AN APPROXIMATE REASONING METHOD BASED ON SET–MAPPING①

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ABSTRACT

In this paper, an approximate reasoning method based on set–mapping is put forward. This method not only can deal with certainty reasoning effectively, but also can deal with uncertainty reasoning reasonably. It can be applied to reasoning machine based–rules expert system.

Keywords: Set–Mapping, Approximate Reasoning,Multi–Implication

1. INTRODUCTION

The various approximate reasoning methods about the various reasoning models based on fuzzy sets have put forward [1–6], and have been used extensively [7–9]. As a rule,these approximate reasoning methods contain certainty reasoning naturally,but it is not. When a certainty information is imported to a reasoning model,but the conclusion is an uncertainty information, that is, fuzzy information. Clearly, it is not proper. For this reason, we start with certainty reasoning directly, with the idea of extension principle [6], and spread the process of certainty reasoning into the process of fuzzy reasoning,and make the reasoning method not

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only can deal with certainty reasoning effectively but also can deal with uncertainty reasoning reasonably.

2.APPROXIMATE REASONING METHOD BASED ON SET–MAPPING

Let X and Y be two sets.The set–mapping

$$R : P(X) \longrightarrow P(Y)$$

is called a reasoning mapping,if

$A_1, A_2 \in P(X), A_1 \subseteq A_2 \Longrightarrow R(A_1) \subseteq R(A_2)$. If $A \in P(X), B \in P(Y), R(A) = B$, then B is called reasoning conclusion of A,and A is called reasoning premise of B.

If we know only these reasoning conclusions of part of the elements in P(X),then we write the set of all these elements as $P^*(X)$.The mapping

$$R^* = R\big|_{P^*(X)} : P^*(X) \longrightarrow P(Y)$$

is called a certain reasoning mapping.

$\forall y \in Y$,set

$$C_y = \{B : y \in B, B \in R^*(P^*(X))\} \tag{1}$$

$$C_y^{-1} = \{A : \exists B \in C_y, R^*(A) = B\} \tag{2}$$

Let $A' \in F(X)$ be an input information, then the reasoning conclusion $B' \in F(Y)$ is defined as

$$\forall y \in Y, B'(y) = \bigvee_{A \in C_y^{-1}} \bigwedge_{x \in A} A'(x) \tag{3}$$

Note that $(C_y^{-1}, \subseteq)$ is a partially ordered set, Let C_y^{-1*} denote the set of all minimal elements of $(C_y^{-1}, \subseteq)$,then we have that

$$\forall y \in Y, B'(y) = \bigvee_{A \in C_y^{-1*}} \bigwedge_{x \in A} A'(x) \tag{4}$$

Clearly, this reasoning method possesses the properties as follows.

(i). The greater $|P^*(X)|$ is,the more reasonable the reasoning method is, since it contains more reasoning information and possesses the character (ii).

(ii). If $A'=A_0\in P^*(X)$, then $B'=R^*(A_0)$.

In fact, $\forall y\in Y$, if $y\in R^*(A_0)$, then

$$B'(y)=\bigvee_{A\in C_y^{-1}}\ \bigwedge_{x\in A} A'(x)\geqslant \bigwedge_{x\in A_0} A_0(x)=1$$

thus, $B'(y)=1$. If $y\bar{\in} R^*(A_0)$, $\forall A\in C_y^{-1*}$, $\exists x_0\in A$, $x_0\bar{\in} A_0$, then $\bigwedge_{x\in A} A_0(x)\leqslant A_0(x_0)=0$, so

$$B'(y)=\bigvee_{A\in C_y^{-1*}}\ \bigwedge_{x\in A} A_0(x)=0$$

(iii). If $A_1, A_2\in F(X)$, $A_1\subseteq A_2$, and $R^*(A_i)=B_i$ $(i=1,2)$, then $B_1\subseteq B_2$.

In fact, $\forall y\in Y$,

$$B_1(y)=\bigvee_{A\in C_y^{-1*}}\ \bigwedge_{x\in A} A_1(x)\leqslant \bigvee_{A\in C_y^{-1*}}\ \bigwedge_{x\in A} A_2(x)=B_2(y)$$

3. MULTI–IMPLICATION REASONING IS CHANGED INTO THE APPOXIMATE REASONING BASED ON SET–MAPPING

The model of multi–implication reasoning is that

Ant.1 :If x_1 is A_{11} and ... and x_n is A_{1n}, then y is B_1, else

Ant.2 :If x_1 is A_{21} and ... and x_n is A_{2n}, then y is B_2, else

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Ant.m−1 :If x_1 is $A_{m-1\,1}$ and ... and x_n is $A_{m-1\,n}$, then y is B_{m-1}, else

Ant.m :If x_1 is A_{m1} and ... and x_n is A_{mn}, then y is B_m.

Ant.m+1 :x_1 is A'_1 and ... and x_n is A'_n

Cons. :y is B' (5)

Where, x_j and y are the names of objects, and A'_j, $A_{ij}\in F(X_j)$, B_i, $B'\in F(Y)$ $(i=1,2,\cdots,m-1,m,j=1,\cdots,n)$.

(i). When "else" is interpreted as "disjunction", set

$$P^{*}\left(\prod_{j=1}^{n} X_j\right)=\left\{\prod_{j=1}^{n}(A_{ij})_\lambda : i=1,2,\ldots,m,\lambda\in[0,1]\right\} \tag{6}$$

$$R^{*}: P^{*}\left(\prod_{j=1}^{n} X_j\right)\longrightarrow P(Y)$$

$$\prod_{j=1}^{n}(A_{ij})_\lambda \longmapsto (B_i)_\lambda$$

If

$$\prod_{j=1}^{n}(A_{i_1 j})_\lambda = \prod_{j=1}^{n}(A_{i_2 j})_\lambda \tag{7}$$

but $(B_{i_1})_\lambda \neq (B_{i_2})_\lambda$,then$R^{*}\left(\prod_{j=1}^{n}(A_{i_1 j})_\lambda\right)$ and B' are given as

$$R^{*}\left(\prod_{j=1}^{n}(A_{i_1 j})_\lambda\right)=(B_{i_1})_\lambda \bigcup (B_{i_2})_\lambda=(B_{i_1}\bigcup B_{i_2})_\lambda \tag{8}$$

$$\forall y\in Y, B'(y)=\bigvee_{A\in C_y^{-1*}}\left[\bigwedge_{(x_1,\cdots,x_n)\in A}\ \bigwedge_{j=1}^{n} A'_j(x_j)\right] \tag{9}$$

(ii).When "else" is interpreted as "conjunction", if (7) holds,

but $(B_{i_1})_\lambda \neq (B_{i_2})_\lambda$,then $R^{*}\left(\prod_{j=1}^{n}(A_{i_1 j})_\lambda\right)$and B' are given as

$$R^{*}\left(\prod_{j=1}^{n}(A_{i_1 j})_\lambda\right)=(B_{i_1})_\lambda \bigcap (B_{i_2})_\lambda=(B_{i_1}\bigcap B_{i_2})_\lambda \tag{10}$$

$$\forall y\in Y, B'(y)=\bigwedge_{i=1}^{m}\ \bigvee_{A\in C_y^{-1*}(i)}\left[\bigwedge_{(x_1,\cdots,x_n)\in A}\ \bigwedge_{j=1}^{n} A'_j(x_j)\right] \tag{11}$$

where, $C_y^{-1*}(i)$ is the set of all minimal elements of $C_y^{-1*}(i)$ and $C_y^{-1}(i)$ is given as

$$C_y(i) = \left\{(B_i)_\alpha : y\in(B_i)_\alpha, (B_i)_\alpha \in R^*\left(\left\{\prod_{j=1}^{n}(A_{ij})_\lambda : \lambda\in[0,1]\right\}\right), \alpha\in[0,1]\right\} \tag{12}$$

$$C_y^{-1}(i) = \left\{\prod_{j=1}^{n}(A_{ij})_\lambda : \lambda\in[0,1], \exists (B_i)_\lambda \in C_y(i), R^*\left(\prod_{j=1}^{n}(A_{ij})_\lambda\right) = (B_i)_\lambda\right\} \tag{13}$$

4.REMARK

We give in (3) by the two points as follows:

(i). $\forall x_1, x_2 \in A$, the logical relation between A_1 and A_2 is conjunction.So, $\bigwedge_{x\in A} A'(x)$ appears in (3).

(ii). $\forall A_1, A_2 \in C_y^{-1}$, the logical relation between A_1 and A_2 is disjunction.So, $\bigvee_{A\in C_y^{-1}}$ appears in (3).

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