# TREATING FUZZY ENVIRONMENT FOR MOBILE ROBOTS

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## ABSTRACT

Mobile robots may work in an uncertain environment. In order to function properly, they must be able to represent, account for and reasoning about the imprecise or fuzzy environment. In this paper, an approach is proposed to model the imprecision by using fuzzy sets. We assume that a fuzzy obstacle is represented by a set of fuzzy half-planes. Based on this representation, a decision function which aims towards finding an acceptable collision-free path is constructed. The local minus gradient information is used to guide the robot to search for its goal. Our approach has been implemented in Turbo Pascal on a simulator running on PC-386.

Key words: triangular fuzzy number, fuzzy half-plane, agreement index, path planning, decision-making.

## 1. INTRODUCTION

Humans encounter various kinds of uncertainties in their everyday life. They put up with imprecise, uncertain or incomplete information and sometimes take the advantage of its flexibility. Robots which is designed for helping or replacing humans in specific tasks also face uncertainty problems [1]. Hardly can a robot obtain the precise description of obstacles in the workspace. This is mainly because 1) robots usually have insufficient or incomplete knowledge about their environment; 2) error in sensing modalities will introduce position and orientation estimation problems in the modeling of robot's workspace; 3) Boundaries of some obstacles such as hills, marsh,etc. can not be clearly defined. Thus we declare that path planning for mobile robots is inherently affected by uncertainty in environment model. But most existing path planning algorithms [2-4] assume that a precise information of the workspace configuration is available. It is seldom researched how to treat an imprecise or 'fuzzy' environment. Only recently has the issue of uncertainty been raised in mobile robotics. C. DongWoo [5] proposed to represent certainty grid for robot navigation by a Bayesian method. However this approach is impractical due to the difficulty of assigning conditional probabilities with little or no statistical information. Beiseds for computational convenience, statistics can be represented in terms of possibility rather than probability [6]. So we advocate representing and handling uncertain environments with fuzzy sets and theory. In [7], we once put forward a fuzzy approach to spatial uncertainty, where uncertain position of a point was described by a cone-shaped fuzzy number. But it didn't concern the explicit ways to represent a fuzzy obstacle as a whole. And path planning problems under uncertainty were not discussed.

In this paper, a new approach to treat an imprecisely described environment is presented. The proposed approach models every object in the workspace as a fuzzy obstacle. A 2-dimensional fuzzy obstacle can be considered as the intersection of a set of fuzzy half-planes. The coefficients of fuzzy half-planes are described by fuzzy numbers and stored as basic world description. Every point in workspace could be assigned a membership value indicating the degree of that point belonging to the obstructing area. Based on this description, a decision function is constructed which represents an aggregate of the goals of the path planning problem. Robot uses the minus gradient information of the decision function to find an acceptable collision-free path from starting point to destination point. Three simulation examples are given to show the capability of the proposed work.

## 2. DESCRIPTION OF UNCERTAIN ENVIRONMENTS

Under uncertainty, obstacles in the workspace could not be modeled precisely. This uncertainty arises from the ill-defined boundaries and location of a fuzzy obstacle. Using the concept of fuzzy sets, any fuzzy obstacle can be represented by a group of fuzzy boundaries. On the other hand, a fuzzy boundary divides the whole plane into two fuzzy half-planes. The interior area of a fuzzy obstacle can be considered as the intersection of several fuzzy half-planes produced by its fuzzy boundaries. In this paper, a fuzzy half-plane is described by a linear inequality whose coefficients are all triangular fuzzy numbers, and its membership function is defined through the concept of agreement index of a fuzzy number with regard to an upper (or lower) bound [8]. The formal definition is given below:

Definition of fuzzy half-planes: A fuzzy half-plane described by $ax+by-c<=0$ (a, b, c are triangular fuzzy numbers) is a fuzzy set $H\subset R^2$, which satisfies:

$$\forall(x, y)\in R^2, \qquad \mu_H(x,y)==f(T, I) ==(\text{area } T\cap I)/(\text{area } T) \qquad (1)$$

where $T==ax+by-c$, $I(x)== 1 \quad x<=0;$
$\qquad == 0 \quad x>0$

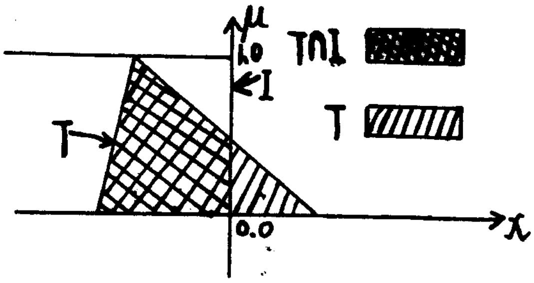

Fig.1 Agreement index for fuzzy number with regard to an upper bound.

Using fuzzy half-planes, an $m_i$-sided fuzzy obstacle Oi can be represented as:

$$Oi: \quad Ai \cdot (x, y)^T - Ci <= 0 \qquad (2)$$

where

$$Ai == \begin{bmatrix} ai(1, 1), & ai(1,2) \\ ai(2, 1), & ai(2,2) \\ \cdots & \cdots \\ ai(m_i, 1), & ai(m_i, 2) \end{bmatrix} \qquad Ci == \begin{bmatrix} ci(1) \\ ci(2) \\ \cdots \\ ci(m_i) \end{bmatrix}$$

For any $j=1..m_i$, the inequality $ai(j,1)x+ai(j,2)y-ci(j)<=0$ defines a fuzzy half-plane Hij. Considering the interior area of this obstacle is a fuzzy set , denoted by Ri, we can write:

$$Ri == \bigcap_{j=1 \cdot\cdot m_i} (Hij) \qquad (3)$$

$$\mu_{Ri}(x,y) == \min_{j=1 \cdot\cdot m_i} [\mu_{H_{ij}}(x,y)] \qquad (4)$$

Finally, imagine the obstructing area R in the environment. It is the union of the interior areas (Ri) of all obstacles in the workspace. Thus

$$R == R1 \cup R2 \ldots\ldots \cup Rp \qquad (5)$$

$$\mu_R(x,y) == \max_{i=1 \cdot\cdot p} [\mu_{R_i}(x,y)] \qquad (6)$$

With Equation 4, Equation 6 can be rewritten as

$$\mu_R(x,y) == \max_{i=1 \cdot\cdot p} \{ \min_{j=1 \cdot\cdot m_i} [\mu_{H_{ij}}(x,y)] \} \qquad (7)$$

## 3. PATH PLANNING UNDER UNCERTAINTY

### 3.1 Formulation of Path Planning Task

The robotic path planning task is modeled as a decision-making problem by considering two objectives (or goals): reach the destination and avoid collisions with fuzzy obstacles. Without any obstacles, the best path is depicted by plotting a straight line from starting point to destination point. However, with one or more obstacles presenting in this path, an acceptable path is the one which avoids colliding with obstacles but still reaches the destination. The above two goals are considered as the goals of the decision-making problem, the variables for which are x and y coordinates. At any point in the workspace, satisfaction of the first goal is examined by computing the distance from this point to the destination. This objective is fully satisfied when the distance is zero, or in other words, when the destination point is reached. The second objective is to prevent robot from interfering with any obstacles. The path planning algorithm should select points on the path such that any collision with fuzzy obstacles may be avoided. Depending on the distribution of obstacles, some points in the workspace might belong to the obstructing area (with certain membership values). Although a few of these points might be closer to the destination point, they should not be selected in constructing an acceptable collision-free path. Thus at some points in the worksapce the two planning objectives might conflict with each other, and we need an aggregation operation to combine the influence of the both together. With this idea, the decision function is constructed and expressed as follows:

$$F(x,y)==w1\cdot\mu(x,y) + w2\cdot\sqrt{(x-xg)^2 + (y-yg)^2} \qquad (8)$$

where $\mu(x,y) \in [0,1]$ is the level of a point belonging to the obstructing area, and (xg, yg)are coordinates of destination point. The weights w1, w2 are used to indicate the preference relationship between two planning goals. The modeling of a preference relationship involves decision-maker's intuition, and for simplication precise numerical weights are used here. In the cases of w1 dominating over w2, a path that is longer but free from possible collisions is preferable. But in the cases of w2 dominating over w1, risk of collision along a selected path is understood and accepted in return for a shorter distance. Hence under the two distinct path planning philosophies of conservative vs. aggressive, different combinations of weights define many interesting trade-offs.

### 3.2 Path Planning Algorithm

The path planning strategy selects points in the workspace by using the minus gradient information to generate an acceptable path. An component of the gradient vector of the decision function is computed by differentiating F(x,y) by variable x or y. So the corresponding minus gradient vector is written as:

$$-\nabla F = \left[-\frac{\partial F(x,y)}{\partial x}, \; -\frac{\partial F(x,y)}{\partial y}\right]^T \qquad (9)$$

where

$$-\frac{\partial F(x,y)}{\partial x} = -w_1 \cdot \frac{\partial \mu(x,y)}{\partial x} + w_2 \cdot \frac{x_g - x}{\sqrt{(x-x_g)^2+(y-y_g)^2}} \qquad (10a)$$

$$-\frac{\partial F(x,y)}{\partial y} = -w_1 \cdot \frac{\partial \mu(x,y)}{\partial y} + w_2 \frac{y_g - y}{\sqrt{(x-x_g)^2+(y-y_g)^2}} \qquad (10b)$$

The partial differentiation of membership function $\mu(x,y)$ is approximated in the algorithm by employing a simple forward differecing technique as given by

$$\frac{\partial \mu(x,y)}{\partial x} = \frac{\mu(x+\Delta x, y) - \mu(x,y)}{\Delta x} \qquad (11a)$$

$$\frac{\partial \mu(x,y)}{\partial y} \quad \frac{\mu(x, y+\Delta y) - \mu(x,y)}{\Delta y} \qquad (11b)$$

Using Equation 9, 10 and 11, the minus gradient vector of the decision function is constructed. A local search direction at a point is determined by identifying this information. At any point in the workspace, the minus gradient vector is used to make a move (i.e. to select the next point on the path). The algorithm terminates when the destination is reached, that is, when the value of decision function is of minimum. A flowchart which gives various steps of the algorithm is shown in Fig.2

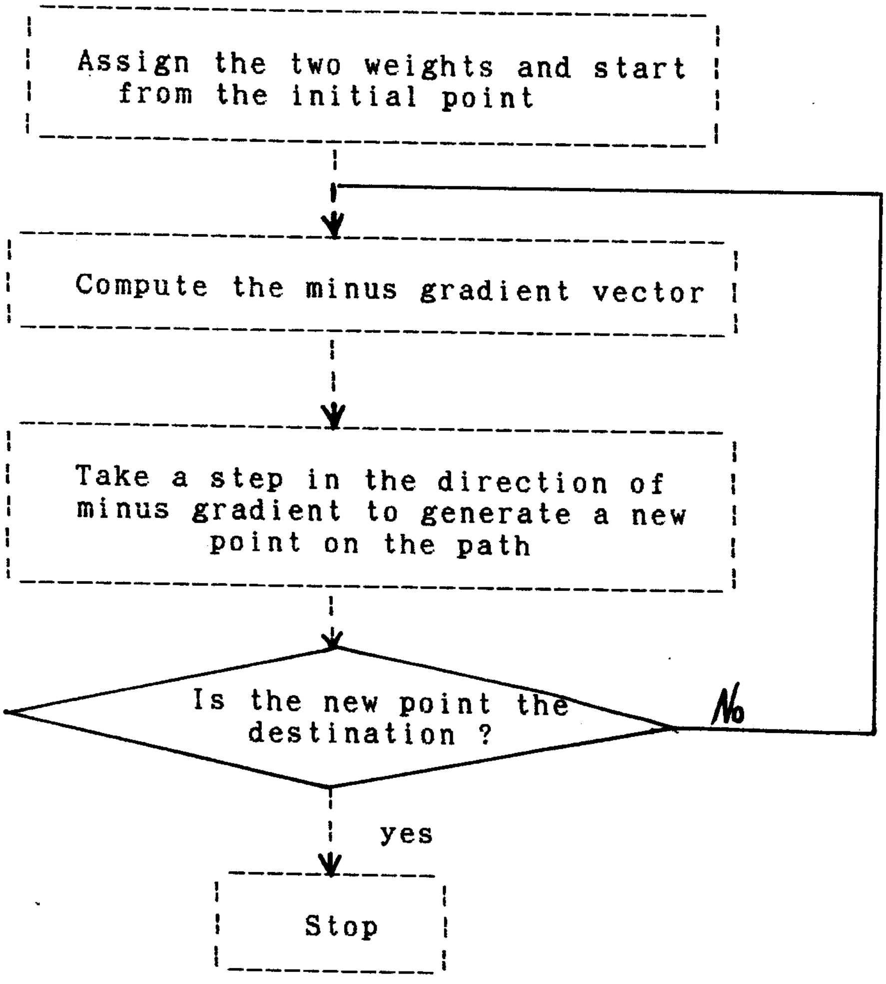

Fig. 2 Flowchart of the algorithm

## 4. SIMULATION EXAMPLES

Simple simulation of the proposed work has been implemented on IBM PC/AT in Turbo Pascal. For simplicity, the workspace is chosen to be a 20X20 square with four fuzzy obstacles which are drawn as polygons approximately. Paths generated (step=0.15) by the path planning algorithm for three different starting and destination points are shown in Fig.3, 4 and 5. In each figure, the line indicates an acceptable path, and coordinates of starting, destination points are given at the upper left-hand corner.

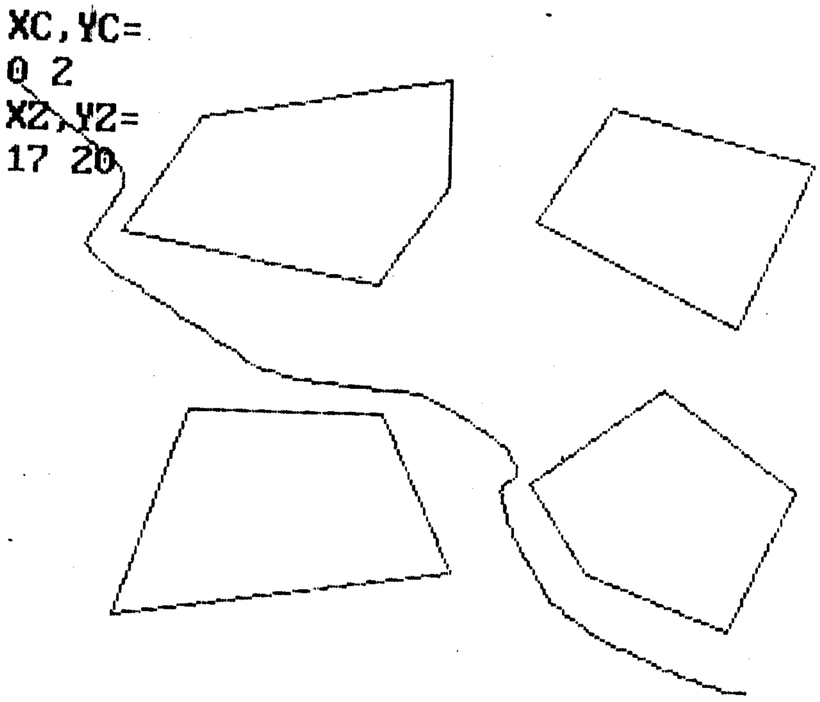

Fig.3 Simulation example 1

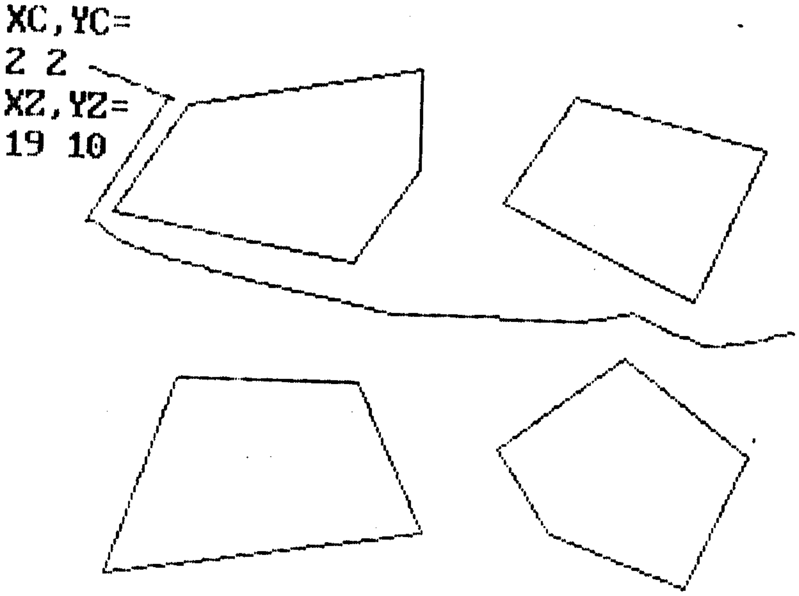

Fig.4 Simulation example 2

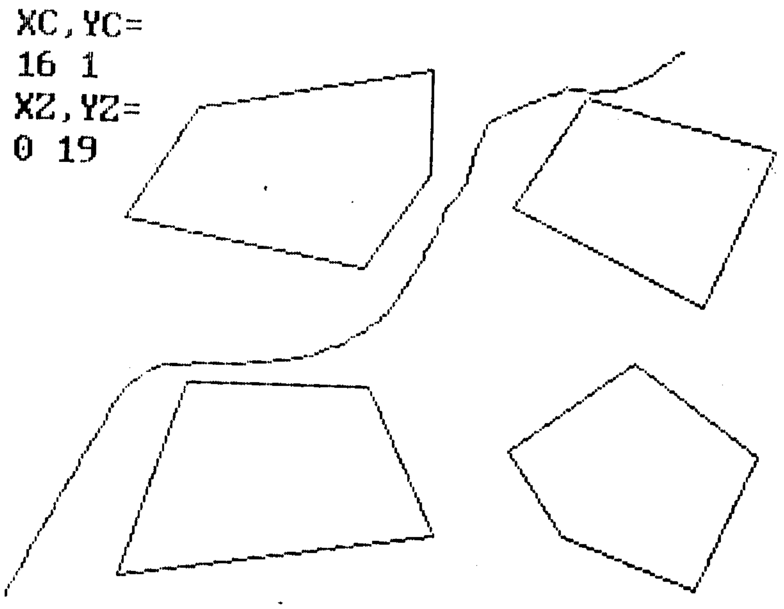

Fig.5 Simulation example 3

## 5. CONCLUSION

Uncertain geometry is an important issue in current mobile robot research. In this paper, we have developed a fuzzy approach to modeling the imprecise description of robot's workspace. The concept of fuzzy half-planes is established to describe a fuzzy obstacle in the environment. And this fuzzy representation is incorporated into the path planning methodology by defining a decision function. The minus gradient information of such a decision function is used to plot an acceptable path. More investigation is underway to extend our approach to three-dimensional environments with uncertainty.

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