Fuzzy Almost Semicontinuity

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Abstract

In this paper we will introduce the concept of fuzzy almost semicontinuous and fuzzy almost semiopen mappings. And we will also discuss the characteristic properties, and relations between those and some other mappings.

Key words

Fuzzy regular open set; Fuzzy semiopen set; Fuzzy semicontinuous mapping; Fuzzy almost continuous mapping; Fuzzy almost semicontinuous mapping; Fuzzy almost semiopen mapping.

In this work, A^{o}, A^{-}, A_{o}, A_{-} and A' will denote respectively the interior, closure, semi-interior[3], semi-closure[3], and complement of the fuzzy set A.

Definition 1. Let $f:(X_1, \delta_1)\to(X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is called,

(1) a fuzzy almost semicontinuous mapping if $f^{-1}(B)$ is a fuzzzy semiopen set of X_1 for each fuzzy regular open set B of X_2.

(2) a fuzzy almost semiopen mapping if f(A) is a fuzzy semiopen set of X_2 for each fuzzy regular open set A of X_1.

Remark 1. For the mapping $f:(X_1, \delta_1)\to(X_2, \delta_2)$, the following statements are valid:

(1) f, fuzzy semicontinuous [2] => f, fuzzy almost semicontinuous,

(2) f, fuzzy almost continuous [2] => f, fuzzy almost semicontinuous,

(3) f, fuzzy semiopen [2] => f, fuzzy almost semiopen,

(4) f, fuzzy almost open [3] => f, fuzzy almost semiopen,

(5) fuzzy weakly continuous [2] and fuzzy almost semicontinuous, mappings are independent notions.

Theorem 1. Let f: $(X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost semicontinuous.

(2) $f^{-1}(B)$ is a fuzzy semiclosed set of X_1 for each fuzzy regular closed set B of X_2.

(3) $f^{-1}(B) \leqslant (f^{-1}(B^{-o}))_o$, for each $B \in \delta_2$.

(4) $(f^{-1}(B^{o-}))_- \leqslant f^{-1}(B)$, for each fuzzy closed set B of X_2.

Definition 2. Let f: $(X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2, f is said to be fuzzy almost semicontinuous at a fuzzy point p in X_1, if fuzzy regular open set B in X_2 and $f(p) \leqslant B$, there exists a fuzzy semiopen set A in X_1 such that $p \leqslant A$ and $f(A) \leqslant B$.

Theorem 2. A mapping f: $(X_1, \delta_1) \to (X_2, \delta_2)$ is fuzzy almost semicontinuous iff f is fuzzy almost semicontinuous for each fuzzy point p in X_1.

Proof. Let f be fuzzy almost semicontinuous, p be a fuzzy point in X_1 and B be a fuzzy regular open set in X_2 such that $f(p) \leqslant B$. Then $p \leqslant f^{-1}(B) = (f^{-1}(B))_o$. Let $A = f^{-1}(B)$, then A is fuzzy semiopen set in X_1, and so $f(A) = ff^{-1}(B) \leqslant B$. Thus f is fuzzy almost semicontinuous for each fuzzy point p in X_1.

Conversely, let B be a fuzzy regular open set in X_2 and p be a fuzzy point in $f^{-1}(B)$. Then $p \leqslant f^{-1}(B)$, i.e., $f(p) \leqslant B$. From hypothesis there exists a fuzzy semiopen set A in X_1 such that $p \leqslant A$ and $f(A) \leqslant B$, hence

$$p \leqslant A \leqslant f^{-1}f(A) \leqslant f^{-1}(B)$$

and

$$p \leqslant A = A_0 \leqslant (f^{-1}(B))_0.$$

Since p is arbitrary and $f^{-1}(B)$ is the union of all fuzzy points in $f^{-1}(B)$, $f^{-1}(B) \leqslant (f^{-1}(B))_0$, i.e., $f^{-1}(B) = (f^{-1}(B))_0$. Thus f is fuzzy almost semicontinuous.

Theorem 3. Let $f:(X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to a fuzzy regular space [2] X_2. Then f is fuzzy almost semicontinuous iff f is fuzzy semicontinuous.

Theorem 4. Let $f:(X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping. Then the following are equivalent:

(1) f is fuzzy almost semiopen.

(2) $f(A) \leqslant (f(A^{-0}))_0$, for each $A \in \delta_1$.

(3) For each fuzzy set B of X_2 and each fuzzy regular closed set A of X_1, when $f^{-1}(B) \leqslant A$, there exists a fuzzy semiclosed set C of X_2, such that $B \leqslant C$ and $f^{-1}(C) \leqslant A$.

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