

A NEW METHOD FOR SELF-LEARNING IN  
FUZZY CONTROLLERS

Da Q. Qian and M. Mizumoto  
Department of Management Engineering  
Osaka Electro-Communication University  
Neyagawa, Osaka 572, Japan

Abstract: In this paper, a new on-line self-learning method in fuzzy controllers is proposed. The simulation results show that the method can effectively improve the performance of the fuzzy controllers.

### 1. FUZZY CONTROLLERS

Today, many fuzzy controllers have been successfully developed (Graham and Newell 1989; Pedrycz 1984; Shao 1988; Xu and Lu 1987). The fuzzy controllers usually have the following form:

$$U(k) = X_1(k) \circ X_2(k) \circ \dots \circ X_n(k) \circ R$$

where  $k$  denotes the instant  $k$ ,  $\circ$  is a max-min composition operator,  $U(k)$  denotes a control action of the fuzzy controller at instant  $k$ ,  $U(k) \in F(U)$ ,  $X_i(k)$  denotes a state of the controlled process at instant  $k$ ,  $X_i(k) \in F(X_i)$  ( $i=1, 2, \dots, n$ ),  $R \in F(X_1 \times X_2 \times \dots \times X_n)$  ( $\times$  is the Cartesian product).  $U, X_i$  are the universes of discourse, and  $F(\cdot)$  stands for the family of the fuzzy sets defined on a particular universe, i.e.,  $U \in F(U)$ ,  $U: U \rightarrow [0, 1]$ .

Each fuzzy set  $X$  is defined in terms of a complete set of fuzzy reference sets  $\{X_1, X_2, \dots, X_m\}$  ( $X_i \in F(X_i)$ ,  $i=1, 2, \dots, m$ ) on the underlying universe of discourse as shown in Fig. 1. Any fuzzy set  $X \in F(X)$  can then be represented by its possibility vector  $[p_1, p_2, \dots, p_m]$  with respect to the set of fuzzy reference sets. Each individual possibility measure  $p_i$  is calculated via

$$p_i = \sup_{x \in X} \min [X_i(x), X(x)]$$

The  $R$  is constructed from a group of fuzzy control rules in terms of the reference sets defined on each universe of discourse. According to the paper (Pedrycz 1984), the  $R$  can be calculated by a recursive formula, i.e., at instant  $k$ , the old fuzzy relation model is  $R(k-1)$ , and the current input-output data set of the fuzzy controller is  $\{U(k), X_1(k), X_2(k), \dots, X_n(k)\}$ , the relation  $R$  is derived via

$$R(k) = R'(k) \circ U \circ R(k-1) \tag{1}$$

where

$$R'(k) = X_1(k) \times X_2(k) \times \dots \times X_n(k) \times U$$

Usually, in the fuzzy controller there are two kinds of input variables: error E between set point S and process output Y, and change in error dE, e.g.,

$$\text{if } E \text{ is } X_1 \text{ and } dE \text{ is } X_2 \text{ then } u \text{ is } U \quad (2)$$

where  $E=S-Y$ .

In this paper, we will propose a new self-learning method that can be used in fuzzy controllers.

## 2. A SELF-LEARNING METHOD

The structure of the fuzzy controller with self-learning is given in Fig. 2. Assume that the rules of the fuzzy controller have the form (2), the self-learning method proposed here can be used to update the fuzzy relation R generated via formula (1) according to the control action of the fuzzy controller and the response of the process to the control action. At instant k, if the current control action is assumed to be  $U(k)$ , the responses of the process to  $U(k)$  and  $U(k-1)$  are respectively assumed to be  $Y(k)$  and  $Y(k-1)$ , the set point of Y is assumed to be S, the error between S and  $Y(k)$  is  $S-Y(k)$ , then, we can define

$$\begin{aligned} E'(k) &= Y(k) - Y(k-1) \\ dE'(k) &= (Y(k) - Y(k)) - (Y(k) - Y(k-1)) \\ &= Y(k-1) - Y(k) \end{aligned}$$

then the R can be updated on-line by the following formula

$$R(k) = R'(k) \cup R(k-1) \quad (3)$$

$R'(k)$  is calculated via

$$R'(k) = E'(k) \times dE'(k) \times U(k) \quad (4)$$

This self-learning method is proposed from the following viewpoint, i.e., since

the control action  $U$  is only dependent on the values of  $E'$  and  $dE'$ , if the process output is assumed to be  $Y(k-1)$  at instant  $k-1$ , and the set point of  $Y$  is assumed to be  $Y(k)$ , then the control action,  $U(k)$ , can drive the process output from  $Y(k-1)$  to  $Y(k)$ , in other words, the error between the process output and its set point will decrease to zero at next instant.

In the case that the fuzzy controller is unable to make a decision  $U(k)$  at instant  $k$  since  $R(k)$  does not contain any information on  $U(k)$  at entry  $E(k)$  and  $dE(k)$ , an auxiliary algorithm can be used, i. e.,  $U(k)$  can be assumed to be  $U(k-1)$ .

### 3. SIMULATION

In the following simulation, the process model is

$$\begin{aligned}
 Y(k) = & (1+2*\exp(-0.2*T))*Y(k-1)-(2*\exp(-0.2*T) \\
 & +\exp(-0.4*T))*Y(k-2)+\exp(-0.4*T)*Y(k-2) \\
 & +2.5*(0.5*U(k-1)+(4.5-5.5*\exp(-0.2*T)) \\
 & *U(k-2)+(-4.5*\exp(-0.2*T)+5*\exp(-0.4*T)) \\
 & *U(k-3))
 \end{aligned}$$

and the conventional controller is:

$$\begin{aligned}
 U(k) = & -(T-0.768)/(T+0.768)*U(k-1) \\
 & +(T+0.832)/(T+0.768)*E(k) \\
 & +(T-0.832)/(T+0.768)*E(k-1)
 \end{aligned}$$

where  $T$  is a sampling time and is assumed to be 0.01. Assume when the set point  $S$  is chosen as 1, and the process is in steady state, i. e.,  $Y$  is also equal 1 and  $U$  is equal to 0, a noise of 1 is added to  $U$  at one instant  $0^-$ , the response of the process controlled by the conventional controller is given in Fig. 3 and the control action is given in Fig. 4. Using the data shown in Fig. 3, a corresponding fuzzy relation  $R$  consisting of 10 reference sets can be produced via formula(1). The process response and the control action of the fuzzy controller with the  $R$  in the same environment, i. e., the same set point and noise, are respectively given in Fig. 5 and Fig. 6. Compared Fig. 3 with Fig. 5, and Fig. 4 with Fig. 6, we can find that the control action of the fuzzy controller is similar to that of the above conventional controller.

By replacing the fuzzy controller with the fuzzy self-learning controller that can modify the fuzzy relation  $R$  via formula (3), the response of the process to the same noise as above is given in Fig. 7. When the set point is changed into 0.6 and 1.4, and the same noise is added, the responses of the process controlled by the fuzzy controller with  $R$  and the self-learning fuzzy controller with  $R$  being updated on-line are given in Fig. 8 to Fig. 11. Compared Fig. 7 with Fig. 3 and Fig. 5, Fig. 9 with Fig. 8, and Fig. 11 and Fig. 10, we can see that the process controlled by the self-learning fuzzy controller can reach the steady state more quickly than the conventional controller and the fuzzy controller without self-learning mechanism.

The further simulation is carried out in the following way. We at first replace the controller with a white noise generator, and the set point of 0 is chosen, therefore, a fuzzy relation  $R_1$  can be generated via formula (3), then, replacing the white noise generator with the conventional controller, the fuzzy relations  $R_2$ ,  $R_3$  and  $R_4$  are generated via formula (1) respectively when the set point and the noise are (1, 1), (-1, 1) and (0.6, -0.8), the new fuzzy relations  $R_j$  are derived by  $R_1 \cup R_i$  ( $j=i+3$ ,  $i=2, 3, 4$ ). Finally, in the same environment, i.e., the same set point and noise, we can get the response in error of the process controlled by the fuzzy controller with  $R_2$  and  $R_3$  as in Fig. 12, the response in error controlled by the conventional controller as in Fig. 13, the response in error controlled by the fuzzy controller with  $R_5$  and  $R_6$  as in Fig. 14, and the response in error controlled by the fuzzy controller with  $R_7$  as in Fig. 15. Comparison of Fig. 12 with Fig. 14, Fig. 13 with Fig. 15 can prove that the formula (3) is able to speed up the convergence of the process to its point set.

#### 4. RELATED WORK AND CONCLUSION

Up to now, most of fuzzy self-learning methods mainly emphasize on identification of fuzzy systems (Pedrycz 1984; Xu and Lu 1988), however, these methods can not be directly used in building adaptive fuzzy controllers. The fuzzy self-organizing controller (Shao 1988) requires the designer to know how to correct the control action according to process output and how to utilize the knowledge to construct the fuzzy self-organizing controller, so that the capacity of the self-organizing controller for improving its control performance

is completely dependent on the designer's priori knowledge. This requirement hinders the practical applications of the fuzzy self-organizing controller. Our work is also different from Graham and Newell's fuzzy adaptive controller(1989), which made use of fuzzy identification techniques for building the fuzzy relation model, and then using the fuzzy relation model, the control action corresponding to the desired process output can be inferred. The major advantages of the fuzzy learning method proposed in this paper are that by it the fuzzy controller can be improved on-line only using its previous control action and response of the process without the request to any complex knowledge on how to do it, and it can also easily be implemented. From the above simulations, it can be seen that by using the method the process can quickly reach the steady state.

The method can further be improved in the following aspects:

- (1). The fuzzy relation  $R(k)$  in (3) can be updated as follows:

$$R(k) = (1-a) * R(k-1) + a * R'(k)$$

where  $0 \leq a \leq 1$ ,  $a$  is determined by the amplitude of  $|E(k)|$  ( obviously a large  $|E(k)|$  should correspond to a large  $a$ ) and the relative contributions of each of control rules to  $U(k)$ -- a rule which contributes more to  $U(k)$  should undergo more modification ( a large  $a$ ) (Xu and Lu 1987).

- (2). In formula (4),  $E'(k)$  can be replaced by

$$E'(k) = (1-b) * (Y(k) - Y(k-1)) + b * (S - Y(k-1))$$

where  $b$  is determined according to the desire for the response speed of the controlled process. A small  $b$  can shorten the setting time of the process but may result in high overshoot of the response.

- (3) In formula (4),  $U(k)$  can also be replaced by the following formula:

$$U(k) = c * U'(k) + (1-c) * U''(k)$$

where  $U'(k)$  is the control action inferred by  $R(k-1)$  at entry  $E'(k)$  and  $dE'(k)$ , and  $U''$  is the control action inferred by  $R'(k)$  at entry  $E'(k)$  and  $dE'(k)$ ,  $c$  is

determined as b.

(4). The  $U(k)$  in the above auxiliary algorithm can be determined by interpolating between neighboring control actions, i.e., by the formula  $d*U' + (1-d)*U''$ ,  $0 \leq d \leq 1$ ,  $U'$  and  $U''$  are the control actions respectively in the two neighboring environments that are most closely similar to the current environment represented by  $E(k)$  and  $dE(k)$ ,  $d$  is determined according to the rate between two similarities of the current environment to the two neighboring environments.

#### REFERENCES

Graham, B. and Newell, R. B., Fuzzy adaptive control of a first-order process. Fuzzy Sets and Systems, Vol. 31, 1989.

Pedrycz, W., An identification algorithm in fuzzy relational systems. Fuzzy Sets and Systems, Vol. 13, 1984.

Shao, S. H., Fuzzy self-organizing controller and its application for dynamic processes. Fuzzy Sets and Systems, Vol. 26, 1988.

Xu C. W. and Lu Y. Z., Fuzzy model identification and self-learning for dynamic systems. IEEE Trans Systems, Man and Cybernetics, Vol. 17, No. 4, 1987.

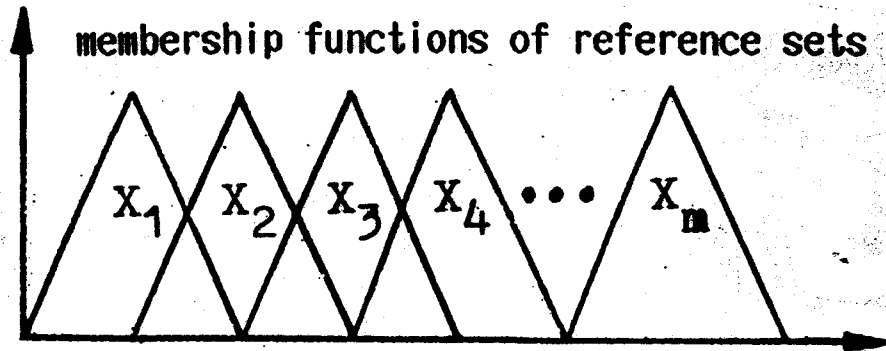


Fig. 1 Reference sets

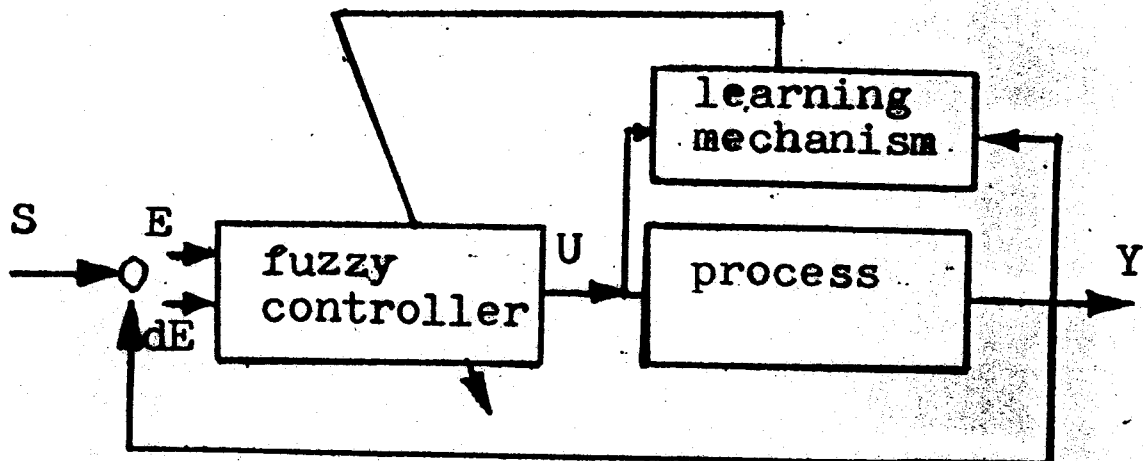


Fig. 2 The structure of the fuzzy controller with self-learning



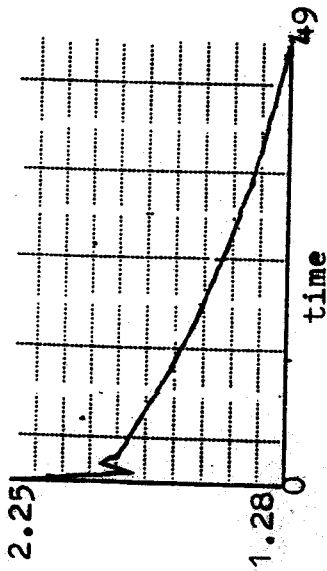


Fig. 3 The response of the process controlled by the conventional controller, set point: 1 noise: 1

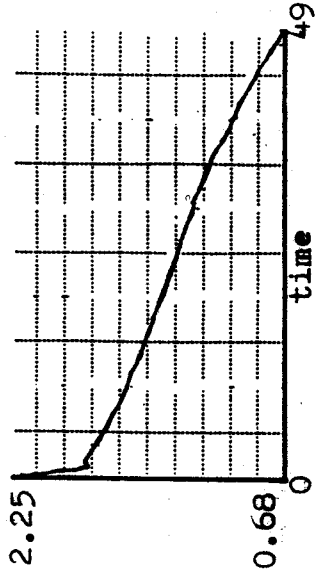


Fig. 5 The response of the process controlled by the fuzzy controller, set point: 1, noise: 1

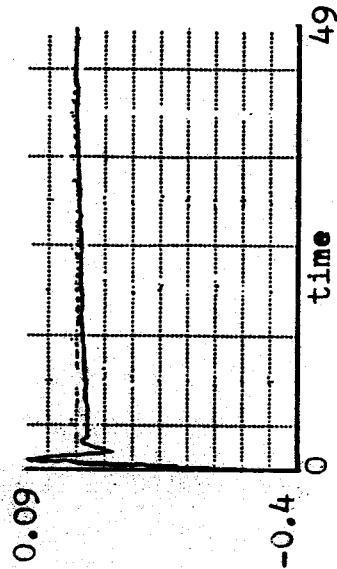


Fig. 4 The control action of the conventional controller, set point: 1, noise: 1

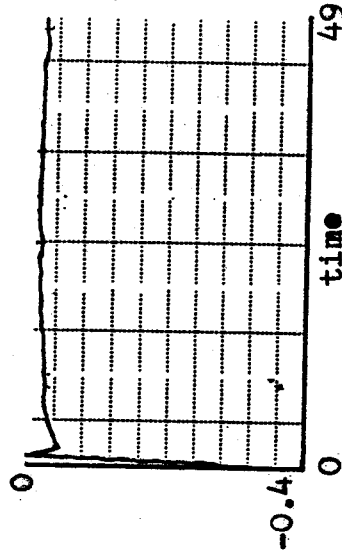


Fig. 6 The control action of the fuzzy controller, set point: 1, noise: 1

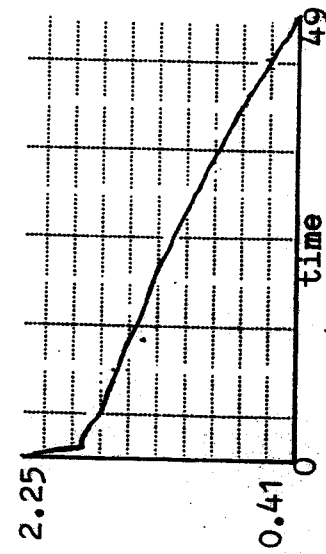


Fig. 7 The response of the process controlled by the fuzzy self-learning controller, set point:1 noise:1

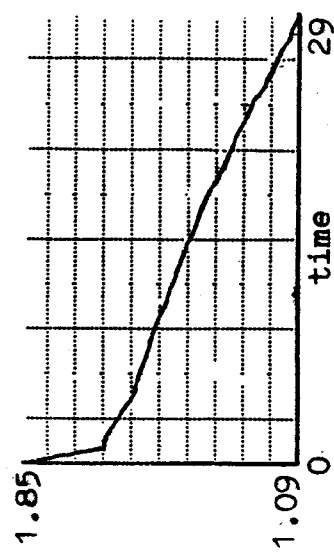


Fig. 9 The response of the process controlled by the fuzzy self-learning controller, set point:0.6, noise:1

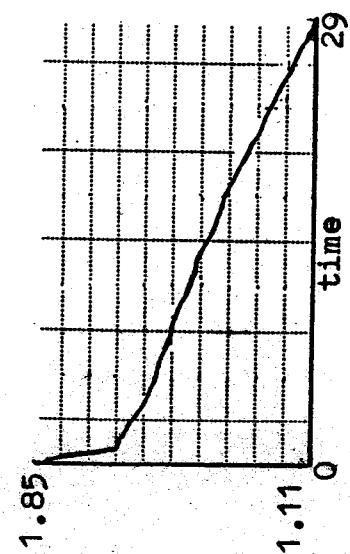


Fig. 8 The response of the process controlled by the fuzzy controller, set point:0.6,noise:1

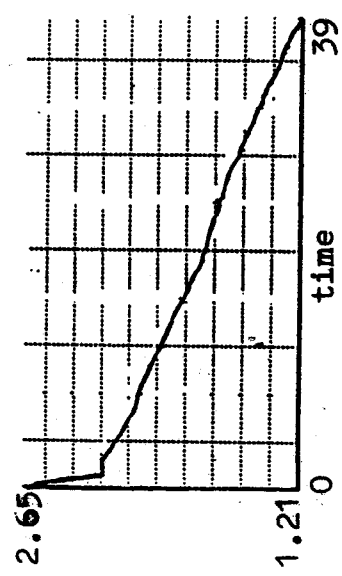


Fig. 10 The response of the process controlled by the fuzzy controller, set point:1.4,noise:1

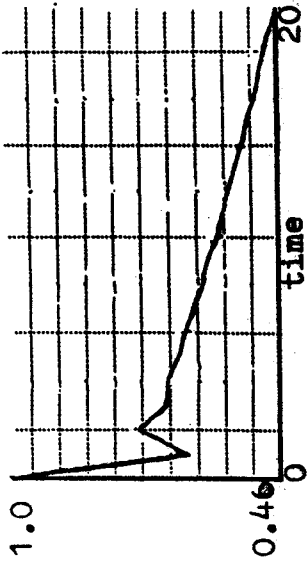


Fig. 13 The response in error controlled by the conventional controller, set point: 0.6, noise: -0.8

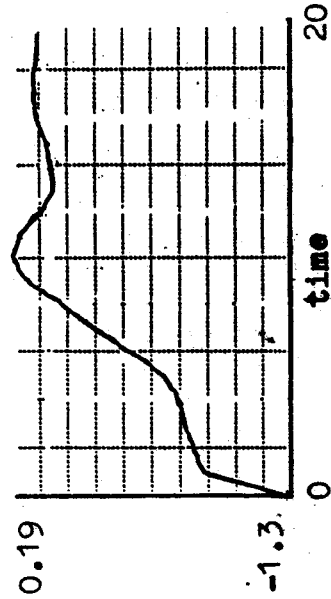


Fig. 14 The response in error controlled by the self-learning fuzzy controller, set point: 1 or -1, noise: 1

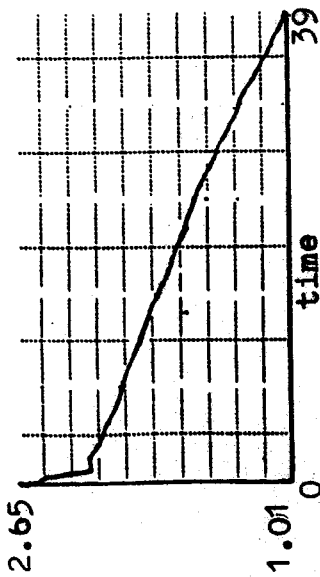


Fig. 11 The response of the process controlled by the fuzzy self-learning controller, set point: 1.4, noise: 1

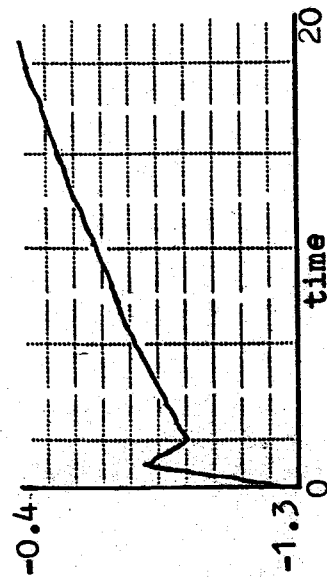


Fig. 12 The response in error controlled by the fuzzy controller, set point: 1 or -1, noise: 1

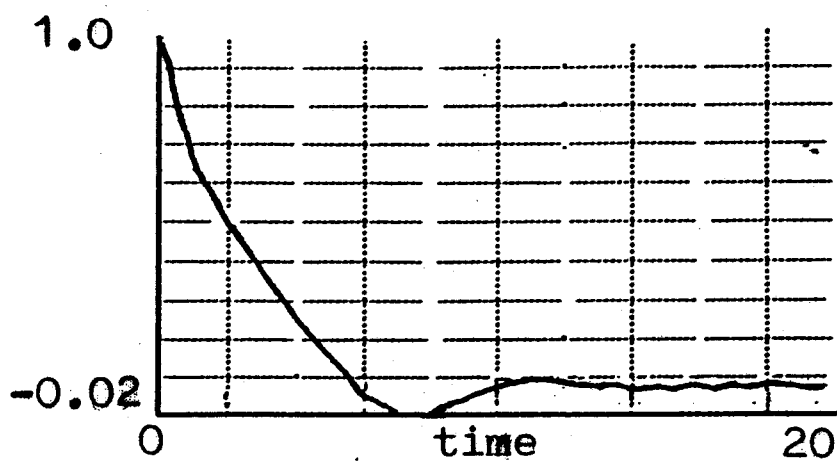


Fig. 15 The response in error controlled by the self-learning fuzzy controller, set point: 0.6, noise: -0.8