FUZZY NUMBER-VALUED FUZZY INTEGRAL OF FUZZY NUMBER-VALUED FUNCTION ON FUZZY SET

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ABSTRACT

The concepts of fuzzy number-valued function and fuzzy number-valued fuzzy integral of fuzzy number-valued function on fuzzy set are introduced and some elementary properties of theirs are given.

1. Introduction

Wang[1] introduced the concepts of the autocontinuity of set function and the fuzzy integral, Wang's integral is real-valued fuzzy integral of real-valued function with respect to the fuzzy measure. Wang and Zhang[2] introduced fuzzy number-valued fuzzy integral of real-valued function on fuzzy set and obtained a series of interesting results simlar to results[1].

In this paper, we introduce the concept of fuzzy number-valued fuzzy integral of fuzzy number-valued function on fuzzy set, and give some elementary properties of this type of fuzzy integral.

This paper is a development of [2]. All concepts and signs not defined in this paper may be found in[1,2]. Throughout this paper, let X be a nonempty set, $\mathcal{B}$ be a σ-algebra of subsets of X, R be the set of all real numbers.

2. Basic definitions

Definition 2.1. Let $F=\{\tilde{a};\ \tilde{a}:R\rightarrow[0,1]\}$, a fuzzy number is a $\tilde{a}\in F$ with the properties:

(FN1) $\tilde{a}$ is normal, i.e., there exists $x\in R$ such that $\tilde{a}(x)=1$, (FN2) Whenever $\lambda\in(0,1]$, then $a_\lambda=\{x;\ \tilde{a}(x)\geq\lambda\}$ is a closed interval, denoted by $[a_\lambda^-,\ a_\lambda^+]$.

Let F^* be a set of all fuzzy numbers.

By decompsition theorem of fuzzy set

$$\tilde{a}=\bigcup_{\lambda\in[0,1]}\lambda[a_\lambda^-,\ a_\lambda^+],$$

for every $\tilde{a}\in F^*$.

If we define $a(x)=1$ iff $x=a$;

$=0$ iff $x\neq a$,

for every $a\in R$, then $a\in F^*$.

Definition 2.2. Let $\tilde{a},\tilde{b}\in F^*$, we say that $\tilde{a}\leq\tilde{b}$, if for every $\lambda\in(0,\ 1]$, $a_\lambda^-\leq b_\lambda^-$ and $a_\lambda^+\leq b_\lambda^+$.

Let $F_+^*=\{\tilde{a};\ \tilde{a}\geq 0,\ \tilde{a}\in F^*\}$.

Definition 2.3. Let $\tilde{a},\tilde{b}\in F^*$, we call that $\tilde{c}=\tilde{a}\vee\tilde{b}$(resp. $\tilde{c}=\tilde{a}\wedge\tilde{b}$), if for every $\lambda\in(0,\ 1]$, $c_\lambda^-=a_\lambda^-\vee b_\lambda^-$ and $c_\lambda^+=a_\lambda^+\vee b_\lambda^+$(resp. $c_\lambda^-=a_\lambda^-\wedge b_\lambda^-$ and $c_\lambda^+=a_\lambda^+\wedge b_\lambda^+$).

Definition 2.4. A nonnegative fuzzy number-valued function is a mapping $f:X\longrightarrow F_+^*$ with the properties: for every $x\in X$, there exists unique $\tilde{y}\in F_+^*$ such that $f(x)=\tilde{y}$.

Definition 2.5. Let f,g be fuzzy number-valued function, we define:

(1) $(f\vee g)(x)=f(x)\vee g(x)$;

(2) $(f\wedge g)(x)=f(x)\wedge g(x)$.

Obviously both $f\vee g$ and $f\wedge g$ are fuzzy number-valued function.

Proposition 2.1. Let f be nonnegative fuzzy number-valued function, then for every $\lambda\in(0,\ 1]$, both $f_\lambda^-(x)=(f(x))_\lambda^-$ and $f_\lambda^+(x)=(f(x))_\lambda^+$ are nonnegative real functions and

$$f(x)=\bigcup_{\lambda\in[0,1]}\lambda[f_\lambda^-(x),\ f_\lambda^+(x)].$$

Definition 2.6. Let f,g be fuzzy number-valued function, we say that $f\leq g$ iff $f(x)\leq g(x)$ for every $x\in X$.

Theorem 2.1. Let f,g be fuzzy number-valued function, then $f\leq g$ if and only if $f_\lambda^-\leq g_\lambda^-$ and $f_\lambda^+\leq g_\lambda^+$ for every $\lambda\in(0,\ 1]$.

Proof. The conclusion is from definition 2.2, 2.3, 2.5 and 2.6.

Definition 2.7.A nonnegative fuzzy number-valued function $f:X\longrightarrow F_+^*$ is called fuzzy measurable, iff for every $\lambda\in(0,\ 1]$

$\alpha\in R$

$$F^-_{\lambda,\alpha}=\{x;\ f^-_\lambda(x)\geq\alpha\}\in\mathcal{B},$$
$$F^+_{\lambda,\alpha}=\{x;\ f^+_\lambda(x)\geq\alpha\}\in\mathcal{B}.$$

3. Fuzzy number-valued fuzzy integral of fuzzy number-valued function on fuzzy set

Let $\mathcal{A}=\{\tilde{A};\ \tilde{A}:X\longrightarrow[0,1]$ and $A_\lambda\in\mathcal{B}$ for every $\lambda\in(0,\ 1]\}$, M^*_+ be the set of all nonnegative fuzzy number-valued measurable function.

Throughout this section, let $f,g\in M^*_+$, $\tilde{A},\tilde{B}\in\mathcal{A}$.

Definition 3.1 Let $f\in M^*_+$, $\tilde{A}\in\mathcal{A}$. The fuzzy number-valued fuzzy integral of fuzzy number-valued function on fuzzy set is defined by

$$\int_{\tilde{A}} f\ d\mu=\bigcup_{\lambda\in[0,1]}[\int_{A_1} f^-_\lambda d\mu,\ \int_{A_\lambda} f^+_\lambda d\mu]$$

where

$$\int_{A_1} f^-_\lambda d\mu=\sup_{\alpha\in[0,1]}\alpha\wedge\mu(A_1\cap\{x;\ f^-_\lambda(x)\geq\alpha\}),$$

$$\int_{A_\lambda} f^+_\lambda d\mu=\sup_{\alpha\in[0,1]}\alpha\wedge\mu(A_\lambda\cap\{x;\ f^+_\lambda(x)\geq\alpha\}).$$

Proposition 3.1. If f is nonnegative real-valued measurable function, then

$$\int_{\tilde{A}} f\ d\mu=(F)\int_{\tilde{A}} f\ d\mu,$$

where

$$(F)\int_{\tilde{A}} f\ d\mu=\bigcup_{\lambda\in[0,1]}\lambda[\int_{A_1} f\ d\mu,\ \int_{A_\lambda} f\ d\mu]$$

is the integral defined in [2].

Proposition 3.2. If $f_1\leq f_2$, then

$$\int_{\tilde{A}} f_1\,d\mu\leq\int_{\tilde{A}} f_2\,d\mu.$$

Proposition 3.3. If $\tilde{A}\subseteq\tilde{B}$, then

$$\int_{\tilde{A}} f\, d\mu \leq \int_{\tilde{B}} f\, d\mu .$$

Proposition 3.4. $\int_{\tilde{A}} (f \vee g) d\mu \geq \int_{\tilde{A}} f\, d\mu \vee \int_{\tilde{A}} g\, d\mu .$

Proposition 3.5. $\int_{\tilde{A}} (f \wedge g) d\mu \leq \int_{\tilde{A}} f\, d\mu \wedge \int_{\tilde{A}} g\, d\mu .$

Proposition 3.6. $\int_{\tilde{A} \cup \tilde{B}} f\, d\mu \geq \int_{\tilde{A}} f\, d\mu \vee \int_{\tilde{B}} f\, d\mu .$

Proposition 3.7. $\int_{\tilde{A} \cap \tilde{B}} f\, d\mu \leq \int_{\tilde{A}} f\, d\mu \wedge \int_{\tilde{B}} f\, d\mu .$

Proposition 3.8. If $f(x)=\tilde{a} \in F_+^*$ for every $x \in X$ and $f \in M_+^*$, then

$$\int_{\tilde{A}} \tilde{a}\, d\mu \leq \tilde{a} \wedge \mu(\mathring{A}),$$

$$\int_{\tilde{A}} \tilde{a}\, d\mu \geq \tilde{a} \wedge \mu(A_1),$$

where $\mathring{A}=\{x;\ \tilde{A}(x)>0\}$.

Theorem 3.1. Whenever $\tilde{A}(x)=\tilde{B}(x)$ a.e., $\int_{\tilde{A}} f\, d\mu = \int_{\tilde{A}} g\, d\mu$ holds, if and only if μ is null-additive [1].

Theorem 3.2. Whenever $f=g$ a.e., $\int_{\tilde{A}} f\, d\mu = \int_{\tilde{A}} g\, d\mu$ holds, if and only if μ is null-additive.

Theorem 3.3. Let $\mathring{A}=\{x;\ \tilde{A}(x)>0\}$, then

$$\int_{\tilde{A}} f\, d\mu = 0$$

if and only if

$$\mu(\{f_\lambda^+ > 0\} \cap \mathring{A})=0.$$

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