FUZZY ALMOST PRECONTINUITY

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ABSTRACT

In this paper we introduce the fuzzy almost precontinuous mapping on fuzzy topological spaces, and also establish some of its characteristic properties, and discuss relations between it and some other mappings.

Key words: Fuzzy regular open set; Fuzzy preopen set; Fuzzy almost continuous mapping; Fuzzy precontinuous mapping; Fuzzy almost precontinuous mapping.

1. PRELIMINARIES

In this work, A^o, A^- and A' will denote respectively the interior, closure and complement of the fuzzy set A. Let A be a fuzzy set of a fuzzy space X. Then A is called (1) a fuzzy preopen set of X iff $A \leqslant A^{-o}$; (2) a fuzzy preclosed set of X iff $A \geqslant A^{o-}$. A mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$ from a fuzzy space X_1 to another fuzzy space X_2 is called a fuzzy precontinuous mapping if $f^{-1}(B)$ is a fuzzy preopen set of X_1 for each $B \in \delta_2$ [4].

2. FUZZY ALMOST PRECONTINUOUS MAPPINGS

Definition 1. A mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$ from a fuzzy space X_1 to another fuzzy space X_2 is called a fuzzy almost precontinuous mapping if $f^{-1}(B)$ is a fuzzy preopen set of X_1 for each fuzzy regular open set B of X_2.

Remark 1. For the mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$, the following statements are valid:

(1) f is fuzzy precontinuous=> f is fuzzy almost precontinuous;

(2) f is fuzzy almost continuous=>f is fuzzy almost precontinuous;

(3) fuzzy almost precontinuous and fuzzy weakly continuous (fuzzy almost semicontinuous[2]) mappings are independent notions.

Theorem 1. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost precontinuous.

(2) $f^{-1}(B)$ is a fuzzy preclosed set of X_1 for each fuzzy regular closed set B of X_2.

(3) $(f^{-1}(B^{o-}))^{o-} \leqslant f^{-1}(B)$ for each fuzzy closed set B of X_2.

(4) $f^{-1}(B) \leqslant (f^{-1}(B^{-o}))^{-o}$ for each $B \in \delta_2$.

(5) there is a base η for δ_2 such that $f^{-1}(B) \leqslant (f^{-1}(B^{-o}))^{-o}$ for each $B \in \eta$.

(6) there is a base η for δ_2 such that $(f^{-1}(B^{o-}))^{o-} \leqslant f^{-1}(B)$ for each $B' \in \eta$.

(7) $(f^{-1}(B))^{o-} \leqslant f^{-1}(B^{-})$ for each fuzzy semiopen set B of X_2.

(8) $f^{-1}(B^{o}) \leqslant (f^{-1}(B))^{-o}$ for each fuzzy semiclosed set B of X_2.

Definition 2. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is said to be fuzzy almost precontinuous at a fuzzy point p in X_1, if fuzzy regular open set B in X_2 and $f(p) \leqslant B$, there exists a fuzzy preopen set A in X_1 such that $p \leqslant A$ and $f(A) \leqslant B$.

Theorem 2. A mapping $f: (X_1, \delta_1) \to (X_2, \delta_2)$ is fuzzy almost precontinuous iff f is fuzzy almost precontinuous for each fuzzy point p in X_1.

Proof. Let f be fuzzy almost precontinuous, p be a fuzzy point in X_1 and B be a fuzzy regular open set in X_2 such that $f(p) \leqslant B$. Then $p \leqslant f^{-1}(B) \leqslant (f^{-1}(B))^{-o}$. Let $A = f^{-1}(B)$, then A is fuzzy preopen set in X_1, and so $f(A) = ff^{-1}(B) \leqslant B$. Thus f is fuzzy almost precontinuous for each fuzzy point p in X_1.

Conversely, let B be a fuzzy regular open set in X_2 and p be a fuzzy point in $f^{-1}(B)$. Then $p \leqslant f^{-1}(B)$, i.e., $f(p) \leqslant B$. From hypothesis there exists a fuzzy preopen set A in X_1 such that $p \leqslant A$ and $f(A) \leqslant B$.

hence $p \leqslant A \leqslant f^{-1}f(A) \leqslant f^{-1}(B)$ and $p \leqslant A \leqslant A^{-o} \leqslant (f^{-1}(B))^{-o}$. Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$, $f^{-1}(B) \leqslant (f^{-1}(B))^{-o}$. Thus f is fuzzy almost precontinuous.

Theorem 3. Let $f: (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to a fuzzy semiregular space X_2[1]. Then f is fuzzy almost precontinuous iff f is fuzzy precontinuous.

Theorem 4. Let X_1, X_2, Y_1 and Y_2 be fuzzy spaces such that X_1 is product related to X_2 and Y_1 is to Y_2. Then the product $f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2$ of fuzzy almost precontinuous mappings $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$ is fuzzy almost precontinuous.

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