REMARK ON NEW OBJECTS RELATED WITH INTUITIONISTIC FUZZY SETS

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Let E be a fixed universe. In the frames of the ordinary set theory we can define for every x, y ∈ E the following three sets:

$$\underline{we}(x, y) = \{X \,/\, x, y \in X \ \&\ X \subset E\},$$

$$\underline{you}(x, y) = \{X \,/\, x \notin X \ \&\ y \in X \ \&\ X \subset E\},$$

$$\underline{they}(x, y) = \{X \,/\, x, y \notin X \ \&\ X \subset E\}.$$

Obviously, if card(E) = n, then:

(a) $\text{card}(\underline{we}(x, y)) = \text{card}(\underline{you}(x, y))$

$$= \text{card}(\underline{they}(x, y)) = 2^{n-2}.$$

(b) $\underline{we}(x, y) \cup \underline{you}(x, y) \cup \underline{you}(y, x) \cup \underline{they}(x, y) = P(E)$,

where $P(X) = \{Y \,/\, Y \subset X\}$.

In the case of Intuitionistic Fuzzy Sets (IFS) [1] (the set A is an IFS in the universe E, if A has the form:

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle \,/\, x \in E\},$$

where $\mu_A(x)$ and $\gamma_A(x)$ are degrees of membership and of non-membership of x ∈ E, respective, and $\mu_A(x) + \gamma_A(x) \le 1$), by analogy with [2], we shall define the following:

the element x ∈ E is sure in A ⊂ E iff $\mu_A(x) \ge 1/2$.

Obviously, for this element are valid:

$$\gamma_A(x) \le 1/2 \text{ and } \mu_A(x) \ge \gamma_A(x)$$

(this is an analogous of the concept "intuitionistic fuzzy tautology" from [2]). By analogy with the above definitions, we can introduce the following objects, related to IFSs:

$$\underline{we}(x, y) = \{X \,/\, \mu_X(x) \ge 1/2 \ \&\ \mu_X(y) \ge 1/2 \ \&\ X \subset E\},$$

$$\underline{you}(x, y) = \{X \,/\, \gamma_X(x) > 1/2 \ \&\ \mu_X(y) \ge 1/2 \ \&\ X \subset E\},$$

$$\underline{they}(x, y) = \{X \,/\, \gamma_X(x) > 1/2 \ \&\ \gamma_X(y) > 1/2 \ \&\ X \subset E\}.$$

Obviously, we(x, y), you(x, y) and they(x, y) are sets of ordinary (non-IFSs) sets. The above properties (a) and (b) are not valid here.

For example, if $E = \{x, y, z\}$ and $A = \{<x, 0.3, 0.3>, <y, 0.3, 0.3>, <z, 0.3, 0.3>$, then $A \in P(E \times [0, 1]^2)$, but $A \notin$ we(x, y) U you(x, y) U you(y, x) U they(x, y). Therefore there exist $x, y \in E$ for which

we(x, y) U you(x, y) U you(y, x) U they(x, y) $\neq P(E \times [0, 1]^2)$.

On the other hand, the above defined sets in the terms of the ordinary fuzzy sets have the forms:

$$\underline{we}(x, y) = \{X / \mu_X(x) \geq 1/2 \ \& \ \mu_X(y) \geq 1/2 \ \& \ X \subset E\},$$

$$\underline{you}(x, y) = \{X / \mu_X(x) < 1/2 \ \& \ \mu_X(y) \geq 1/2 \ \& \ X \subset E\},$$

$$\underline{they}(x, y) = \{X / \mu_X(x) < 1/2 \ \& \ \mu_X(y) < 1/2 \ \& \ X \subset E\},$$

and the above property (b) is valid for these sets.

Therefore, there exist ordinary set properties which are not valid for IFSs.

THEOREM 1: For every $x, y \in E$

(a) we(x, y) = we(y, x),

(b) you(x, y) ∩ you(y, x) = ∅,

(c) they(x, y) = they(y, x).

THEOREM 2: For every $x, y \in E$

(a) we(x, y) is a filter

(b) they(x, y) is an ideal

in the sense of [3].

Proof: (a) Let $Y, Z \in$ we(x, y). Then

$$\mu_Y(x) \geq 1/2 \ \& \ \mu_Y(y) \geq 1/2 \ \& \ Y \subset E$$

$$\mu_Z(x) \geq 1/2 \ \& \ \mu_Z(y) \geq 1/2 \ \& \ Z \subset E$$

and therefore for the IFS $X = Y \cap Z$ follows, that

$$\mu_X(x) = \min(\mu_Y(x), \mu_Z(x)) \geq 1/2,$$

$$\mu_X(y) = \min(\mu_Y(y), \mu_Z(y)) \geq 1/2.$$

Hence $Y \cap Z \in$ <u>we</u>(x, y).

Let $X \in$ <u>we</u>(x, y) and $X \subset Y$. Then for every $x, y \in E$:

$$1/2 \le \mu_X(x) \le \mu_Y(x),$$

$$1/2 \le \mu_X(y) \le \mu_Y(y),$$

and therefore $Y \in$ <u>we</u>(x, y).

(b) Let $Y, Z \in$ <u>they</u>(x, y). Then

$$\gamma_Y(x) > 1/2 \ \& \ \gamma_Y(y) > 1/2 \ \& \ Y \subset E$$

$$\gamma_Z(x) > 1/2 \ \& \ \gamma_Z(y) > 1/2 \ \& \ Z \subset E$$

and therefore for the IFS $X = Y \cup Z$ follows, that

$$\gamma_X(x) = \min(\gamma_Y(x), \gamma_Z(x)) > 1/2,$$

$$\gamma_X(y) = \min(\gamma_Y(y), \gamma_Z(y)) > 1/2.$$

Hence $Y \cup Z \in$ <u>they</u>(x, y).

Let $X \in$ <u>they</u>(x, y) and $Y \subset X$. Then for every $x, y \in E$:

$$1/2 < \gamma_X(x) \le \gamma_Y(x),$$

$$1/2 < \gamma_X(y) \le \gamma_Y(y),$$

and therefore $Y \in$ <u>they</u>(x, y). ◊

In [4] are defined two operators which are similar to the operators "necessity" and "possibility" defined in some modal logics. They have the forms for every IFS A:

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle / x \in E\};$$

$$\Diamond A = \{\langle x, 1-\gamma_A(x), \gamma_A(x)\rangle / x \in E\}.$$

Let A is the IFS which corresponds to the ordinary set $\bar{A}$ (see e.g., [5]).

Obviously, it is valid

<u>THEOREM 3</u>: For every $x, y \in E$

(a) if $\bar{X} \in$ <u>we</u>(x, y), then $\overline{\Diamond X} \in$ <u>we</u>(x, y),

(b) if $\bar{X} \in$ <u>they</u>(x, y), then $\overline{\Box X} \in$ <u>they</u>(x, y).

It is valid more general assertion too.

THEOREM 4: For every $x, y \in E$

$$\text{(a) if } \overline{X} \in \underline{we}(x, y), \text{ then } \overline{\Box X} \in \underline{we}(x, y),$$

$$\text{(b) if } \overline{X} \in \underline{they}(x, y), \text{ then } \overline{\Diamond X} \in \underline{they}(x, y).$$

Proof: (a) Let $\overline{X} \in \underline{we}(x, y)$. Then

$$\mu_X(x) \geq 1/2 \ \& \ \mu_X(y) \geq 1/2 \ \& \ \overline{X} \subset E.$$

Hence for the IFS $\Box X$ is valid that

$$\mu_{\Box X}(x) \geq 1/2 \ \& \ \mu_{\Box}(y) \geq 1/2 \ \& \ \overline{\Box X} \subset E$$

and therefore for the IFS $\Box X$ follows, that $\overline{\Box X} \in \underline{we}(x, y)$.

(b) is proved abalogously. $\Diamond$

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