LIMIT OF THE SUM AND THE DIFFERENCE OF COMPLEX FUZZY FUNCTIONS

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Basing on the concept of complex fuzzy limit, We would like to discuss the limits of the sum and the difference of two complex fuzzy functions in the present paper.

Theorem 1 If $\lim\limits_{x\to x_0} f(x)=A$, $\lim\limits_{x\to x_0} g(x)=b$,

then, $\lim\limits_{x\to x_0}[f(x)+g(x)]=A+B$.

Proof:

$$\lim_{x\to x_0} P[f(x)+g(x)]=\lim_{x\to x_0}(P[f(x)]+P[g(x)])$$
$$=P[A]+P[B]=P[A+B]$$

$$\lim_{x\to x_0} Q[f(X)+g(X)]=\lim_{x\to x_0}(Q[f(x)]+Q[g(x)])$$
$$=Q[A]+Q[B]=Q[A+B]$$

As for $\lim\limits_{x\to x_0} \inf(f(x)+g(x))=\inf(A+B)$, We shall prove it separately.

First, when at least one of the equations $\inf A=0$, and $\inf B=0$ is true, we may suppose $\inf A=0$ holds.

Since $\lim\limits_{x\to x_0} f(x)=A$, then $\lim\limits_{x\to x_0} \inf f(x)=\inf A=0$.

Therefore, for $\varepsilon=1>0$, there exists real number $\delta>0$, so that when $d[x,x_0]<\delta$, the following will be true:

$$d[\inf f(x),0]=|\inf f(x)-0|=|\inf f(x)|<1,$$

Notice that $\inf f(x)$ takes only 1 or 0, we have $\inf f(x)\equiv 0$.

When $d[x,x_0]<\delta$, $\inf(f(x)+g(x))\equiv 0$,

Thus $\lim\limits_{x\to x_0} \inf(f(x)+g(x))=0$

On the other hand, as $\inf A=0$, so $\inf(A+B)=0$.

Thus $\lim\limits_{x\to x_0} \inf(f(x)+g(x))=\inf(A+B)$.

In the second case, when $\inf A\neq 0$, and $\inf B\neq 0$ are true, both $\inf A=1$ and $\inf B=1$ will also be true.

Since $\lim\limits_{x\to x_0} f(x)=A$, $\lim\limits_{x\to x_0} g(x)=B$,

Then $\lim\limits_{x\to x_0} \inf f(x)=\inf A=1$

$\lim\limits_{x\to x_0} \inf g(x)=\inf B=1$

Thus for $\varepsilon=1>0$, there exists $\delta>0$, so that when $d(x,x_0)<\delta$,

then $d(\inf f(x),1)=|\inf f(x)-1|<1$

$d(\inf g(x),1)=|\inf g(x)-1|<1$.

Again, as $\inf f(x)$ and $\inf g(x)$ take only the real number 1 or 0, there must be $\inf f(x)=1$, and $\inf g(x)=1$ so that the two inequalities could be true. Thus when $d(x,x_0)<\delta$, then

$\lim\limits_{x\to x_0} \inf(f(x)+g(x))=1$

$\inf(f(x)+g(x))=1$ (derived from the definition of complex fuzzy number.)

On the other hand, $\inf A=1$, $\inf B=1$, we have $\inf(A+B)=1$.

then $\lim\limits_{x\to x_0} \inf(f(x)+g(x))=\inf(A+B)$

from Theorem 2 in our previous paper "complex fuzzy limit", we have

$$\lim_{x\to x_0}[f(x)+g(x)]=\lim_{x\to x_0} f(x)+\lim_{x\to x_0} g(x)=A+B$$

Theorem 2 If $\lim\limits_{x\to x_0} f(x)=A$, $\lim\limits_{x\to x_0} g(x)=B$, then

$$\lim_{x\to x_0}[f(x)-g(x)]=A-B$$

proof: $\lim\limits_{x\to x_0} f(x)=A$, $\lim\limits_{x\to x_0} g(x)=B$,

$\lim\limits_{x\to x_0} P[f(x)]=P[A]$, $\lim\limits_{x\to x_0} P[g(x)]=P[B]$

$\lim\limits_{x\to x_0} Q[f(x)]=Q[A]$, $\lim\limits_{x\to x_0} Q[g(x)]=Q[B]$

$$\lim_{x\to x_0} \inf f(x)=\inf A,\quad \lim_{x\to x_0} \inf g(x)=\inf B$$

$$\lim_{x\to x_0} P[f(x)-g(x)]=\lim_{x\to x_0}[P[f(x)]-Q[g(x)]]=$$

$$\lim_{x\to x_0} P[f(x)]-\lim_{x\to x_0} Q[g(x)]=P[A]-Q[B]=P[A-B].$$

$$\lim_{x\to x_0} Q[f(x)-g(x)]=\lim_{x\to x_0}[Q[f(x)]-P[g(x)]]=$$

$$\lim_{x\to x_0} Q[f(x)]-\lim_{x\to x_0} P[g(x)]=Q[A]-P[B]=Q[A-B].$$

The following will be proved separately:

$$\lim_{x\to x_0} \inf[f(x)-g(x)]=\inf[A-B]$$

1. When at least one of the two equations $\inf A=0$ or $\inf B=0$ is tenable, we may assume $\inf A=0$. Since $\lim_{x\to x_0} f(x)=A$, we have $\lim_{x\to x_0} \inf f(x)=\inf A=0$. Then for $\varepsilon=1>0$, there exists real number $\delta>0$, so that when $d(x,x_0)<\delta$,

$$d(\inf f(x),0)=|\inf f(x)|<1 \text{ will hold.}$$

As $\inf f(x)$ takes only 1 or 0, therefore, $\inf f(x)\equiv 0$.

Thus when $d(x,x_0)<\delta$, $\inf[f(x)-g(x)]\equiv 0$

thus $\lim_{x\to x_0} \inf[f(x)-g(x)]=0$.

In addition, since when $\inf A=0$, $\inf[A-B]=0$,

therefore $\lim_{x\to x_0} \inf[f(x)-g(x)]=\inf[A-B]$

2. If both $\inf A=1$ and $\inf B=1$ are true, notice that $\lim_{x\to x_0} f(x)=A$, $\lim_{x\to x_0} g(x)=B$, we have $\lim_{x\to x_0} \inf f(x)=\inf A=1$, $\lim_{x\to x_0} \inf g(x)=\inf B=1$

thus for $\varepsilon=1>0$, there exists real number $\delta>0$, so that when $d(x,x_0)<\delta$

$$d(\inf f(x),\inf A)=|\inf f(x)-\inf A|=|\inf f(x)-1|<1$$

$$d(\inf g(x),\inf B)=|\inf g(x)-\inf B|=|\inf g(x)-1|<1$$

As $\inf f(x)$ and $\inf g(x)$ take only 0 or 1, there must be $\inf f(x)\equiv 1$ and $\inf g(x)\equiv 1$ so that the above-mentioned inequalities could hold. then when $d(x,x_0)<\delta$,

$\inf[f(x)-g(x)] \equiv 1$, thus $\lim_{x\to x_0} \inf[f(x)-g(x)]=1$.

On the other hand, when $\inf A=1$ and $\inf B=1$, $\inf(A-B)=1$.

Therefore, $\lim_{x\to x_0} \inf[f(x)-g(x)]=\inf(A-B)$

To sum up , we obtain

$$\lim_{x\to x_0}[f(x)-g(x)]=\lim_{x\to x_0} f(x)-\lim_{x\to x_0} g(x)=A-B.$$

Reference

[1] Wu Heqin, Yue Changan, Grey functions — complex fuzzy functions, n°42 issue 90 of BUSEFAL.

[2] Wu Heqin, Yue Changan, Grey Limit — limit of complex fuzzy function, n°46 issue 91 of BUSEFAL