THE NOTES ON THE ORTHOGONAL F-MATRIX

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ABSTRACT

The concept of the orthogonal F-matrix was first put forward in the document [1].In this paper we supplemented the properties of an orthogonal F-matrix, and gave a simple and convenient decision method that to judge a F-matrix whether or not is an orthogonal F-matrix, and same time gave a method computed I_A.

Keywords: Orthogonal F-matrix, Decision theorem of the orthogonal F-matrix.

1. PRELIMINARIES

We shall study and discuss on the lattice $L=[0,1]$ in this paper. $L^{n\times m}$ is a set of all $n\times m$ matrices on the lattice L, its every element is called a fuzzy matrix or F-matrix.

Definition 1.1 [1] Let $A=(a_{ij})\in L^{n\times n}$, $n\geq 2$. If there is $\lambda\in[0,1]$ such that

$$A_\lambda (A^T)_\lambda = I_n, \tag{1.1}$$

where I_n is the n-order unit matrix, and $A_\lambda = (a_{ij}(\lambda))_{n\times n}$ is the λ -cut matrix of the A.(Its definition is

$$a_{ij}(\lambda) = \begin{cases} 1, & \text{if } a_{ij}\geq\lambda , \\ 0, & \text{if } a_{ij}<\lambda . \end{cases}$$

Therefore A_λ is a permutation matrix). Then the A is called the orthogonal F-matrix. And λ is called the characteristic number of the A, denoted by λ_A. And the set of all λ_A is called the characteristic interval, denoted by I_A. If a F-matrix B is not an orthogonal, we define its characteristic number is zero. i.e. $\lambda_B = 0$. And its characterstic interval is $I_B = \{0\}$.

Definition 1.2 [1] Let $A \in L^{n\times n}$ $(n \geq 2)$. The A is called the orthogonal F-matrix, if there is $\lambda \in [0,1]$ such that the λ-cut matrix of the A, A_λ , is a permutation matrix.

Wang. X. P. and Liu W.J. proved that the definition 1.1 is equaivalent to the definition 1.2 in the document [1]. A_λ is called the permutation matrix of the A in (1.1).

2.THE COMPLEMENT OF THE PROPERTIES OF AN ORTHOGONAL F-MATRIX

Theorem 2.1 Any invertible F-matrix is orthogonal.

On the definition of an invertible F-matrix see [2,3] .

Theorem 2.2 The permutation matrix A_λ of any orthogonal F-matrix A is only.

Definition 2.1 Let $A \in L^{n\times n}$, $n \geq 2$. the A is called the quasi-permutation matrix. if every row and column of the A have only one non-zero element.

Theorem 2.3 Let $A \in L^{n\times n}$ is a quasi-permutation matrix. Then AA^T and A^TA are the diagonal F-matrix, respectively.

Theorem 2.4 Let $A \in L^{n\times n}$ is a quasi-permutation matrix. Then the A is an orthogonal F-matrix.

Theorem 2.5 Let $A \in L^{n\times n}$ is an orthogonal F-matrix. Then the A is a full row rank F-matrix, a full column rank F-matrix, a nonsingular F-matrix and a full rank F-matrix.

On the concepts of a full row rank, a full column rank, a nonsingular and a full rank F-matrix see [2,3] .

3.THE DECISION THEOREM OF AN ORTHOGONAL F-MATRIX A AND THE COMPUTATIONAL METHOD OF I_A

Theorem 3.1(The decision theorem of an orthogonal F-matrix) Let $A = (a_{ij}) \in L^{n\times n}, n \geq 2$. And let

$$\lambda = \min_i \{ \max_j \{ a_{ij} \} \},$$

$$\mu = \max_i \{ \max_j{}^2 \{ a_{ij} \} \},$$

where $\max_j^2 \{a_{ij}\}$ denotes second maximum element between $a_{i1}, a_{i2}, \ldots,$ and a_{in}. Then

(1) for any $\eta, \xi \in (\mu, \lambda]$. have

$$A_\eta = A_\xi = A_\lambda ;$$

(2) The A is an orthogonal F-matrix iff to have only one element is greater that and equal to λ in every row and column of the A.

(3) If the A is an orthogonal, then $I_A = (\mu, \lambda.]$

Thus we obtain following method:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \quad \begin{matrix} \text{max} & \text{① num.} & \text{max}^2 \\ a_{1j_1} & k_1 & a_{1t_1} \\ \cdots & \cdots & \cdots \\ a_{nj_n} & k_n & a_{nt_n} \end{matrix}$$

② num. $h_1 \ \ldots \ h_n$ $\quad \lambda =$ $\qquad \mu =$

Where (i) In the column "max" fill in the maximun element $a_{ij_i} (1 \leq i \leq n)$ in every row of the A.

(ii) To compute $\lambda = \min \{a_{1j_1}, a_{2j_2}, \ldots, a_{nj_n}\}$, and to fill in the formular.

(iii) In column "① num. " to fill in the number k_i $(1 \leq i \leq n)$ of the element(s) which is greater that and equal to λ in every row of the A.

(iv) In row "② num." of the element(s) which is greater that and equal to λ in every column of the A.

(v) If there is $i \in \{1, \ldots, n\}$ such that $h_i \neq 1$ or $k_i \neq 1$, then this time $I_A = 0$, and the A is not an orthogonal.

(vi) If for every $i \in \{1, \ldots, n\}$, have $h_i = 1$ and

$k_i = 1$, then A is an orthogonal. This time in column "max^2 " to fill in second maximum element a_{it_i} $(1 \leq i \leq n)$ for every row of the A. And compute

$$\mu = \max_i \{ a_{it_i} \} ,$$

then $I_A = (\mu, \lambda]$.

Example 3.1 By

$$A = \begin{pmatrix} .1 & .1 & .2 & .4 & .2 \\ .7 & .2 & .1 & .3 & .2 \\ .2 & .1 & .4 & .2 & .3 \\ .3 & .2 & .1 & .3 & .6 \\ .1 & .8 & .3 & .3 & .1 \end{pmatrix}$$

						max	① num.	max^2
	.1	.1	.2	.4	.2	.4	1	.2
	.7	.2	.1	.3	.2	.7	1	.3
A =	.2	.1	.4	.2	.3	.4	1	.3
	.3	.2	.1	.3	.6	.6	1	.3
	.1	.8	.3	.3	.1	.8	1	.3
② num.	1	1	1	1	1	$\lambda=.4$		$\mu=.3$

,

we know that the A is an orthogonal, and obtains that $I_A=(.3,.4]$. And

$$A_\lambda = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} .$$

Example 3.2 By

						max	① num.
	.1	.4	.2	.4	.2	.4	2
	.7	.2	.5	.3	.2	.7	2
B =	.2	.1	.4	.2	.3	.4	1
	.3	.2	.1	.3	.6	.6	1
	.1	.8	.3	.3	.1	.8	1
② num.	1	2	2	1	1		

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we know the B is not an orthogonal. And $I_B = \{0\}$.

REFERENCES

[1] Wang Xiuping & liu Wangjin, The Orthogonal F-Matrix, '90 CHENGDU, 222--224

[2] Wang Hongxu, Invertible F-Matrix, Full Rank F-Matrix, And Nonsingular F-Matrix, ibid, 217--219

[3] Wang Hongxu, The Fuzzy Nonsingular Matrix, BUSEFAL, No.34, 1988, 107--116