence from X_0 to X'. In order to evaluate the control H, we set up a concept of grade of optimal control.

Definition 7. Suppose that optimal control step lengths of a HFSFAS equal to k, for a given control H, T is a minimum time index satisfying the inequality

$$\| (X_{t-1} \circ H) \circ (X_{t-1} \circ \delta) - X' \| \leqslant \varepsilon,$$

then

$$\rho = \frac{k}{T}$$

is called the grade of optimal control (for time) of H from X_0 to X'.

It is easy to see that $0 \leqslant \rho \leqslant 1$.

References

[1] L.A.Zadeh, Toward a theory of fuzzy systems, New York, 1971.

[2] S.S.Chang and L.A.Zadeh, On fuzzy mapping and control, IEEE Trans. Systems, Man Cybernet, SMC 2 (1972), 30-34.

[3] C.V.Negoita and D.A.Ralescu, Application of fuzzy sets to systems analysis, Birkhauser-Verlag, Basel (1975).

[4] R.M.Tong, Analysis and control of fuzzy system using finite discrete relations, Int. J.Control 27, No.3, (1978) 431-440.

[5] D.Dubois and H.Prade, Fuzzy sets and systems: Theory and application, Academic press, New York, 1980.

[6] M.de Glas, A mathematical theory of fuzzy systems, Fuzzy Information and Decision Processes , M.M. Gupta and E.Sanchez eds., 401-410, North-Holland Publishing Company, 1982.

[7] M.Mizumoto, Fuzzy algebra and application (Chinese translation), Science Publishing Company, Bei Jing, (1986).

[8] E.H.Mamdani, Application of fuzzy logic to approximate reasoning using linguistic synthesis, IEEE Trans. Comput., 26 (1977), 1182-1191.

[9] R.M.Tong, Some properties of fuzzy feedback systems, IEEE Trans. Syst., Man and Cyberentics, Vol, SMC-10, No.6, (1980), 327-330.