A DIRECT ALGORITHM FOR FUZZY CONTROL

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[ABSTRACT]

In this paper, at first, with the help of a simple method of defuzzification, called Direct Method, we simplified the process of fuzzy inference with Max-Min reasoning operator.

As the result of the simplification mentioned above, we give a direct algorithm for fuzzy control with which the calculating amount of fuzzy controller can be largely reduced, so that its control strategy can be regulated on-line, which will increase its flexibility to practical applicational problems.

Finally, a numerical example is given in order to explain how to use the direct algorithm for fuzzy control proposed in this paper. And the result shows us that the a fuzzy controller based on the direct algorithm is superior to a classical PID controller.

[KEYWORDS] FUZZY CONTROL, FUZZY REASONING

I. INTRODUCTION

Since the first fuzzy controller (FC for short) was proposed in 1974 by Mamdani[7], there already have been many successful applicational achievements[1,2]. The reason for this is that FC makes use of some properties of thinking in man-kind brain so that its control policy isn't depended on exactal mathematical model of controlled object. These advantages of FC make up some defects of modern control theory[1,2,3].

Of course, at present, there are a lot of problems with studies and applications of fuzzy control. Firstly, there doesn't exist a perfect theory about FC so that the design of FC is still suffering from being lack of systematicness and the exactal theorical analyses of the control system. Secondly, in order to realize FC, almost all fuzzy controller utilizes the technique of look-up control table, which is obtained off-line, but the calculating amount is very large, specially with the increasement of the discrete level of Universe and the number of input and output. This problem is mainly caused by the calculating of fuzzy relation matrix R. For this reason, some algorithms of FC cannot be executed on a micro computer. Thus, it is difficult to modify its control policy on-line so that its flexibility is not so good.

In this paper, we are trying to propose a new control algorithm with which FC can realize its control strategy without calculating fuzzy relation matrix R .

At first, with the help of a simple method of defuzzification, called Direct Method, we simplified the process of fuzzy inference with Max-Min reasoning operator.

As the result of the simplification mentioned above, we give a direct algorithm for fuzzy control with which the calculating amount of fuzzy controller can be largely reduced, so that its control strategy can be regulated on-line, which will increase its flexibility to practical applicational problems.

Finally, a numerical example is given in order to explain how to use the direct algorithm for fuzzy control proposed in this paper. And the result shows us that the a fuzzy controller based on the direct algorithm is superior to a classical PID controller.

II. A DIRECT METHOD OF DEFUZZIFICATION [4]

There already have been a lot of methods of defuzzification from a fuzzy output [2,3,5,6]. Although the influence of the methods of defuzzification on the reasoning result of FC is small, the method proposed by Peng and Wang [4] is a simple method called Direct MethoD (DMD for short). The method can be described as following formula:

$$y = \sum_{i=1}^{l} H(y^{\sim})*y^{\sim} \Big/ \sum_{i=1}^{l} H(y^{\sim}) \qquad (1)$$

where, $H_i \in F(Y)$, $i=1,\dots,l$

$y_i^{\sim}$ is the center of fuzzy set H_i, y is the defuzzificational output of fuzzy set H, $H \supset (H_1, \dots, H_l)$.

The method is the same as the common gravity method, but the summation is only over the centers of the fuzzy sets $H_i (i=1,\dots, l)$. Therefore, the direct method is simpler than the gravity method. Fourthermore, the results given by [4] show us that the DMD is superior to the common gravity method on the considering the fuzzy compostional operators as Max-*, Sum-* and Max-min.

III. SIMPLIFICATION OF FUZZY REASONING BASED ON DMD [10]

The max-min reasoning operator has been widely used in practical applications, especially in control system, since proposed by Mamdani [7,8].

A lot of work has also been done about the simplification of max-min reasoning, but here we can simplify it with the help of DMD advancedly.

Generally, we give the form of rules as follows:

if x_1 is A_1 AND x_2 is B_1 AND x_3 is C_1 then y is D_{111}
else
......
else
if x_1 is A_n AND x_2 is B_m AND x_3 is C_l then y is D_{nml} (2)

where, all of x_1, x_2, x_3 & y are linguistic variables with the universes of discourse X_1, X_2, X_3 & Y respectively.

$A_i \in F(X_1)$, $B_j \in F(X_2)$, $C_k \in F(X_3)$ & $D_{ijk} \in F(Y)$

for all i=1,2,...n; j=1,2,...m; k=1,2,...l;

We can easily obtained:

$$R(x_1, x_2, x_3, y) = \bigvee_{i,j,k} \{(A_i(x_1) \wedge B_j(x_2) \wedge C_k(x_3)) \wedge D_{ijk}(y)\} \tag{3}$$

for all $x_1 \in X_1, x_2 \in X_2, x_3 \in X_3$ & $y \in Y$;

Suppose that there are input A*, B*, C*, where $A^* \in F(X_1)$, $B^* \in F(X_2)$, $C^* \in F(X_3)$, then the output of FC is:

$$D^* = (A^* \times B^* \times C^*) \circ R \tag{4}$$

Here the Max-min composition operator " o " is used. That is to say:

$$D^*(y) = \bigvee_{X1,X2,X3} [A^*(x_1) \wedge B^*(x_2) \wedge C^*(x_3) \wedge R(x_1, x_2, x_3, y)]$$

$$= \bigvee_{i,j,k} \{[\bigvee_{X1} (A^*(x_1) \wedge A_i(x_1))] \wedge [\bigvee_{X2} (B^*(x_2) \wedge B_j(x_2))]$$

$$\wedge [\bigvee_{X3} (C^*(x_3) \wedge C_k(x_3))] \wedge D_{ijk}(y)\} \tag{5}$$

where, $x_1 \in X_1$, $x_2 \in X_2$, $x_3 \in X_3$, $y \in Y$,

i=1,2,...n; j=1,2,...m; k=1,2,...l;

Let:

$$\alpha_i = \bigvee_{X1} (A^*(x_1) \wedge A_i(x_1)) \tag{6}$$

$$\beta_j = \bigvee_{X2} (B^*(x_2) \wedge B_j(x_2)) \tag{7}$$

$$\gamma_k = \bigvee_{X3} (C^*(x_3) \wedge C_k(x_3)) \tag{8}$$

then , (5) can also be denoted as:

$$D^*(y) = \bigvee_{i,j,k} (\xi_{ijk} \wedge D_{ijk}(y)) \quad \text{for all } y \in Y \tag{9}$$

where, $\xi_{ijk} = \alpha_i \wedge \beta_j \wedge \gamma_k$ (10)

With the help of DMD of defuzzification mentioned above, we can simplify Equ.(9):

suppose that $D_{ijk} \in D = (D_1, D_2, ..., D_q)$ (11)
for all $D_s \in F(Y)$, s=1,2,...q

for all i,j k, $i=1,2,\dots n$; $j=1,\dots,m$; $k=1,\dots,l$;

then

$$D^*(y) = \bigvee_{i,j,k} (\xi_{ijk} \wedge D_{ijk}(y))$$

$$= [\bigvee_{I1} (\xi_{i1} \wedge D_1(y))] \vee \dots \vee [\bigvee_{Iq} (\xi_{iq} \wedge D_l(y))]$$

$$= [(\bigvee_{I1} \xi_{i1}) \wedge D_1(y)] \vee \dots \vee [(\bigvee_{Iq} \xi_{iq}) \wedge D_l(y)]$$

$$= [\xi_1 \wedge D_1(y)] \vee \dots \vee [\xi_q \wedge D_l(y)]$$

$$= \bigvee_{s=1}^{q} (\xi_s \wedge D_s(y)]$$

$$= \bigvee_{s=1}^{q} (D_s^*(y)] \qquad (12)$$

where,

$$\xi_s = \bigvee_{Is} \xi_i \ , \quad s=1,2,\dots,q; \qquad \text{for all } i_s \in I_s \qquad (13)$$

$$\xi_{i_s} = \xi_{ijk}, \text{ if } D_{ijk} = D_s \qquad (14)$$

$$I_1 + I_2 + \dots + I_l = n * m * l \qquad (15)$$

$$D_s^*(y) = \xi_s \wedge D_s(y) \qquad \text{for all } y \in Y \qquad (16)$$

According to (1), we have defuzzificational output of $D^*(y)$:

$$y = \sum_{s=1}^{q} D_s^*(y_s^*) * y_s^* / \sum_{s=1}^{q} D_s^*(y_s^*) \qquad (17)$$

where, y_s^* is the center of fuzzy set D_s^* , $s=1,2,\dots,q$. So, for given fuzzy set distribution D_s $(s=1,2,\dots,q)$, if we can obtain $D_s^*(y_s^*)$, and then we have y .

According to (16), we have that:

$$D_s^*(y_s^*) = \xi_s \wedge D_s(y_s^*) \qquad s=1,2,\dots,q, \quad \xi_s \in [0,1] \qquad (18)$$

When we consider (18), it is obvious that the following theorem is held:

THEOREM: For given fuzzy set $D_s \in F(Y)$, $\xi_s \in [0,1]$. If D_s is normal convex fuzzy set, then

$$D_s^*(y_s^*) = \xi_s \qquad (19)$$

Proof: According to the definition of the normal convex fuzzy set, the proof is obvious.

finished.

So, give D satisfied the condition in THEOREM, $s=1,2,\dots,q$, we have that :

$$y = \sum_{s=1}^{q} \xi_s * y_s^* \Big/ \sum_{s=1}^{q} \xi_s \qquad (20)$$

That is to say, when we adopt a simple method of defuzzification, Max-min inference process will become simpler. We describe it again as follows:

For given inputs A*, B*, C*, then

Step 1. From (6)-(8), we can get α_i, β_j, γ_k.

Step 2. Solving: $\xi_{ijk} = \alpha_i \wedge \beta_j \wedge \gamma_k$

Step 3. Finding: $\xi_s = \bigvee_{Is} \xi_{is}$ $\qquad s=1,2,...,q$

where, ξ_{is} satisfies (14) and ($\sum_{s=1}^{q} Is$) is satisfies (15)

Step 4. From (20), we can get a final output y

Specially, if the input is a single-point fuzzy set, the reasoning process is the same as mentioned above, but seems much simpler.

IV. A DIRECT ALGORITHM FOR FUZZY CONTROL

When the result mentioned above is used to FC, the step1--step4 becomes a direct algorithm for fuzzy control (DAFC for short). In practical applications, most of inputs of fuzzy controller are singleton fuzzy set, i.e.,

$$A^*(x_1) = \begin{cases} 1 & x_1 = x_{10} \\ 0 & x_1 \neq x_{10} \end{cases} \qquad B^*(x_2) = \begin{cases} 1 & x_2 = x_{20} \\ 0 & x_2 \neq x_{20} \end{cases} \qquad C^*(x_3) = \begin{cases} 1 & x_3 = x_{30} \\ 0 & x_3 \neq x_{30} \end{cases}$$

where, $x_i \in X_i$, x_{i0} is the practical input of control system (i=1,2,3).

So, in such case, Equ.(6)--(8) can be advancedly simplified into as follows:

$$\alpha_i = A_i(x_{10}), \qquad \beta_j = B_j(x_{20}), \qquad \gamma_k = C_k(x_{30}) \qquad (21)$$

In other hand, error and error_in_change are often considered as two inputs of FC. The configuration of fuzzy control system based on DAFC is shown in Fig.1, where, K_e and $K_{\dot{e}}$ are all input scalers for error and error_in_change, K_u is a output scaler for control u. E, $\dot{E}$ and U are fuzzy linguistic variables of error(e), error_in_change($\dot{e}$) and control(u) respectively.

From Fig.1, compared with conventional FC system, the off_line calculating step for control table is omitted and the technique of look-up control table isn't used in the fuzzy controller based on DAFC. So, the calculating amount is largely decreased and the flexibility of the control system is enforced. For example, the control policy of the controller can be easily and quickly modified on-line as the control-rule is changed.

V. A NUMERICAL EXAMPLE

In this section, let us give a numerical example to explain how to use the direct algorithm for fuzzy control presented in this paper.

The controlled object is as follows:

$$G(s) = \exp(-2s) \;/\; (30s+1) \qquad (22)$$

The value of fuzzy linguistic variable E and $\dot{E}$ are shown in Fig.2, Fig.3 respectively. The control-rule is shown in table1. In fact, it is another merit of DAFC not to define Membership function about control U in practical applications but only if the center positions of fuzzy distributions in control rule table such as NB, PB,etc., are given:

NB = -1, NM = -0.5, NS = -0.25, ZR = 0
PB = 1, PM = 0.5, PS = 0.25

We can easily obtain a block diagram of operating procedure of control system based on DAFC, shown in Fig.4, according to step1--step4 in section 3, Equ.(21) and Fig.1.

where, $\alpha_i = A_i(e)$, $\beta_j = B_j(\dot{e})$, $\xi_{ij} = \alpha_i \wedge \beta_j$

On the other hand, a classical PID controller is designed by use of CHR method to deteremine its three optimal parameters as follows:

$$Kp = 0.6*T \;/\; K*L = 9, \quad Ti = T = 30\ (s), \quad Td = 0.5*L = 1\ (s)$$

The sampling time is 0.1s, and the reference value r is 100°C. Three parameters for the FC in Fig.1 are follows:

$$Ke = 120, \quad K\dot{e} = 20, \quad Ku = 200$$

The numerical result shown in Fig.5 explains the fact that fuzzy controller based on DAFC also has superior property to a classical PID controller whose parameters is optimal.

VI. A CONCLUSION

There have already been a lot of research papers about the simplification of fuzzy control algorithm[2,9,11]. But, the control method is almost depended on the technique of look-up table. So, such control system, in spite of somewhat adventages, is lack of enough flexibility needed in practical applications and the calculating amount for obtaining a control table is too large, which sets a limitation to some extent on fuzzy control to be used into industrial applications widely.

The direct algorithm for fuzzy control given by author can be avoid of calculating control table and can curry out fuzzy reasoning on-line. The algorithm is simple and can be easily operated so that the flexibility of the control system will be increased, which will enwide the applicational range of fuzzy control at the same time. On the other hand, making use of the DAFC, it is easy to construct an on-line self-learning fuzzy controller, which is an interesting topic.

REFERENCES

[1] Zhang Zhi-Ming, Studies on the Implementation of Fuzzy Inference Machine on Hardware and Its Application, Ph.D thesis, Beijing Normal University, 1990.

[2] Wu Zhi-Qiao, A research on some aspects of fuzzy controller and its applications, Master Thesis, Beijing Inst. of Light Industry, 1987.

[3] Zhang Zhi-Ming, Studies on the Application of Fuzzy Controller to Robot, Master Thesis, Beijing Inst. of Tech., 1987

[4] Xian-Tu Peng, Pei-Zhuang Wang, On generating linguistic rules for fuzzy models, Proc. of IPMU-88, Urbian, Italy, 1988.

[5] Yamazaki, T., An Improved Algorithm for a Self-Organising Controller and Its Experimental Analysis, Ph.D Thesis, London Univ., 1982.

[6] E.Czogala and W.Pedrycz, Control problems in fuzzy systems, Fuzzy Sets and Systems, Vol.7, pp. 257-273, 1982.

[7] E.H. Mamdani, Assilian, Application of Fuzzy Algorithms for Control of a Simple Dynamic Plant, Proc. IEE 212, 1974, pp1585-1588.

[8] E.H.Mamdani, Application of Fuzzy Logic to Approximate reasoning using linguistic system, IEEE Trans. Comput. 26(1977), 1182-1191.

[9] J.F.Baldwin and B.W.Pilsworth, A Model of Fuzzy Reasoning through Multi-valued Logic & Set Theory, Internat.J.Man-Machine Studies 11(1979) 351-380.

[10] Zhang Zhi-Ming, Huang Zhong-Gao, Wang Pei-Zhuang, Several Fuzzy Parallel Inference Models with Max-min Operations, the 1st Inter. Symposium on Uncertainty Modelling & Analysis, Maryland, USA, DEC.3-5, 1990.

[11] Chen Yongyi, Wang Peizhuang, Optimum Fuzzy Implication and Direct Method of Approximate Reasoning, Fuzzy Mathematics, No.1, 1985.

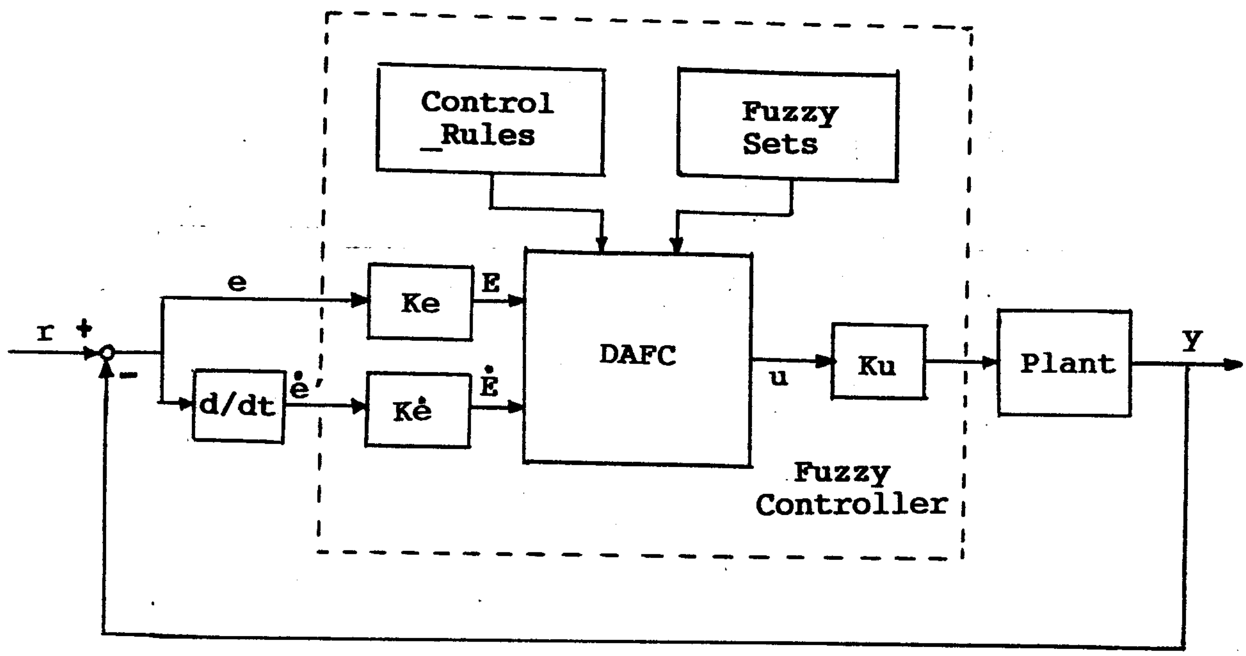


Fig.1 A Configuration of Fuzzy Control System Based on DAFC

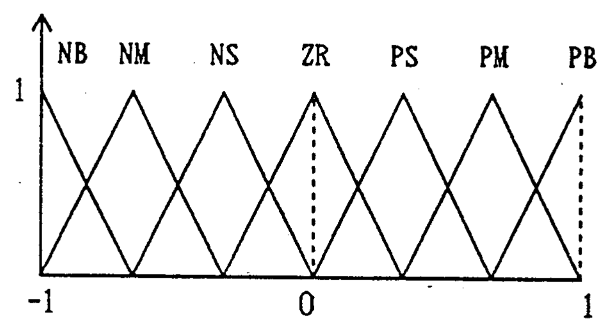


Fig.2 The value of membership function of E

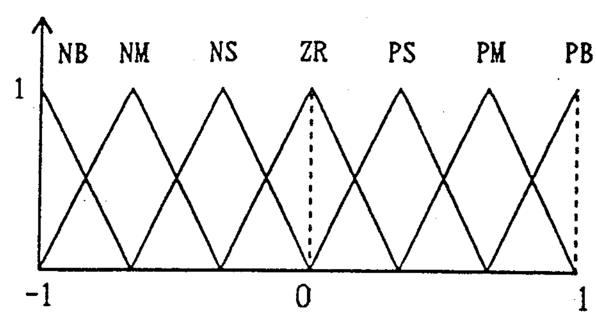
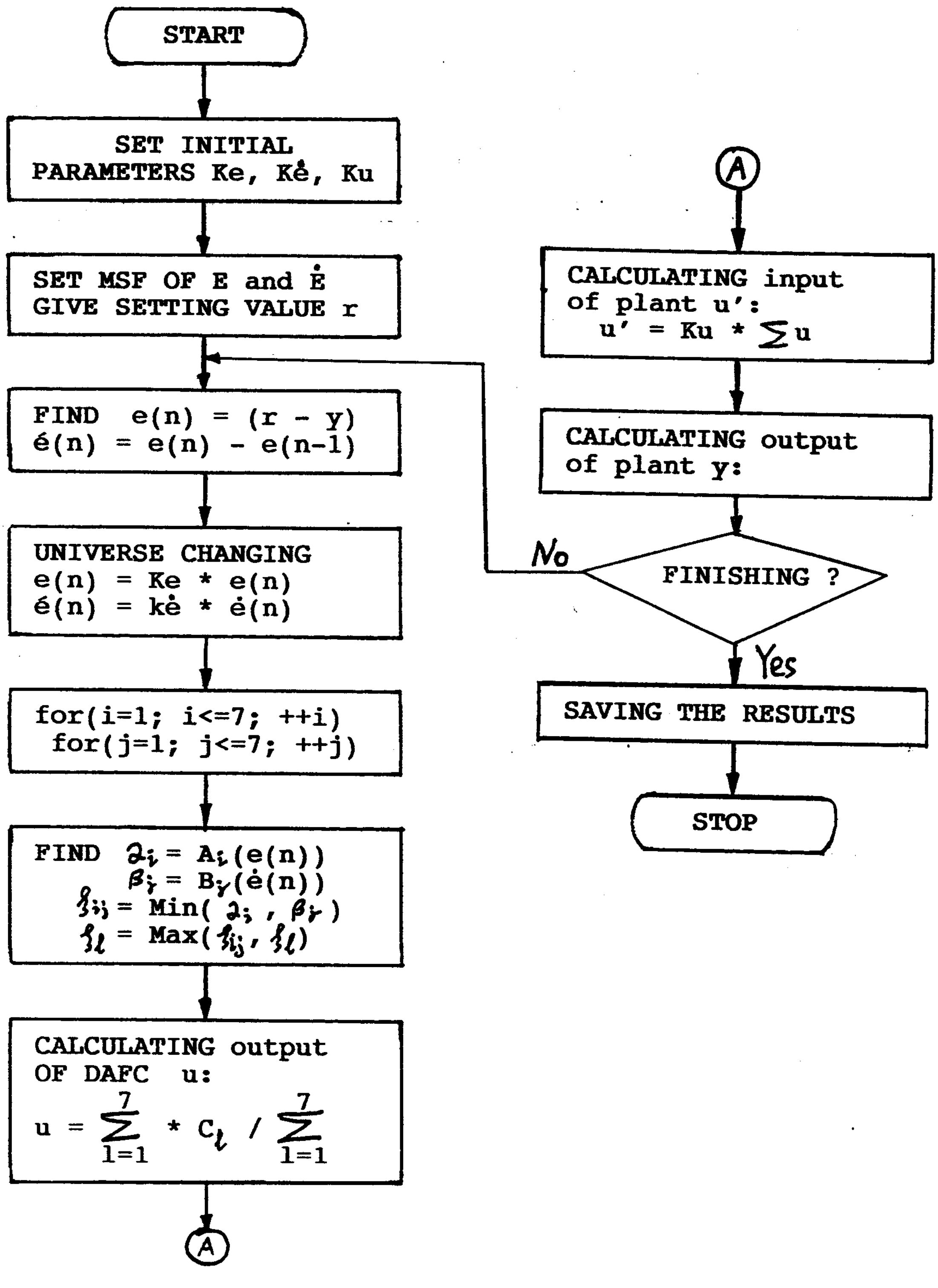


Fig.3 The value of membership function of $\dot{E}$



where, the form of rules from Table 1 is:

if e(n) is A_i and e'(n) is B_j then u is C_l

i = 1, 2, ..,7, j = 1, 2, ..., 7, l = 1, 2, ..., 7

Fig.4 A Block Diagram of Operating Process of DAFC

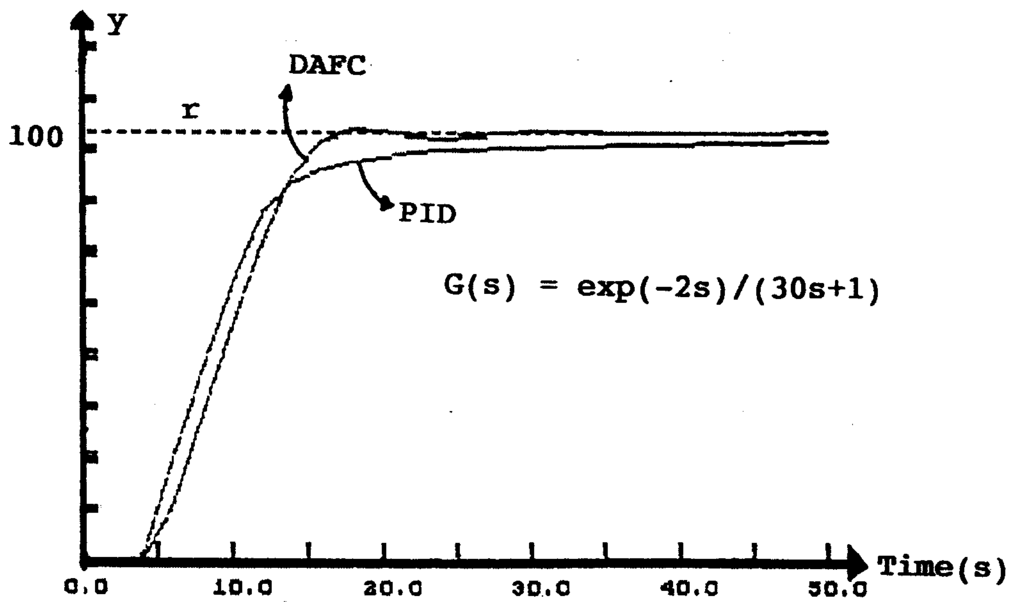


Fig.5 Step Response of a Plant controlled by DAFC

Table1. The Fuzzy Control Rules for DAFC

e \ ė (u)	NB	NM	NS	ZR	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZR
NM	NB	NB	NB	NM	NS	ZR	PS
NS	NB	NB	NM	NS	ZR	PS	PM
ZR	NB	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PM	PB	PB
PM	NS	ZR	PS	PM	PB	PB	PB
PB	ZR	PS	PM	PB	PB	PB	PB