A Note On Fuzzy Topological Maps

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Abstract: The notion of a fuzzy topological map is introduced and it is shown that the necessity part of the fuzzy version of Theorem2.1[2] is not true.

Let X be a set and $I=[0,1]$.

Unles otherwise mentioned as regards fuzzy notions and notations we follow Ming and Ming[3].

Let $X=\{x_1, x_2, \ldots, x_n\}$ and $a_i \in I$ for $i=1,2,\ldots n$. By $(a_1, a_2, \ldots a_n)$ we shall mean the fuzzy subset A of X such that $A(x_i) = a_i$.

Let (X,T) and (Y,T') be two fuzzy topological spaces.

Let $\mathcal{B}$ be the family of all fuzzy points $\{x_p, p \in I-\{0\}, x \in X\}$ in X. We define two relations $\underset{T}{\leqslant}$ and $\underset{T}{=}$ in $\mathcal{B}$ by

$x_p \underset{T}{\leqslant} y_q$ iff $\forall\ O \in T$, $y_q \in O \implies x_p \in O$.

$x \underset{T}{=} y$ iff $\forall\ O \in T$, $x_p \underset{T}{\leqslant} y_q$ and $y_q \underset{T}{\leqslant} x_p$.

The reletion $\underset{T}{=}$ is an equivalence reletion in $\mathcal{B}$. The equivalence class containing the fuzzy point $x_p \in \mathcal{B}$ will be denoted by (x_p). Then $\forall\ O \in T$, $O = \bigcup_{x_p \in O} (x_p)$.

The set of all fuzzy continuous maps of (X,T) into (Y,T') is denoted by $C(X,Y;T,T')$.

Theorem 1: $\forall\ f \in C(X,Y;T,T')$ and $\forall\ x_p, y_q \in \mathcal{B}$, $x_p \underset{T}{\leqslant} y_q \implies f(x_p) \underset{T'}{\leqslant} f(y_q)$.

Proof: Let $x_p \underset{T}{\leqslant} y_q$ and let $O \in T'$.

Since $f \in C(X,Y;T,T')$, $f^{-1}(O) \in T$.

Since $x_p \underset{T}{\leqslant} y_q$, $y_q \in f^{-1}(O) \implies x_p \in f^{-1}(O)$.

So $f(y_q) \in O \implies f(x_p) \in O$.

Therefore $f(x_p) \lesssim_{T'} f(y_q)$.

Corollary:1.1: $\forall f \in C(X,Y;T,T')$, $x_p \equiv_T y_q$ ===> $f(x_p) \equiv_{T'} f(y_q)$.

Definition 1.2: $f, g \in C(X,Y:T,T')$ are said to be fuzzy topologically equivalent written as $f \underset{(T,T')}{=} g$ iff $f(x_p) \equiv_{T'} g(x_p)\ \forall x_p \in \mathcal{B}$.

Theorem 1.3: $\forall f, g \in C(X,Y:T,T')$, $f \underset{(T,T')}{=} g$ iff $f^{-1}(O) = g^{-1}(O)$ $\forall O \in T'$.

Proof: Let $f \underset{(T,T')}{=} g$ and let there be $O \in T'$ such that $f^{-1}(O) \neq g^{-1}(O)$. Then there exist $x_p \in \mathcal{B}$ such that $x_p \in f^{-1}(O)$ but $x_p \notin g^{-1}(O)$ or $x_p \in g^{-1}(O)$ but $x_p \notin f^{-1}(O)$.

That is, $f(x_p) \in O$ but $g(x_p) \notin O$ or $g(x_p) \in O$ but $f(x_p) \notin O$.

So $f(x_p) \not\equiv_T g(x_p)$, a contradiction.

Therefore $f^{-1}(O) = g^{-1}(O)$.

Conversely let $f^{-1}(O) = g^{-1}(O)$.

If possible let $f \underset{(T,T')}{\neq} g$. Then there exists a $x_p \in \mathcal{B}$ such that $f(x_p) \not\equiv_{T'} g(x_p)$.

Therefore there exists a $O \in T'$ such that $f(x_p) \in O$ but $g(x_p) \notin O$ or $g(x_p) \in O$ but $f(x_p) \notin O$.

That is, $x_p \in f^{-1}(O)$ but $x_p \notin g^{-1}(O)$ or $x_p \in g^{-1}(O)$ but $x_p \notin f^{-1}(O)$

So $f^{-1}(O) \neq g^{-1}(O)$, a cotradiction.

Hence $f \underset{(T,T')}{=} g$ holds.

Definition 1.4: A map $F : T' \text{----}> T$ is said to be a fuzzy topological map if it satisfies the following conditions:

(i) $F(0_Y) = 0_X$, $F(1_Y) = 1_X$.

(ii) $F(\bigcup_{O \in T_1} O) = \bigcup_{O \in T_1} F(O)$ for all $T_1 \subset T'$.

(iii) $F(\bigcap_{O \in T_1} O) = \bigcap_{O \in T_1} F(O)$, if $T_1 \subset T'$ & $\bigcap_{O \in T_1} O \in T'$.

Remark 1.5 : Dib[2] has shown that if (X,T) and (Y,T') be topological spaces, then $F: T' \text{---}> T$ is a topological map iff there exists a continuous function $f: (X,T) \text{---}> (Y,T')$ such that $F(O) = f^{-1}(O)$, $\forall O \in T'$.

The sufficiency of the fuzzy version of the above result is obvious. However the following example will show that the necessity part of the fuzzy version of the above result is not true.

Example:1.6 Let X = {x,y,z} and Y = {a,b}

T={0,1,(7/8,7/8,0),(3/4,3/4,0),(0,0,1/4),(3/4,3/4,1/4),(7/8,7/8,1/4)

T={0,1,(3/4,0),(0,1/4),(3/4,1/4)}

Let F: T′---T be defined by

F(3/4,0) = (7/8,7/8,0)

F(0,1/4) = (0,0,1/4)

F(3/4,1/4) = (7/8,7/8,1/4)

F(0) =0, F(1) = 1

Then F is a fuzzy topological map.

The only fuzzy continuous map of (X,T) into (Y,T′) is f where f: X-->Y is defined by f(x) = f(y) =a, f(z) =b.

But if O =(3/4,0), then O∈T′, but f^{-1}(O) = (3/4,3/4,0) ≠ F(O).

References

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