FUZZY ALMOST WEAK CONTINUITY

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ABSTRACT

In this paper we introduce and study a new weaker form of fuzzy continuity---fuzzy almost weak continuity. And it is also weaker forms of both fuzzy weak continuity and fuzzy almost semicontinuity.

Key words : Fuzzy semiopen set; Fuzzy almost semicontinuous mapping; Fuzzy weakly semicontinuous mapping; Fuzzy weakly continuous mapping. Fuzzy almost weakly continuous mapping.

1. PRELIMINARIES

In this work, A°, A^{-}, A_{o}, A_{-} and A' will denote respectively the interior, closure, semi-interior, semi-closure and complement of the fuzzy set A. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is called (1) a fuzzy weakly semicontinuous mapping if $f^{-1}(B) \leqslant (f^{-1}(B_{-}))_{o}$ for each $B \in \delta_2$ [3]. (2) a fuzzy almost semicontinuous mapping if $f^{-1}(B)$ is a fuzzy semiopen set of X_1 for each fuzzy regular open set B of X_2 [2].

2. FUZZY ALMOST WEAKLY CONTINUOUS MAPPINGS

Definition 1. A mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$ from a fuzzy space X_1 to another fuzzy space X_2 is called a fuzzy almost weakly continuous mapping if $f^{-1}(B) \leqslant (f^{-1}(B^{-}))_{o}$ for each $B \in \delta_2$.

Theorem 1. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost weakly continuous.

(2) $(f^{-1}(B^{\circ}))_{-} \leqslant f^{-1}(B)$ for each fuzzy closed set B of X_2.

(3) $f^{-1}(B^{\circ}) \leqslant (f^{-1}(B^{-}))_{\circ}$ for each fuzzy set B of X_2.

(4) $(f^{-1}(B^{\circ}))^{-} \leqslant f^{-1}(B^{-})$ for each fuzzy set B of X_2.

(5) there is a base η in δ_2, $f^{-1}(B) \leqslant (f^{-1}(B^{-}))_{\circ}$ for each $B \in \eta$.

(6) there is a base η in δ_2, $(f^{-1}(B^{\circ}))^{-} \leqslant f^{-1}(B)$ for each $B' \in \eta$.

Definition 2. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is said to be fuzzy almost weakly continuous at a fuzzy point p in X_1 if $B \in \delta_2$ and $f(p) \leqslant B$, there exists a fuzzy semiopen set A in X_1 such that $p \leqslant A$ and $f(A) \leqslant B^{-}$.

Theorem 2. A mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$ is fuzzy almost weakly continuous iff f is fuzzy almost weakly continuous for each fuzzy point p in X.

Proposition 1. Let $f: X \to Y$ be a mapping. Then the following statements are valid:

(1) f is fuzzy weakly continuous. Then f is also fuzzy almost weakly continuous.

(2) f is fuzzy weakly semicontinuous. Then f is also fuzzy almost semicontinuous.

(3) f is fuzzy almost semicontinuous. Then f is also fuzzy almost weakly continuous.

Proposition 2. Let $f: X \to Y$ and $g: Y \to Z$ be mappings. If f is fuzzy almost weakly continuous and g is fuzzy continuous, then $g \circ f$ is a fuzzy almost weakly continuous.

Theorem 3. Let $f: X \to Y$ be a mapping from a fuzzy space X to a fuzzy regular space Y. Then f is fuzzy almost weakly cotinuous iff f is fuzzy semicontinuous.

Lemma 1. Let X and Y be fuzzy spaces such that X is product related to Y. Then the product $A \times B$ of a fuzzy semiclosed set A of X and a fuzzy semiclosed set B of Y is a fuzzy semiclosed set of the fuzzy product space $X \times Y$.

Theorem 4. Let X and Y be fuzzy spaces such that X is product related to Y. Then for a fuzzy set A of X and a fuzzy set B of Y,

(1) $A_{\circ} \times B_{\circ} \leqslant (A \times B)_{\circ}$.

(2) $(A\times B)^{-}\leqslant A^{-}\times B^{-}$.

Theorem 5. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2 and Y_1 is to Y_2. Then the product $f_1\times f_2$: $X_1\times X_2\to Y_1\times Y_2$ of fuzzy almost weakly continuous mapping $f_1: X_1\to Y_1$ and $f_2: X_2\to Y_2$ is fuzzy almost weakly continuous.

Theorem 6. Let X, X_1 and X_2 be fuzzy spaces and $p_i: X_1\times X_2\to X_i$ $(i=1, 2)$ be the projection of $X_1\times X_2$ onto X_i. Then if $f: X\to X_1\times X_2$ is a fuzzy almost weakly continuous, $p\circ f$ is also a fuzzy almost weakly continuous.

Theorem 7. Let X and Y be fuzzy spaces such that X is product related to Y and let $f: X\to Y$ be a mapping. Then, if the graph $g: X\to X\times Y$ of f is fuzzy almost weakly continuous, f is also a fuzzy almost weakly continuous.

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