Connectedness on L-Fuzzy Syntopogenous Spaces *)

Liu Wang-jin and Mo Zhi-wen

Department of Mathematics, Sichuan Normal University

Chengdu, P.R.China

In this paper, it is studied that the relations of L-fuzzy syntopogenous spaces and L-fuzzy topological spaces. And the connectedness of L-fuzzy syntopogenous spaces is defined; moreover, it is showed that the agreement of the corresponding properties in L-fuzzy topological, L-fuzzy proximity and L-fuzzy uniformity spaces. And also some of properties concerning connected set in L-fuzzy syntopogenous spaces are obtained.

Keywords : Fuzzy lattice, fuzzy real line, fuzzy syntopogenous space, LFS-connected set.

1. Introduction

A.Császár [1] gave the concept of syntopogenous structure for the unified theory of topology, proximity and uniformity. A.K.Katsaras and C.G.Petalas [2,3,4] introduced the fuzzy syntopogenous structure and studied the unified theory of fuzzy topology, fuzzy proximity and fuzzy uniformity and obtained some similar properties. In this paper, we shall study the relations of L-fuzzy syntopogenous spaces and L-fuzzy topological spaces. And we shall study the LFS-connectedness in L-fuzzy syntopogenous spaces, the

---

*) Projects Supported by the Science Fund of the Chinese Academy of Sciences

LFP- connectedness in L-fuzzy proximity spaces and LFU-connectedness in L-fuzzy uniformity spaces.

2. Preliminaries

In this paper, $L = \langle L, \leq, \wedge, \vee, ' \rangle$ always denotes a completely distributive lattice with order-reversing involution "'" (i.e. fuzzy lattice). Let o be the least element and 1 be the greatest one in L. Suppose X is a nonempty (usual) set, an L-fuzzy set in X is a map $A: X \longrightarrow L$, and L^X will denote the family of all L-fuzzy sets in X. It is clear that $L^X = \langle L^X, \leq, \wedge, \vee, ' \rangle$ is a fuzzy lattice, which has the least element $\underline{o}$ and the greatest one $\underline{1}$, where $\underline{o}(x) = o$, $\underline{1}(x) = 1$ for any $x \in X$.

The following principal definitions and lemmas about fuzzy syntopogenous structure are similar to [2,3,4] they can be expanded to function domain which is L.

Definition 2.1. A binary relation $\ll$ on L^X is called L-fuzzy semi-topogenous order if it satisfies the following axioms :

(1) $\underline{o} \ll \underline{o}$ and $\underline{1} \ll \underline{1}$;

(2) $A \ll B$ implies $A \leq B$;

(3) $A_1 \leq A \ll B \leq B_1$ implies $A_1 \ll B_1$.

The complement of an L-fuzzy semi-topogenous order $\ll$ is the L-fuzzy semi-topogenous order $\ll^c$ which is defined by

$$A \ll^c B \text{ iff } B' \ll A' .$$

An L-fuzzy semi-topogenous order $\ll$ is called :

(a) symmetrical if $\ll = \ll^c$;

(b) topogenous if $A_1 \ll B_1$ and $A_2 \ll B_2$ imply $A_1 \vee A_2 \ll B_1 \vee B_2$ and $A_1 \wedge A_2 \ll B_1 \wedge B_2$;

(c) perfect if $A_j \ll B_j$, $j \in J$, implies $\bigvee A_j \ll \bigvee B_j$;

(d) biperfect if $A_j \ll B_j$, $j \in J$, implies $\bigvee A_j \ll \bigvee B_j$ and $\bigwedge A_j \ll \bigwedge B_j$.

Definition 2.2. An L-fuzzy syntopogenous structure on X is a nonempty set S of L-fuzzy topogenous orders on X having the following properties :

(LFS1) S is directed in the sense that given any two members $\ll_1$, $\ll_2$ of S there exists $\ll$ in S finer than both $\ll_1$ and $\ll_2$, i.e. $\forall$ A, $B \in L^X$, $A \ll_1 B$ (or $A \ll_2 B$) implies $A \ll B$;

(LFS2) For each $\ll$ in S there exists $\ll_1$ in S such that $A \ll B$ implies the existence of an L-fuzzy set D with $A \ll_1 D \ll_1 B$.

The pair (X, S) is called an L-fuzzy syntopogenous space.

Lemma 2.1. Let S be L-fuzzy syntopogenous structure on X, then the mapping $A \longmapsto A^o = \bigvee \{B : B \ll A,$ for some $\ll \in S\}$ is an interior operator and so it defines an L-fuzzy topology $T_1(S)$. If $\ll_s = \bigcup_{\ll \in S} \ll$, then $A \in T_1(S)$ iff $A \ll_s^{?} A$. Conversely, for every L-fuzzy topology on X there exists a perfect L-fuzzy syntopogenous structure $S_1(T) = \{\ll\}$, where $A \ll B$ iff there exists $D \in T$ with $A \leq D \leq B$.

Proof. See [2].

3. Fuzzy Syntopogenous Structure and Fuzzy Topology

Proposition 3.1. Let (X,S) be L-fuzzy syntopogenous space, $A \in L^X$, then the mapping $A \longmapsto A^* = \bigwedge \{B : A \ll B$ for some $\ll \in S\}$ is a closure operator and so it defines an L-fuzzy topology $T_2(S)$:

$$T_2(S) = \{A' : A^* = A, A \in L^X\}.$$

The proof is similar to Lemma 2.1 and hence omitted.

Theorem 3.2. Let S be L-fuzzy syntopogenous structure on X then $T_1(S) \neq T_2(S)$ in general.

Proof. It follows from example 1.

Example 1. Let $\tilde{R}$ denote the set of all decreasing function $\lambda : R \to [0 , 1]$ such that $\lim_{s\to -\infty} \lambda(s) = 1$ and $\lim_{s\to\infty} \lambda(s) = 0$. On $\tilde{R}$ we consider the equivalence relation $\sim$ defined by $\lambda \sim \mu$ iff $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ for all $t \in R$. The fuzzy real line R_φ is the set $\tilde{R}/\sim$ of all equivalence classes. For each $\lambda \in \tilde{R}$, we will denote by $[\lambda]$ the equivalence class. For $t \in R$, we define L_t , R_t to be the fuzzy sets in R_φ defined by $L_t[\lambda] = 1 - \lambda(t-)$ and $R_t[\lambda] = \lambda(t+)$, then the collections $T_R = \{ R_t : t \in R\} \cup \{ \underline{0} , \underline{1}\}$ and $T_L = \{L_t : t \in R\} \cup \{\underline{0} , \underline{1}\}$ are fuzzy topologies on R_φ. For $\varepsilon > 0$, we define the order relation $\ll_\varepsilon$ on I^R by $A \ll_\varepsilon B$ iff either $A = \underline{0}$ or $B = \underline{1}$ or there exists $t \in R$ such that $A \leq L'_t \leq R_{t-\varepsilon} \leq B$. From proposition 4.3 of [4], the families $S_R = \{ \ll_\varepsilon : \varepsilon > 0\}$ and $S_L = \{\ll_\varepsilon^c : \varepsilon > 0\}$ are biperfect fuzzy syntopogenous structures on R_φ, $T_1(S_R) = T_R$, $T_1(S_L) = T_L$. Now we prove $T_1(S_L) \subseteq T_2(S_R)$. Indeed, if $A \in T_1(S_L)$, i.e. $A = \bigvee\{B : B \ll_\varepsilon^c A$, for some $\ll_\varepsilon^c \in S_L , \varepsilon > 0\}$, it follows that $[\bigwedge\{B : A' \ll_\varepsilon B$, for some $\ll_\varepsilon \in S_R ; \varepsilon > 0\}]' = \bigvee \{B' ; A' \ll_\varepsilon B$, for some $\ll_\varepsilon \in S_R, \varepsilon > 0\}$ $= \bigvee\{B_1 : B_1 \ll_\varepsilon^c A$, for some $\ll_\varepsilon^c \in S_L, \varepsilon > 0\} = A$, thus $A \in T_2(S_R)$. From the same reason $T_2 (S_R) \leq T_1 (S_L)$. And thus $T_2(S_R) = T_1(S_L)$. Similarly we have $T_2(S_L) = T_1(S_R) = T_R$. Because $T_L \neq T_R$, thus $T_1(S_R) \neq T_2(S_R)$, $T_1(S_L) \neq T_2(S_L)$.

Definition 3.1. An L-fuzzy syntopogenous spaces (X , S) is called preserved complement if $\forall \ll_1 \in S$, there exists $\ll \in S$ such that $\ll_1^c$ coarser than $\ll$ (i.e. $\ll$ finer than $\ll_1^c$).

Proposition 3.3. If $f : X \to Y$ is a mapping and (Y , S') is preserved complement, then (X , $f^{-1}(S')$) is preserved complement, where $f^{-1}(S') = \{ f^{-1}(\ll) : \ll \in S'\}$.

Proof. This proof can be verified directly.

Proposition 3.4. If $\{(X, S_j) : j \in J\}$ is a preserved complement family, then $(X, \bigvee_{j \in J} S_j)$ is preserved complement.

The proof is straight forward and hence omitted.

Proposition 3.5. If (X, S) is preserved complement, then $T_1(S) = T_2(S)$.

Proof. See example 1.

Definition 3.2. L-fuzzy semi-topogenous order $\ll$ is called co-perfect, if $A_j \ll B_j$, $j \in J$, implies $\bigwedge A_j \ll \bigwedge B_j$.

Proposition 3.6. For any L-fuzzy topogenous order $\ll$ on X, there exists a co-perfect L-fuzzy topogenous order $\ll^i$ finer than $\ll$ and coarser than any co-perfect L-fuzzy semi-topogenous order on X which is finer than $\ll$. It is defined by : $A \ll^i B$ iff there is a family $\{B_j : j \in J\}$ of L-fuzzy sets such that $B = \bigwedge_{j \in J} B_j$ and $B \ll B_j$ for each $j \in J$.

Proof. It follows easily from Definition 3.2.

We omit the proof of the following two propositions easily established.

Proposition 3.7. Let (X, S) be L-fuzzy syntopogenous space, then $A \in T_2(s)$ iff $A' \ll^i_s A'$. If $S = \{\ll\}$ is co-perfect, and thus $A \in T_2(S)$ iff $A' \ll A'$.

Proposition 3.8. Let (X, T) be L-fuzzy topological space, then T corresponds a co-perfect L-fuzzy syntopogenous structure $S_2(T) = \{\ll\}$. It is defined by : $A \ll B$ iff there exists $C \in L^X$, $C' \in T$, and $A \leq C \leq B$. If $T_1 \neq T_2$ then $S_2(T_1) \neq S_2(T_2)$.

Proposition 3.9. (1) Let (X, S) be L-fuzzy syntopogenous space, then a) $S_1(T_1(S)) = \{\ll^p_s\}$, b) $S_1(T_2(S)) = \{\ll^{ic}_s\}$,

c) $S_2(T_1(S)) = \{\ll_s^{pc}\}$, d) $S_2(T_2(S)) = \{\ll_s^{i}\}$.

(2) Let (X , T) be L-fuzzy topological space, then

a) $T_1(S_1(T)) = T$, b) $T_2(S_1(T)) = \{\bigvee_j A_j : A_j' \in T\}$,

c) $T_1(S_2(T)) = \{\bigvee_j A_j : A_j' \in T\}$, d) $T_2(S_2(T)) = T$.

Proof. (1) c) Let $S_2(T_1(S)) = \{\ll_o\}$, if $A \ll_o B$, then there exists $C \in L^X$ such that $C' \ll_s^p C'$, $A \leq C \leq B$, thus $A \leq C \ll_s^{pc} C \leq B$, so $A \ll_s^{pc} B$. Conversely, if $A \ll_s^{pc} B$, then $B' \leq \bigvee\{B : B \ll_s^p A'\} \leq A'$, and $\bigvee\{B : B \ll_s^p A'\} \ll_s^p \bigvee\{B : B \ll_s^p A'\}$, i.e. $\bigvee\{B : B \ll_s^p A'\} \in T_1(S)$, so $A \leq (\bigvee\{B : B \ll_s^p A'\})' \leq B$, $A \ll_o B$. Similarly, we can prove a), b), d).

(2) b) Let $S_1(T) = \{\ll_o\}$, if $A \in T_2(S_1(T))$, since $A \in T_2(S_1(T))$ iff there exists $\{A_j : j \in J\}$ such that $A' = \bigwedge A_j$, $A' \ll_o A_j$, $j \in J$, so $A' \leq C_j \leq A_j$, $C_j \in T$, $j \in J$, and thus $A' = \bigwedge C_j$, $A = \bigvee C_j'$, $C_j \in T$. Conversely, Let $A = \bigvee_{j \in J} A_j$, $A_j' \in T$, then $A' = \bigwedge A_j'$, $A' \leq A_j' \leq A_j'$, so $A' \ll_o A_j'$, $A' \ll_o^i \bigwedge A_j' = A'$, i.e. $A \in T_2(S_1(T))$. Similarly, we can prove a), c), d).

4. Connectedness in L-fuzzy Syntopogenous Space

In this section, the connectedness of L-fuzzy syntopogenous space will be put forward. Not only it is the expanding situation of [5] , but also it gives a unified treatment mothed of L-fuzzy set's con--nectedness in L-fuzzy topological space, proximity space, uniformity space. And it provides a feasible frame for the further research of its properties.

Definition 4.1. Let (X , S) be L-fuzzy syntopogenous space. $C \in L^X$ is called LFS-connected iff there are no $A \neq \underline{0}$, $B \neq \underline{0}$, such that (1) $A \ll A$, $B \ll B$ for some $\ll \in S$; (2) $C' \geq A \wedge B$, $C' \vee A \vee B = \underline{1}$, $C' \vee B \neq \underline{1}$, $C' \vee A \neq \underline{1}$.

For example, Let $L = \{0, 1\}$, $C \in L^X$ (i.e. $C \leq X$), then C is S-connected(see [5]) iff C is LFS-connected.

Definition 4.2.([7]) Let (X, T) be an L-fuzzy topological space, $C \in L^X$ is called connected iff there are no elements $A \neq \underline{0}$ and $B \neq \underline{0}$ in T such that $C' \geq A \wedge B$, $C' \vee A \vee B = \underline{1}$, $C' \vee A \neq \underline{1}$ and $C' \vee B \neq \underline{1}$.

Proposition 4.1. (1) Let (X, S) be a down-perfect L-fuzzy syntopogenous space, and $S = \{\ll\}$, $D \in L^X$. If D is connected in $(X, T_2(S))$, then D is LFS-connected in (X, S).

(2) Let (X, T) be L-fuzzy topological space and $D \in L^X$. If D is LFS-connected in $(X, S_2(T))$, then D is connected in (X, T).

Proposition 4.2. Any molecule in the topological molecular lattice L^X is LFS-connected.

Theorem 4.3. If $f : (X, S) \to (Y, S')$ is (S, S')-continuous(i.e. for each $\ll' \in S$ there exists $\ll \in S$ finer than $f^{-1}(\ll')$), and D is LFS-connected in (X, S), then f(D) is LFS'-connected.

Proof. If not, there must be $A \neq \underline{0}$, $B \neq \underline{0}$ such that (1) $A \ll A$, $B \ll B$, for some $\ll \in S'$, (2) $[f(D)]' \geq A \wedge B$, $[f(D)]' \vee A \vee B = \underline{1}$, $[f(D)]' \vee A \neq \underline{1}$, $[f(D)]' \vee B \neq \underline{1}$. So $f^{-1}(A)$, $f^{-1}(B) \in L^X$, $f^{-1}(A) \neq \underline{0}$, $f^{-1}(B) \neq \underline{0}$, and f is (S, S')-continuous, $\ll \in S'$, then there must be $\ll_0 \in S$ such that $f^{-1}(\ll)$ coarser than $\ll_0$. From Definition 2.1 (3) and Lemma 2.1, it follows that $f^{-1}(A) \ll_0 f^{-1}(A)$, $f^{-1}(B) \ll_0 f^{-1}(B)$ and $D' \geq [f^{-1}(f(D))]' \geq f^{-1}(A) \wedge f^{-1}(B)$. Let M denote the collection of all molecules in L^X(see [6]), so $A' \vee B' \geq f(D) = \bigvee \{ f(b) : b \in M, b \leq D\}$. Thus $b \in M$, $b \leq D$ implies $f(b) \leq A'$ or $f(b) \leq B'$. If $f(b) \nleq A'$ holds for any $b \in M$, $b \leq D$, then $f(D) \leq B'$, $[f(D)]' \vee A \vee B = [f(D)]' \vee A \neq \underline{1}$, but it is im--possible. So we may choose a molecule $b \in M$, $b \leq D$ with $f(b) \leq A'$. Then $D \wedge f^{-1}(A') \geq b \neq \underline{0}$, $D' \vee f^{-1}(A) \neq \underline{1}$. Similarly $D' \vee f^{-1}(B) \neq \underline{1}$.

From this, we obtain the contradiction that D is not LFS-connected.

Theorem 4.4. Let (X, S) be L-fuzzy syntopogenous space, $F \in L^X$, $\widetilde{F} = [(F')^{o}]'$, ($(F')^{o}$ see Lemma 2.1) if D is LFS-connected and $D \leq E \leq \widetilde{D}$, then E is LFS-connected.

Proof. If E is not LFS-connected, then there must be $A \neq \underline{0}$ and $B \neq \underline{0}$, such that (1) $A \ll A$, $B \ll B$ for some $\ll \in S$, (2) $E' \geq A \wedge B$, $E' \vee A \vee B = \underline{1}$, $E' \vee A \neq \underline{1}$, $E' \vee B \neq \underline{1}$. Now we prove $D' \vee A \neq \underline{1}$, i.e. $D \wedge A' = \underline{0}$. In fact, we may choose $b \in M$, $b \leq D$ such that $b \leq A'$, be--cause if $b \nleq A'$ holds for any $b \in M$, $b \leq D$, then $D \leq B'$, and implies that $\widetilde{D} \leq \widetilde{B'} = B'$, and so $E' \vee A \vee B = E' \vee A \neq \underline{1}$. Thus we must have $D \wedge A' \neq \underline{0}$, i.e. $D' \vee A \neq \underline{1}$. Similarly $D' \vee B \neq \underline{1}$. Hence $D' \geq A \wedge B$, $D' \vee A \vee B = \underline{1}$, $D' \vee A \neq \underline{1}$ and $D' \vee B \neq \underline{1}$, but it is impossible.

Proposition 4.5. Let $\{D_j : j \in I\}$ be a family of LFS-connected in (X , S) such that $D_j \wedge D_k \neq \underline{0}$ holds for any j and k in I. Then $D = \bigvee\{D_i : i \in I\}$ is LFS-connected.

Proof. If D is not LFS-connected, there must be $A \neq \underline{0}$, $B \neq \underline{0}$ such that for some $\ll \in S$, $A \ll A$, $B \ll B$, $D' \geq A \wedge B$, $D' \vee A \vee B = \underline{1}$, $D' \vee B \neq \underline{1}$, $D' \vee A \neq \underline{1}$, thus we must have $D_i' \geq A \wedge B$ and $D_i' \vee A \vee B = \underline{1}$ for/any $i \in I$, and $D_j' \vee A \neq \underline{1}$ and $D_k' \vee B \neq \underline{1}$ for some j, k in I, then $D_j' \vee B = D_k' \vee A =$ because both D_j and D_k are LFS-connected. Letting fuzzy point (see[8] $x_\lambda \in D_j \wedge D_k$, we must have $x_\lambda \bar{\in} A'$, $x_\lambda \bar{\in} B'$ by $D_k \wedge A' = D_j \wedge B = \underline{0}$. That is to say that $D_j \wedge D_k \nleq A' \vee B'$, thus $D' \ngeq A \wedge B$, but it is impossible. From the contradiction we have that $D = \bigvee\{D_i : i \in I\}$ is LFS-connecte

Corollary 4.6. The union of any family $\{D_i : i \in I\}$ Of LFS-connected in (X , S) with $\bigwedge\{D_i : i \in I\} \neq \underline{0}$ is LFS-connected.

Let (X , S) be L-fuzzy syntopogenous space. We call a maximal LFS-connected set a component in (X , S). From Proposition 4.2, Theorem 4.4, Corollary 4.6 we have the following proposition.

Proposition 4.7. Let (X, S) be L-fuzzy syntopogenous space, then

(1) For each molecule b in L^X, there exists a component C_b with $b \leq C_b$.

(2) For any component A, $\widetilde{A} = A$.

(3) If A and B are two different componehts in (X, S), then $A \wedge B = \underline{0}$.

Theorem 4.8. Let (X, S) be L-fuzzy syntopogenous space, then $D \in L^X$ is LFS-connected iff $\forall\, x_\lambda, y_\mu \in D$, then there exists LFS-connected set E in (X, S) such that $x_\lambda, y_\mu \in E \leq D$.

Proof. If D is not LFS-connected, then there must be $A \neq \underline{0}$, $B \neq \underline{0}$ such that for some $\ll \in S$, $A \ll A$, $B \ll B$, $D \leq A' \vee B'$, $D \wedge A' \wedge B' = \underline{0}$, $D \wedge A' \neq \underline{0}$, $D \wedge B' \neq \underline{0}$. So there are fuzzy points $x_\lambda, y_\mu \in E$, $x_\lambda \in D$, $x_\lambda \in A'$, $y_\mu \in D$, $y_\mu \in B'$, and E is LFS-connected. Thus then $E \wedge A' \neq \underline{0}$, $E \wedge B' \neq \underline{0}$ and it is clear that $E' \geq A \wedge B$, $E' \vee A \vee B = \underline{1}$. So E is not LFS-connected. Conversely, the proof is straight forward and hence omitted.

Theorem 4.9. Let $\{(X_i, S_i) : i \in I\}$ be a family of L-fuzzy syntopogenous spaces.

(1) If $(\prod_{i \in I} X_i, \prod_{i \in I} S_i)$ is LFS-connected, then (X_i, S_i) is LFS_i-connected for any $i \in I$, where $S = \prod_{i \in I} S_i$.

(2) If (X_i, S_i) is preserved complement and LFS_i-connected for any $i \in I$, then $(\prod_{i \in I} X_i, S)$ is LFS-connected.

Proof. (1) Since the canonical projection are (S, S_i)-continuous, each (X_i, S_i) is LFS_i-connected by Theorem 4.2.

(2) Let $\{z_i\}$ be a fixed usual point of $\prod_{i \in I} X_i$, the subset E_j of the product space consisting of all ucual points $\{x_i\}$ such that $x_i = z_i$

if $i \neq j$ while x_j may be any point of X_j is isomorphic to X_j, and hence is LFS-connected set containing $\{z_i\}$. Clearly, for any finite number of indices $j_1, j_2, j_3, \dots, j_m$, $E_{j_1} \times E_{j_2} \times \dots \times E_{j_m} = E$, i.e. the set of all usual points $\{x_i\}$ such that $x_i = z_i$ if $i \neq j_1, j_2, \dots j_m$, is LFS-connected and contains $\{z_i\}$. Since $\{(X_i , S_i) : i \in I\}$ is preserved complement, so ($\prod_{i \in I} X_i$, $\prod_{i \in I} S_i$) is preserved complement by Proposition 3.3, 3.4. Thus $\bar{A}$ in ($\prod_{i\in I} X_i$, $T_2(\prod_{i \in I} S_i)$) is equal to $\tilde{A}$ in ($\prod_{i\in I} X_i$, $\prod_{i\in I} S_i$). By the proof of Theorem 3.7 [7] , we can get $\tilde{E} = \bar{E} = \prod_{i\in I} X_i$, by Theorem 4.4 it follows that ($\prod_{i\in I} X_i$, $\prod_{i\in I} S_i$) is LFS-connected.

5. Connectedness in L-fuzzy Proximity and Uniformity Space

Definition 5.1. Let (X , δ) be L-fuzzy proximity space [8] , $D \in L^X$ is called LFP-connected iff there are no $A \neq \underline{0}$, $B \neq \underline{0}$ such that (1) $A \bar{\delta} A'$, $B \bar{\delta} B'$, (2) $D' \geq A \wedge B$, $D' \vee A \vee B = \underline{1}$, $D' \vee A \neq \underline{1}$, $D' \vee B \neq \underline{1}$.

By Definition 4.1, we can get following proposition.

Proposition 5.1. (1) Let (X , S) be a symmetrical L-fuzzy syntopogenous space and $S = \{\ll\}$, $\delta_{\ll}$: $A \bar{\delta}_{\ll} B$ iff $A \ll B'$. If $D \in L^X$ is LFP-connected in $(X , \delta_{\ll})$, then D is LFS-connected in (X , S).

(2) Let (X , δ) be L-fuzzy proximity space and $\ll_{\delta}$: $A \ll_{\delta} B$ iff $A \bar{\delta} B'$. If $D \in L^X$ is LFS-connected in $(X , S = \{\ll_{\delta}\})$, then D is LFP--connected in (X , δ).

Definition 5.2. Let (X , U) be L-fuzzy quasi-uniformity space [9] , $D \in L^X$ is called LFU-connected iff there are no $A \neq \underline{0}$ and $B \neq \underline{0}$ such that (1) $A = \alpha(A)$, $B = \alpha(B)$ for some $\alpha \in U$. (2) $D' \geq A \wedge B$, $D' \vee A \vee B = \underline{1}$, $D' \vee A \neq \underline{1}$, $D' \vee B \neq \underline{1}$.

Lemma 5.2. To every biperfect L-fuzzy topogenous order $\ll$ on X corresponds $w(\ll) = \alpha = \alpha_{\ll} : L^X \to L^X$ defined by $\alpha(A) = \wedge \{B : A \ll B\}$. If S is a biperfect L-fuzzy syntopogenous structure on X, then the family $w(S) = \{w(\ll) : \ll \in S\}$ is a base for L-fuzzy quasi-uniformity on X, then $w^{-1}(B)$ is a biperfect L-fuzzy syntopogenous structure on X.

Proof. See [2].

From Definition 4.1, 5.2, we get the following proposition.

Proposition 5.3. (1) Let (X , S) be L-fuzzy biperfect syntopogenous space. $D \in L^X$ is LFU-conected in (X , U(w(S))), where w(S) is the base of U(w(S)) , then D is LFS-connected in (X ,S).

(2) Let (X , U) be L-fuzzy quasi-uniformity space, B be the base of U. If $D \in L^X$ is LFS-connected in $(X , S = w^{-1}(B))$, then D is LFU--connected in (X , U).

References

1. A.Csazar, Foundation of general topology, Pergamon Elmsford, New York 1963.

2. A.K.Katsaras and C.G.Petalas, A unified theory of fuzzy topologies, fuzzy proximities and fuzzy uniformities, Rev. Roumaine Math. Pures Appl. 28(1983) 845-856.

3. A.K.Katsaras and C.G.Petalas, On fuzzy syntopogenous structures, J.Math.Anal.Appl. 99(1984) 219-236.

4. A.K.Katsaras, On fuzzy syntopogenous structures, Rev. Roumaine Math. Pures Appl. 30(1985) 419-431.

5. J.L.Sieber and W.J.Pervin, Connectedness in syntopogenous spaces, Amer.Math.Monthly, 16(1965) 550-559.

6. Zhao Xiao-dong, Connectedness on fuzzy topological spaces, Fuzzy Sets and Systems, 20(1986) 223-240.

7. Zhao Dong-sheng and Wang Guo-jun, A new fuzzy connectedness, Fuzzy Mathematics, 4(1984) 15-22 (in Chinese).

8. Liu Wang-jin, Fuzzy proximity redefined, Fuzzy Sets and Systems, 15(1985) 241-249.

9. B.W.Hutton, Uniformities on fuzzy topological spaces, J.Math.Anal.Appl. 50(1975) 75-79.