A NEW MODEL FOR DEFAULT REASONING BASED ON POSSIBILITY THEORY

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Abstract:

The purpose of this paper is to discuss the issue of default reasoning in the light of the theory of approximate reasoning. We showed how it is possible to represent pure default knowledge (involving inexact concepts) with the help of possibility theory. We discuss the procedure briefly and then apply the same to a simple yet concrete example.

Key-words:

Default reasoning, approximate reasoning, fuzzy sets.

1. Introduction:

Default logic as found in [1,2,3] was developed aiming at considering the simultaneous representation of the availability of supporting evidences and the absence of contradictory conditions in expert systems. In considering the tentative nature of human thinking and reasoning , that frequently involve inexact concepts , expert systems should be able to cope with such inexactness. It is well-known today that fuzzy logic provides a tool for the representation of uncertainty in expert systems and also for the development of rules of combination of premises which are imprecise, incomplete or not totally reliable [4] . Now approximate reasoning makes it possible for the expert to decide in uncertain environments [5] . Hence, very recently, AI researchers from different parts of the world proposes to represent default reasoning in the framework of the theory of possibility [6] . This paper proposes a model for default reasoning that takes both exact and inexact concepts simutaneously yet works effectively. The proposed method is first described and then a rigorous mathematical formulation is presented with a simple example to illustrate the computations involved in it.

2. Mathematical formulation of the problem:

In this section we discuss the rule of inference in the proposed default logic which, like any other logic, infers a proposition, r, from a set of prerequisite premise $\{p_1, p_2, \ldots, p_n\}$ and a set of conditional premises $\{q_1, q_2, \ldots, q_m\}$ [2] .

In this model we consider a default rule

p: if X_1 is A_1 and X_2 is A_2 and ... and X_n is A_n
and it is consistent to believe that
Y_1 is B_1 and Y_2 is B_2 and ... and Y_m is B_m
then Z is C

where the variables X_i takes their values from the universes U_i; i = 1,2,..,n ; Y_i takes their values from the universes V_i; i = 1,2,..,m ; and the variable Z takes its values from the universe of discourse W respectively .

Also A_i is a fuzzy subset of U_i; i = 1,2,...,n ;
B_i is a fuzzy subset of V_i; i = 1,2,...,m ; and
C is a fuzzy subset of W .

And the set of facts

q: X_1 is A_1'

X_2 is A'_2

$\vdots$

X_n is A'_n

Y_1 is B'_1

Y_2 is B'_2

$\vdots$

Y_k is B'_k $\quad k \leqq m$.

Here A'_i is a fuzzy subset of U_i ; $i = 1,2,\ldots,n$ and
B'_i is a fuzzy subset of V_i ; $i = 1,2,\ldots,k \leqq m$.
It should be mentioned here that the fuzzy sets may as well be crisp charactered and still they may induce a possibility distribution , like the fuzzy ones, where the possibility of an element of the respective universe to belong to the underlying set is exactly 1 or Ø.

The translation of p may be expressed as

$$p \dashrightarrow \Pi_{(X_1,X_2,..,X_n,Y_1,Y_2,..,Y_m,Z)} = R \text{ (,say)}$$

which is an (n+m+1) – dimensional relational matrix defined over the universe $U_1\times U_2\times .. \times U_n\times V_1\times V_2\times .. \times V_m\times W$ where

$$\mu_R(u_1,u_2,..,u_n,v_1,v_2,..,v_m,w)$$
$$= \min \{ \mu_{A_1}(u_1),\mu_{A_2}(u_2),..,\mu_{A_n}(u_n),\mu_{B_1}(v_1),\mu_{B_2}(v_2),..,\mu_{B_m}(v_m),\mu_C(w)\}.$$

And the translation of q may be expressed as

$$q \dashrightarrow \Pi_{(X_1,X_2,..,X_n,Y_1,Y_2,..,Y_k)} = S \text{ (,say)}$$

which is an (n+k)-dimensional relational matrix defined over the universe $U_1\times U_2\times .. \times U_n\times V_1\times V_2\times .. \times V_k$ where

$$\mu_S(u_1,u_2,..,u_n,v_1,v_2,..,v_k) = \min \{\mu'_A(u_1),\mu'_A(u_2),\ldots,$$
$$\mu'_A(u_n),\mu'_B(v_1),\mu'_B(v_2),\ldots,\mu'_B(v_k) \}.$$

Now keeping in mind the most optimistic values for the default variables about which the expert has no information we form the cylindrical extension of S over $V_{k+1} \times V_{k+2} \times \ldots \times V_m \times W$. Let

$$\bar{S} = S \times V_{k+1} \times V_{k+2} \times V_m \times W$$

then $$\mu_{\bar{S}}(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_m,w) = \mu_S(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_k).$$

And the particularisation of R by S, denoted by T, will be given by $T = R \sqcap \bar{S}$ and is such that

$$\mu_T(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_m,w) = \{\mu_R(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_m,w) \wedge \mu_S(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_k)\}$$

where $\wedge$ stands for the well_known min operator.

The required default inference can then be given by projecting T on W . It, therefore, follows that the desired inference will be

$$r \longleftarrow Z \text{ is } C' = Proj_W T$$

where

$$\mu_{C'}(w) = \sup_{(u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_m)} \mu_T(u_1,\ldots,u_n,v_1,\ldots,v_m)$$

$$= \sup \{ \mu_R \wedge \mu_{\bar{S}} \}$$

$$= \sup_{(u_1,\ldots,u_n,v_1,\ldots,v_m)} \{ \mu_R \wedge \mu_S(u_1,\ldots,u_n,v_1,\ldots,v_k)\}$$

3. Numerical Example:

In this section we consider a simple example, where the variables involved in the premises range over finite sets or can be approximated by variables ranging over such sets [5] , to illustrate the computations involved in it. For that , let us consider a default rule

p: if X is A and it is consistent to believe that Y is B
then Z is C

together with the fact that

q: X is A'.

The translation of p may be expressed as

$$p \dashrightarrow \Pi_{(X,Y,Z)} = R \text{ (,say)}$$

where

$$R = A \cap B \cap C$$

such that

$$\mu_R(u,v,w) = \min \{ \mu_A(u), \mu_B(v), \mu_C(w) \}.$$

Let X, Y, Z range respectively over U, V, W given by

$$U = u_1 + u_2 + u_3 + u_4$$

$$V = v_1 + v_2 + v_3 + v_4$$

$$W = w_1 + w_2 + w_3.$$

Now in p and q A and A' are fuzzy subsets of the universe U. B and C are fuzzy subets of V and W respectively. Let them be

$$A = 1./u_1 + .7/u_2 + .4/u_3 + .1/u_4$$

$$B = .5/v_1 + .8/v_2 + 1./v_3 + .6/v_4$$

$$C = .25/w_1 + .65/w_2 + 1/w_3$$

and $A' = \text{very } A = 1./u_1 + .49/u_2 + .16/u_3 + .Ø1/u_4.$

Then $R = .1 / [u_4v_1w_1 + u_4v_1w_2 + u_4v_1w_3 + u_4v_2w_1 + u_4v_2w_2 + u_4v_2w_3 +$
$u_4v_3w_1 + u_4v_3w_2 + u_4v_3w_3 + u_4v_4w_1 + u_4v_4w_2 + u_4v_4w_3] + .25 /$
$[u_1v_1w_1 + u_1v_2w_1 + u_1v_3w_1 + u_1v_4w_1 + u_2v_1w_1 + u_2v_2w_1 + u_2v_3w_1 +$
$u_2v_4w_1 + u_3v_1w_1 + u_3v_2w_1 + u_3v_3w_1 + u_3v_4w_1] + .4 / [u_3v_1w_2 +$
$u_3v_1w_3 + {}_3u_2v_2w + u_3v_2w_3 + u_3v_3w_2 + {}_3u_3v_3w + u_3v_4w_2 + u_3v_4w_3] +$
$.5 / [u_1v_1w_2 + u_1v_1w_3 + u_2v_1w_2 + u_2v_1w_3] + .6 / [u_1v_4w_2 +$
$u_1v_4w_3 + u_2v_4w_2 + u_2v_4w_3] + .65 / [u_1v_2w_2 + u_1v_3w_2 + u_2v_2w_2 +$
$u_2v_3w_2] + .7 / [u_2v_2w_3 + u_2v_3w_3] + .8 / u_1v_2w_3 + 1 / u_1v_3w_3 .$

And

$$\bar{S} = S \times V \times W$$

will be given by

$$\bar{S} = .01 / [\, u_4v_1w_1 + u_4v_1w_2 + u_4v_1w_3 + u_4v_2w_1 + u_4v_2w_2 + u_4v_2w_3 + u_4v_3w_1 + u_4v_3w_2 + u_4v_3w_3 + u_4v_4w_1 + u_4v_4w_2 + u_4v_4w_3 \,] + .16 / [\, u_3v_1w_1 + u_3v_1w_2 + u_3v_1w_3 + u_3v_2w_1 + u_3v_2w_2 + u_3v_2w_3 + u_3v_3w_1 + u_3v_3w_2 + u_3v_3w_3 + u_3v_4w_1 + u_3v_4w_2 + u_3v_4w_3 \,] + .49 / [\, u_2v_1w_1 + u_2v_1w_2 + u_2v_1w_3 + u_2v_2w_1 + u_2v_2w_2 + u_2v_2w_3 + u_2v_3w_1 + u_2v_3w_2 + u_2v_3w_3 + u_2v_4w_1 + u_2v_4w_2 + u_2v_4w_3 \,] + 1 / [\, u_1v_1w_1 + u_1v_1w_2 + u_1v_1w_3 + u_1v_2w_1 + u_1v_2w_2 + u_1v_2w_3 + u_1v_3w_1 + u_1v_3w_2 + u_1v_3w_3 + u_1v_4w_1 + u_1v_4w_2 + u_1v_4w_3 \,] .$$

And then

$$T = R \sqcap \bar{S}$$

will be given by

$$T = .01 / [\, u_4v_1w_1 + u_4v_1w_2 + u_4v_1w_3 + u_4v_2w_1 + u_4v_2w_2 + u_4v_2w_3 + u_4v_3w_1 + u_4v_3w_2 + u_4v_3w_3 + u_4v_4w_1 + u_4v_4w_2 + u_4v_4w_3 \,] + .16 / [\, u_3v_1w_1 + u_3v_1w_2 + u_3v_1w_3 + u_3v_2w_1 + u_3v_2w_2 + u_3v_2w_3 + u_3v_3w_1 + u_3v_3w_2 + u_3v_3w_3 + u_3v_4w_1 + u_3v_4w_2 + u_3v_4w_3 \,] + .25 / [\, u_1v_1w_1 + u_1v_2w_1 + u_1v_3w_1 + u_1v_4w_1 + u_2v_1w_1 + u_2v_2w_1 + u_2v_3w_1 + u_2v_4w_1 \,] + .49 / [\, + u_2v_1w_2 + u_2v_1w_3 + u_2v_2w_2 + u_2v_2w_3 + u_2v_3w_2 + u_2v_3w_3 + u_2v_4w_2 + u_2v_4w_3 \,] + .5 / [\, u_1v_1w_2 + u_1v_1w_3 \,] + .6 / [\, u_1v_4w_2 + u_1v_4w_3 \,] + .65 / [\, u_1v_2w_2 + u_1v_3w_2 \,] + .8 / u_1v_2w_3 + 1 / u_1v_3w_3 .$$

Now let $\mathrm{Proj}_W\, T = C'$ (,say). Then

$$\mu_{C'}(w) = \sup_{(u,v)} \mu_T(u,v,w) .$$

Which at once gives

$$C' = .25 / w_1 + .65 / w_2 + 1 / w_3 .$$

And the required inference will be

$$Z \text{ is } C' .$$

4. Conclusion:

The present model for default reasoning in Expert systems based on the idea of approximate reasoning , as is presented in this paper , may be taken as another attempt to represent inexactness , uncertainty , imprecision or incompleteness of the real knowledges . Certainly it is not as precise as classical reasoning simply because of its dependancy on fuzzy logic . Although it is well_ known that the real world is full of indecision yet it is not getting crippled . Rather it gives us another oppurtunity to define the most complicated intelligent system in a way as simple as is possible and whose behaviour would be as close to the behaviour of any humanistic system .

5. **References:**

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