ON SYMMETRIC DECREASING DIAGONALLY DOMINANT FUZZY MATRIX

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ABSTRACT

In this paper , we first introduced the concept of symmetric decreasing diagonally dominant fuzzy matrix, gived a construct method of its realization matrix and point out its content not is greater than the order of itself.

The main results of this paper are as follows:

Definition 1. Let $B = (b_{ij}) \in L^{n\times n}$ such that

(1) $\forall i,j \quad b_{ij} = b_{ji}$;

(2) $\forall i,j \quad b_{ii} \wedge b_{jj} \geqslant b_{ij}$

then we call B the symmetric diagonally dominant fuzzy matrix.

Theorem 1.[1] Let $B = (b_{ij}) \in L^{n\times n}$, then B is a realizable iff

(1) $\forall i,j \quad b_{ij} = b_{ji}$;

(2) $\forall i,j \quad b_{ii} \geqslant b_{ij}$.

Theorem 2.[1] Let $B = (b_{ij}) \in L^{n\times n}$, $\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{pmatrix}$

is a permatation, and $B^* = (b^*_{ij}) \in L^{n\times n}$ here $b^*_{ij} = b_{\sigma(i)\sigma(j)}$ $(i,j=1,2,\ldots,n)$, then the following results hold:

(1) If B is a realizable, then B^* is also a realizable;

(2) If $A = (a_{ij}) \in L^{n\times m}$ such that aA $A' = B$ and $A^* = (a^*_{ij}) \in L^{n\times m}$, where $(a^*_{ij}) = a_{\sigma(i)\, j}$ $(i=1,2,\ldots,n;\ j=1,2,\ldots,m)$, then $B^* = A^*(A^*)'$;

(3) If B is a realizable then $r(B) = r(B^*)$, where $r(B)$ denotes the content of B.

Theorem 3.[2] Let $L = [0,1]$, $B = (b_{ij}) \in L^{n\times n}$ is a realizable, if there is a $B^* = (b^*_{ij}) \in L^{n\times n}$ such that $b^*_{i\ i+1} = \max\{b^*_{ij}|$

$j = i+1, \ldots, n\}$ for $i=1,2, \ldots, n-2$, then B^* is a realizable and $r(B) = r(B^*)$.

Theorem 4.[1] Let $B = (b_{ij}) \in L^{n\times n}$ then B is a realizable iff any its submatrices that is obtained by striking out certain rows and columns with same ordinal numbers is realizable.

Theorem 5. Let $B = (b_{ij}) \in L^{n\times n}$, then B is a realizable iff

(1) $\forall\, i,j \quad b_{ij} = b_{ji}$;

(2) $\forall\, i,j \quad b_{ii} \wedge b_{jj} \geqslant b_{ij}$.

Theorem 6. Let $B = (b_{ij}) \in L^{n\times n}$ $(n \geqslant 3)$ is a realizable and B such that

$$
\begin{array}{l}
b^*_{12} \geqslant b^*_{13} \geqslant \ldots \geqslant b^*_{1n} \\
b^*_{23} \geqslant b^*_{24} \geqslant \ldots \geqslant b^*_{2n} \\
\ldots \quad \ldots \quad \ldots \quad \ldots \\
b^*_{n-2\ n-1} \geqslant b^*_{n-2\ n}
\end{array}
$$

then

$$
A^* = \begin{pmatrix}
b^*_{11} & b^*_{12} & \ldots & b^*_{1\ n-1} & 0 \\
0 & b^*_{22} & \ldots & b^*_{2\ n-1} & b^*_{2n} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & b^*_{n-1\ n-1} & b^*_{n-1\ n} \\
b^*_{1n} & 0 & \ldots & 0 & b^*_{n\ n}
\end{pmatrix}
$$

is a realizable of B^*.

Definition 2 . If $B = (b_{ij}) \in L^{n\times n}$ satisfied:

(1) $\forall\, i,j \quad b_{ij} = b_{ji}$;

(2) $\forall\, i,j \quad b_{ii} \wedge b_{jj} \geqslant b_{ij}$;

(3) $b_{12} \geqslant b_{13} \geqslant \ldots \geqslant b_{1n}$,

$$b_{13} \geqslant b_{23} \geqslant \dots \geqslant b_{2n},$$

$$\dots \quad \dots \quad \dots \quad \dots,$$

$$b_{n-2\ n-1} \geqslant b_{n-2\ n}$$

then B is called by a symmetric decreasing diagonally dominant fuzzy matrix.

Theorem 7. A symmetric decreasing diagonally dominant fuzzy matrix is a realizable, and its content not is greater than order of itself.

Definition 3. For $B = (b_{ij}) \in L^{n\times n}$ if there is a permatation $\sigma = \begin{pmatrix} 1 & 2 & \dots & n \\ i_1 & i_2 & \dots & i_n \end{pmatrix}$ such that $B^* = (b^*_{ij}) \in L^{n\times n}$ where $b^*_{ij} = b_{\sigma(i)\,\sigma(j)}$ $(i,j=1,2,\dots,n)$ is a symmetric decreasing diagonally dominant fuzzy matrix, then B is called subsymmetric decreasing diagonally dominant matrix.

Theorem 8. If $B = (b_{ij}) \in L^{n\times n}$ is a sub-symmetric dereasing diagonally dominat, fuzzy matrix then $r(B) \leq n$.

Theorem 9. Algorithm for finding a realization of B, where B is a sub-symmetric decreasing diagonally dominant fuzzy matrix.

(1) By exchaging some columns of B we make B into decreas by rows upon diagonal and exchage corresponding rows, we get matrix B^*.

(2) On the basis of the proof of theorem 6 write A^*.

(3) By exchanging some rows those are exchanged in (1), we get a realization A of B.

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