GENERALIZED APPROXIMATE REASONING

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ABSTRACT

In this paper, we generalize the concept of ordinary and extended approximate reasoning. We demonstrate that under certain conditions the generalization actually reduces respectively to the ordinary and extended approximate reasoning and agrees with the classical concept of modus-ponens. We offer a simple method of computation, the generalized approach is applied on a numerical example and very promising results are obtained.

Keywords : Approximate Reasoning, Extended Fuzzy Reasoning.

1. INTRODUCTION

In ordinary approximate reasoning as suggested by Zadeh [1], we make inferences of the form

$$\begin{array}{ll} p: & \text{if } X \text{ is } A \text{ then } Y \text{ is } B \\ q: & \underline{\quad X \text{ is } A' \qquad\qquad} \\ r \leftarrow & \qquad\qquad Y \text{ is } B' \end{array}$$

where the variables X, Y take values in universes of discourse, U, V respectively and A, A', B, B' are inexact concepts which are approximated by fuzzy sets over U,U,V,V respectively.

In extended fuzzy reasoning as suggested by Mizumoto [2], we make inferences of the form

$$
\begin{array}{ll}
p: & \text{if } X_1 \text{ is } A_1 \text{ and } X_2 \text{ is } A_2 \text{ and} \ldots X_n \text{ is } A_n \text{ then } Y \text{ is } B \\
q: & \underline{\quad X_1 \text{ is } A_1' \text{ and } X_2 \text{ is } A_2' \text{ and} \ldots X_n \text{ is } A_n' \quad} \\
r \leftarrow & \hfill Y \text{ is } B'
\end{array}
$$

where X_i's are variables taking values in the universes of discourse U_i; $i = 1, 2, \ldots, n$ and Y is a variable taking values in V, A_i and A_i' are fuzzy subsets of U_i $(i = 1, 2, \ldots, n)$, B and B' are fuzzy subsets of V.

In both models the similarity is that the conclusions are simple fuzzy statements that define the possible state of a single variable Y. Thus in order to obtain a conclusion, in both fuzzy models discussed so far we require prior information about all variables that appear in the body of the rule. Hence the above method is applicable to deducing conclusions when the expert does not have adequate information regarding one or more variables but does have information regarding the other variables that appear in the body of the rule. Also in this case the existence of the information in the set of facts induces a possibility distribution which is implied by the dependence between the variables expressed by the fuzzy relation(s) as obtained from the translation of premise(s) p.

Here we shall consider the derivation of a conclusion (, a relation) from a rule of the form if X_1 is A_1 and X_2 is A_2 and .... X_n is A_n then Y is B consisting of a number of simple fuzzy propositions in its body combined using the connective " and " together with a number of premises of the form

$$X_i \text{ is } A'_i \; ; \quad i = 1, 2, \ldots, m$$

or a single premise of the form

$$X_1 \text{ is } A'_1 \text{ and } X_2 \text{ is } A'_2 \text{ and } \ldots X_m \text{ is } A'_m$$

where $m \leq n < \infty$.

Finally, the procedure to deduce a rule from the above relation is discussed, in which the body of the rule must consist of the variables about which no prior information was found. And thus in our proposed technique, we are going to make inferences of the form r given by

$$\begin{array}{ll} p: & \text{if } X_1 \text{ is } A_1 \text{ and } X_2 \text{ is } A_2 \text{ and } \ldots X_n \text{ is } A_n \text{ then } Y \text{ is } B \\ q: & X_1 \text{ is } A'_1 \text{ and } X_2 \text{ is } A'_2 \text{ and } \ldots X_m \text{ is } A'_m \\ \hline r \leftarrow & \text{if } X_{m+1} \text{ is } A'_{m+1} \text{ and } X_{m+2} \text{ is } A'_{m+2} \text{ and} \ldots X_n \text{ is } A'_n \text{ then } Y \text{ is } B'. \end{array}$$

Hence the proposed technique is a sharp extension of Mizumoto's extended fuzzy reasoning [2].

2. MATHEMATICAL FORMULATION OF THE PROBLEM

For some $n < \infty$, let

$$X_1, X_2, \ldots, X_n, Y$$

be n+1 linguistic variables taking values in universes of discourse

$$U_1,\ U_2,\ \ldots,\ U_n,\ V$$

respectively, and let there be two typical premises expressed as

$$p:\ \text{if } X_1 \text{ is } A_1 \text{ and } X_2 \text{ is } A_2 \text{ and } \ldots\ X_n \text{ is } A_n \text{ then } Y \text{ is } B$$

$$q:\ \quad X_1 \text{ is } A_1' \text{ and } X_2 \text{ is } A_2' \text{ and } \ldots\ X_m \text{ is } A_m' .$$

Here A_i ($i = 1, 2, \ldots, n$), A_i' ($i = 1, 2, \ldots, m$), B are inexact concepts that when approximated by fuzzy sets over their respective universes takes the form

$$A_i = \sum_{j=1}^{j_i} \mu_{A_i}(u_i^j) / u_i^j \subset U_i = \sum_{j=1}^{j_i} u_i^j\ ; \quad i = 1, 2, \ldots, n$$

$$A_i' = \sum_{j=1}^{j_i} \mu_{A_i'}(u_i^j) / u_i^j \subset U_i\ ; \quad i = 1, 2, \ldots, m$$

$$B = \sum_j \mu_B(v_j) / v_j \subset V = \sum_j v_j .$$

In the case where the variables appearing in q are not in the said order, we can rearrange the appearance of the variables in both premises pand qand then rename them to obtain the same result.

The translation of the logical relation between sentences appearing in premises pand q into mathematical relations R and S respectively, will be given by

$$p \longrightarrow \Pi(X_1, X_2, \ldots, X_n, Y) = R \subset U_1 \times U_2 \times \ldots \times U_n \times V$$

where

$$\mu_R(u_1, u_2, \ldots, v) = \min \left\{ \mu_{A_1}(u_1), \mu_{A_2}(u_2), \ldots, \mu_{A_n}(u_n), \mu_B(v) \right\}$$

and

$$q \longrightarrow \Pi(X_1, X_2, \ldots, X_m) = S \subset U_1 \times U_2 \times \ldots \times U_m$$

where

$$\mu_S(u_1, u_2, \ldots, u_m) = \min \left\{ \mu_{A'_1}(u_1), \mu_{A'_2}(u_2), \ldots \mu_{A'_m}(u_m) \right\}.$$

Thus we have two relational matrices R and S induced by the propositions p and q. The particularization of R by S can be easily obtained according to

$$\Pi(X_1, X_2, \ldots, X_n, Y) \left[\Pi(X_1, X_2, \ldots, X_m) = S \right] = R \cap \bar{S}$$

where $\bar{S} = S \times U_{m+1} \times U_{m+2} \times \ldots \times U_n \times V$ is the cylindrical extension of S. Projecting $R \cap \bar{S}$ on $U_{m+1} \times U_{m+2} \times \ldots \times U_n \times V$

we have the required inference

$$r \longleftarrow \Pi(X_{m+1}, X_{m+2}, \ldots, X_n, Y) = \mathrm{Proj}_{U_{m+1} \times U_{m+2} \times \ldots \times U_n \times V} \left[R \cap \bar{S} \right]$$

where

$$\mu_{(X_{m+1}, X_{m+2}, \ldots, X_n, Y)}(u_{m+1}, u_{m+2}, \ldots, u_n, v) = \sup_{u_1, u_2, \ldots, u_m} \left\{ \mu_R(u_1, \ldots, u_n, v) \wedge \mu_S(u_1, u_2, \ldots, u_m) \right\}$$

Thus from premises pand q we can have a conclusion r that is a $(n-m+1)^{th}$ order relational matrix corresponding to (n-m+1) dependent variables, viz. $X_{m+1}, X_{m+2}, \ldots, X_n, Y$. To convert it to a rule we first find the induced possibility distributions of every variable independently, using the projection principle, and obtain

$$r_i \longleftarrow X_i \text{ is } A'_i \;; \quad i = m+1, m+2, \ldots, n$$

and

$$r \longleftarrow Y \text{ is } B'$$

where A_i', B are fuzzy sets defined over U_i, ($i = m+1, m+2, \ldots, n$) and V respectively and are given by

$$B = \text{Proj}_V \left[\Pi(X_{m+1}, X_{m+2}, \ldots, X_n, Y) \right]$$

and

$$A_i' = \text{Proj}_{U_i} \left[\Pi(X_{m+1}, X_{m+2}, \ldots, X_n, Y \right] ; \quad i = m+1, \ldots, n.$$

Hence from propositions p and q we obtain

$r \leftarrow$ if X_{m+1} is A_{m+1}' and X_{m+2} is A_{m+2}' and ... X_n is A_n' then Y is B'.

Now the obvious demand that the consequence be

if X_{m+1} is A_{m+1} and X_{m+2} is A_{m+2} and ... X_n is A_n then Y is B

whenever

$$A_i' \leftarrow A_i ; \quad i = 1, 2, \ldots, m$$

will be met if we choose the underlying fuzzy sets normal.

4. CONCLUSION

The above fuzzy mathematical model for approximate reasoning is completely a new one aiming at application in many engineering problems. Throughout this paper we use only one rule of inference but we can surely use other methods in fuzzy logic for the translation of propositions. We implement the concept of generalization through computationally simple procedures. The results obtained are very promising.

5. REFERENCES

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