Fuzzy Reliability for Fuzzy event – Fuzzy probability Mode

Li Tingjie
(Beijing University of Aeronautics and Astronautics)

Abstract: In this paper a preliminary research of the fuzzy reliability for Fuzzy event – Fuzzy probability Mode is proceeded. Some new concepts about fuzzy reliability, fuzzy failure rate, and fuzzy mean life are defined, and a series of related mathematical models are established.

1. Introduction

In daily life,there is a certain kind of question,like this one,"Is this product reliable? " and to answer it, replying "Its reliability is 0.99" is certainly not more directly and understood than saying "It is very reliable" or "It is extremely reliable". Because in many situations what people's concern is not the understanding to the accurate date but only the whole conception about a certain product, and it is more convenient to be expressed with fuzzy language. The "event" and the "probability" above–mentioned are fuzzy, and we call this kind of question Fuzzy event – Fuzzy probability Mode question , simply denoted by FF mode or FFM.

2. Definition of Fuzzy Reliability for FF Mode

We call reliability for FF mode fuzzy reliability, as it is named, a FF mode has a fuzzy event and a fuzzy probability, and its answer should be described only with a fuzzy linguistic value. So from the properties and the correlation between FF mode and general mode, we can extend the definition of reliability for general mode to fuzzy reliability for FF mode as follows: fuzzy reliability for FF mode is ability described with fuzzy linguistic value of a product to perform a required performance under stated conditions for a stated period of time.

3. Main Index of Fuzzy Reliability for FF Mode

Now only one main index which is fuzzy reliability is extended from the theory of general reliability and studied in the sequel.

We define that fuzzy reliability is the fuzzy linguistic probability of a product to perform a required performance under stated conditions for a stated period of time, and denoted by $\underset{\sim}{R}(\underset{\sim}{\omega})$, in which $\underset{\sim}{\omega}$ represents a fuzzy event.

By definition, a fuzzy set representing linguistic value of probability is called a linguistic–valued fuzzy set, which is additionally divided into nine basic fuzzy subsets, that is, "extremely reliable", "very reliable", "reliable", "comparatively reliable", "critically reliable", "comparatively unreliable", "unreliable", "very unreliable", "extremely unreliable", which are simply denoted by $\pi_1, \pi_2, \ldots, \pi_9$ in turn. Using these nine basic fuzzy subsets, we can character the linguistic–valued space of fuzzy reliability entirely. Then, if we want to know the fuzzy reliability for FF mode toward a certain product, we can describe it by using one of these fuzzy subsets. But note that these nine basic fuzzy subsets are exactly divided, that is, among them these is not any relation of inclusion.

Now we can give the expression from the value of accurate reliability to the linguistic value of fuzzy reliability as follow:

$$\underset{\sim}{R}(\underset{\sim}{\omega}) = \pi_j, \text{ for } \pi_j \in \varepsilon \quad \text{and } j = 1, 2, \ldots, 9.$$

in which

$\underset{\sim}{\omega}$ — a fuzzy event, its value of accurate probability is $P(\underset{\sim}{\omega})$,

ε — the linguistic–valued space of fuzzy reliability,

π_j — a basic fuzzy subset, or a linguistic value of fuzzy reliability.

The next thing we must resolved is determining the membership function of the nine basic fuzzy subsets. After studying the relation between a probability and the definition of reliability, we come to the conclusion that values of reliability have to be described through values of probability completely, or say, values of reliability are just what values of probability are, that is also to say, values of probability and corresponding values of reliability are equivalent, such as , "extremely probable" and "extremely reliable" , "very probable" and "very reliable", and so on.

According to the theory of fuzzy probability, we know that the membership function of the fuzzy linguistic value, "very probable" is defined by

$$\mu(P|\text{very probable}) = \begin{cases} 0 , & \text{for } 0 \leqslant P \leqslant a, \\ 2\left(\dfrac{P-a}{1-a}\right)^2 , & \text{for } a \leqslant P \leqslant \dfrac{a+1}{2}, \\ 1-2\left(\dfrac{P-1}{1-a}\right)^2 , & \text{for } \dfrac{a+1}{2} \leqslant P \leqslant 1, \end{cases}$$

in which a is a parameter limited by $\frac{1}{2} \langle a \langle 1$. Then, if we replace the value of probability P by the value of reliability $P(\underset{\sim}{\omega})$, of course we obtain the formula for the definition of the term "very reliable", temporarily add a symbol" * " on it, as the following

$$\mu[P(\underset{\sim}{\omega})|\text{very reliable}^*] = \mu[P(\underset{\sim}{\omega})|\pi_2^*]$$

$$= \begin{cases} 0\ , & \text{for } 0 \leqslant P(\underset{\sim}{\omega}) \leqslant a\ , \\ 2\left[\dfrac{P(\underset{\sim}{\omega}) - a}{1-a}\right]^2\ , & \text{for } a \leqslant P(\underset{\sim}{\omega}) \leqslant \dfrac{a+1}{2}\ , \\ 1 - 2\left[\dfrac{P(\underset{\sim}{\omega}) - a}{1-a}\right]^2\ , & \text{for } \dfrac{a+1}{2} \leqslant P(\underset{\sim}{\omega}) \leqslant 1\ . \end{cases}$$

After using the definitions of the mood operators in fuzzy mathematics and notations π_1, π_2, ..., π_9, we can obtain the expression of $\mu[P(\underset{\sim}{\omega})|\pi_j^*]$, $j = 1, 3, ..., 9$, they are omitted in this paper.

Now it seems that we can easily judge in which degree a value of accurate reliability $P(\underset{\sim}{\omega})$ ($\in [0,1]$) belongs to every basic fuzzy subset. But when we arbitrarily give a value of parameter a , of course limited by $1/2 < a < 1$, we will find that there is a relation of fuzzy inclusion among them, clearly, the relation can be represented by

$$\pi_1^* \subseteq \pi_2^* \subseteq \pi_3^* \subseteq \pi_4^*\ ,$$
$$\pi_9^* \subseteq \pi_8^* \subseteq \pi_7^* \subseteq \pi_6^*\ .$$

Certainly the conclusion above is contradictory to the notion of the linguistic–valued space of fuzzy reliability defined formerly. The question is that how to convert these functions with inclusion into the ones without any inclusion which describes the bassic fuzzy subsets really.

The method is "excavating", suth as we excavate π_1^* from out of π_2^* , and π_2^* from out of π_3^* , and so on, then we have the following equations

$$\mu[P(\underset{\sim}{\omega})|\pi_2] = \mu[P(\underset{\sim}{\omega})|\pi_2^*] \wedge \mu[P(\underset{\sim}{\omega})|\pi_1^{*c}]\ ,$$
$$\mu[P(\underset{\sim}{\omega})|\pi_3] = \mu[P(\underset{\sim}{\omega})|\pi_3^*] \wedge \mu[P(\underset{\sim}{\omega})|\pi_2^{*c}]\ ,$$
$$\mu[P(\underset{\sim}{\omega})|\pi_4] = \mu[P(\underset{\sim}{\omega})|\pi_4^*] \wedge \mu[P(\underset{\sim}{\omega})|\pi_3^{*c}]\ ,$$

and some others

$$\mu[P(\underset{\sim}{\omega})|\pi_1] = \mu[P(\underset{\sim}{\omega})|\pi_1^*], \qquad \mu[P(\underset{\sim}{\omega})|\pi_5] = \mu[P(\underset{\sim}{\omega})|\pi_5^*]\ ,$$
$$\mu[P(\underset{\sim}{\omega})|\pi_6] = \mu[1 - P(\underset{\sim}{\omega})|\pi_4], \qquad \mu[P(\underset{\sim}{\omega})|\pi_7] = \mu[1 - P(\underset{\sim}{\omega})|\pi_3]\ ,$$
$$\mu[P(\underset{\sim}{\omega})|\pi_8] = \mu[1 - P(\underset{\sim}{\omega})|\pi_2], \qquad \mu[P(\underset{\sim}{\omega})|\pi_9] = \mu[1 - P(\underset{\sim}{\omega})|\pi_1]\ .$$

So we can obtain the correct membership functions which are not with symbol "∗" from now on, and give by

$$\mu[P(\underset{\sim}{\omega})|\pi_1]=\begin{cases}0, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant a,\\ 4\left[\dfrac{P(\underset{\sim}{\omega})-a}{1-a}\right]^4, & \text{for } a\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(a+1),\\ \left[1-2\left[\dfrac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2\right]^2, & \text{for } 0.5(a+1)\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_2]=\begin{cases}0, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant a,\\ 2\left[\dfrac{P(\underset{\sim}{\omega})-a}{1-a}\right]^2, & \text{for } a\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(a+1),\\ 1-2\left[\dfrac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2, & \text{for } 0.5(a+1)\leqslant P(\underset{\sim}{\omega})\leqslant 1-0.437(1-a),\\ 1-\left[1-2\left[\dfrac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2\right]^2, & \text{for } 1-0.437(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_3]=\begin{cases}0, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant a,\\ \sqrt{2}\dfrac{P(\underset{\sim}{\omega})-a}{1-a}, & \text{for } a\leqslant P(\underset{\sim}{\omega})\leqslant a+0.437(1-a),\\ 1-2\left[\dfrac{P(\underset{\sim}{\omega})-a}{1-a}\right]^2, & \text{for } a+0.437(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(a+1),\\ 2\left[\dfrac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2, & \text{for } 0.5(a+1)\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_4]=\begin{cases}0, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant a,\\ 2^{3/8}\left[\dfrac{P(\underset{\sim}{\omega})-a}{1-a}\right]^{3/4}, & \text{for } a\leqslant P(\underset{\sim}{\omega})\leqslant a+0.318(1-a),\\ 1-\sqrt{2}\dfrac{P(\underset{\sim}{\omega})-a}{1-a}, & \text{for } a+0.318(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(a+1),\\ 1-\sqrt{1-2\left[\dfrac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2}, & \text{for } 0.5(a+1)\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_5]=\begin{cases}1-\left[1-2\left[\frac{P(\underset{\sim}{\omega})}{1-a}\right]^2\right]^{3/8}, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(1-a),\\ 1-2^{3/8}\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^{3/4}, & \text{for } 0.5(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1-a,\\ 1, & \text{for } 1-a\leqslant P(\underset{\sim}{\omega})\leqslant a,\\ 1-2^{3/8}\left[\frac{P(\underset{\sim}{\omega})-a}{1-a}\right]^{3/4}, & \text{for } a\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(a+1),\\ 1-\left[1-2\left[\frac{P(\underset{\sim}{\omega})-1}{1-a}\right]^2\right]^{3/8}, & \text{for } 0.5(a+1)\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_6]=\begin{cases}1-\sqrt{1-2\left[\frac{P(\underset{\sim}{\omega})}{1-a}\right]^2}, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(1-a),\\ 1-\sqrt{2}\frac{1-P(\underset{\sim}{\omega})-a}{1-a}, & \text{for } 0.5(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 0.682(1-a),\\ 2^{3/8}\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^{3/4}, & \text{for } 0.682(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1-a,\\ 0, & \text{for } 1-a\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_7]=\begin{cases}2\left[\frac{P(\underset{\sim}{\omega})}{1-a}\right]^2, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(1-a),\\ 1-2\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^2, & \text{for } 0.5(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 0.563(1-a),\\ \sqrt{2}\frac{1-P(\underset{\sim}{\omega})-a}{1-a}, & \text{for } 0.563(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1-a,\\ 0, & \text{for } 1-a\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_8]=\begin{cases}1-\left[1-2\left[\frac{P(\underset{\sim}{\omega})}{1-a}\right]^2\right]^2, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant 0.437(1-a),\\ 1-2\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^2, & \text{for } 0.437(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(1-a),\\ 2\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^2, & \text{for } 0.5(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1-a,\\ 0, & \text{for } 1-a\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

$$\mu[P(\underset{\sim}{\omega})|\pi_9]=\begin{cases}\left[1-2\left[\frac{P(\underset{\sim}{\omega})}{1-a}\right]^2\right]^2, & \text{for } 0\leqslant P(\underset{\sim}{\omega})\leqslant 0.5(1-a),\\ 4\left[\frac{1-P(\underset{\sim}{\omega})-a}{1-a}\right]^4, & \text{for } 0.5(1-a)\leqslant P(\underset{\sim}{\omega})\leqslant 1-a,\\ 0, & \text{for } 1-a\leqslant P(\underset{\sim}{\omega})\leqslant 1.\end{cases}$$

The last question for us is to find out what the practical concrete method converting a value of accurate reliability into a linguistic value of fuzzy reliability is. Naturally, we utilize the principle about maximum grade of membership in fuzzy mathematics, then we can derive the mathematical model

$$\underset{\sim}{R}(\underset{\sim}{\omega}) = \pi_j \ , \quad \text{for} \quad \pi_j \in \varepsilon \quad \text{and} \quad j = 1, 2, ..., 9 \ ,$$

if

$$\mu[P(\underset{\sim}{\omega})|\pi_j] = max\{\mu[P(\underset{\sim}{\omega})|\pi_1], \ ..., \ \mu[P(\underset{\sim}{\omega})|\pi_9]\} \ .$$

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