LOGIC PROGRAMMING WITH INTUITIONISTIC FUZZYNESS

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Abstract This paper presents a definition of the logic programming language InF-Prolog, which is designed to represent various degrees of uncertainty. The approach described is based on the combination of the Intuitionistic Fuzzy Set (IFS) Theory and Logic Programming paradigm. Two different real numbers presenting the degree of truth and the degree of false of the facts and rules are processed during the derivation of goals. Calculation of the goal's truth and false degrees during the inference is based on the Intuitionistic Fuzzy Logic (IFL). Modal operators necessity and possibility from IFL defined over every IFS are implemented.

Keywords **Logic programming, Intuitionistic fuzzy sets, Expert systems**

1. Introduction

The solutions to many real-world problems require reasoning under uncertainty (also known as inexact) and there are several well-known techniques in drawing inferences under uncertainty. Various forms of fuzzy logic have been proposed, some of which have been used to solve problems in control and in expert systems. The notion of fuzziness has variety of interpretations. The problem at hand is that of designing programming system to reason with and about uncertainty. Toward this aim we present the foundations of a programming language based upon intuitionistic fuzziness and the logic programming paradigm which generalises logic programming to the case in which various forms of uncertainty can be included. Much of the present research in logic programming concentrates on extensions to Prolog and on the basis of the research [1-7] we shall describe a new variant of PROLOG, which will include the elements of the intuitionistic fuzzy logic.

Two real numbers α and β which satisfy the following constraint: $0 \le \alpha + \beta \le 1$ are associated to every fact in knowledge base. They mark the **degree of truth** (first number) and the **degree of false** (second number) of the fact. How it is seen these degrees are not probabilities and for a given statement do not in general add up to 1.

In the clauses body logical operations can be included: "," (conjunction) and ";" (disjunction), the modal operators "necessity" and "possibility" (which we shall denote by " " and " ", respectively) and other operators from IFL.

2. Theoretical Background

Here we will give an brief exposition of ideas for IFS [1,2] and interval valued IFS [6].

Let a set E be fixed. An intuitionistic fuzzy set A^* in E is an object having the form:

$$A^* = \{ <x, \mu_A(x), \tau_A(x)> \mid x \in E \},$$

where the functions $\mu_A: E \to [0,1]$ and $\tau_A: E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to A, which is a subset of E and respectively for every $x \in E$:

$$0 \le \mu_A(x) + \tau_A(x) \le 1.$$

Obviously, every fuzzy set [15] has the form:

$$A = \{ <x, \mu_A(x), 1-\mu_A(x)> \mid x \in E \}.$$

We shall write A instead of A^* for simplicity ahead.

An interval valued [6] IFS A over E is an object which is of the form :

$$A = \{ <x, M_A(x), N_A(x)> \mid x \in E \},$$

where the intervals M_A and N_A are subsets of the interval [0,1] and for every $x \in E$:

$$\sup M_A(x) + \sup N_A(x) \le 1.$$

Let $\alpha, \beta \in [0,1]$ be a fixed number. For every IFS A the following operators are defined:

$$D_\alpha(A) = \{ <x, \mu_A(x)+\alpha.\pi_A(x), \tau_A(x)+(1-\alpha).\pi_A(x)> \mid x \in E \},$$

where $\pi_A(x) = 1 - \mu_A(x) - \tau_A(x)$.

Let $\alpha, \beta \in [0,1]$ and $\alpha + \beta \le 1$. The operators $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ for the IFS A are defined as:

$$F_{\alpha,\beta}(A) = \{ <x, \mu_A(x)+\alpha.\pi_A(x), \tau_A(x)+\beta.\pi_A(x)> \mid x \in E \},$$

where $\alpha + \beta \le 1$;

$$G_{\alpha,\beta}(A) = \{ <x, \alpha.\mu_A(x), \beta.\tau_A(x)> \mid x \in E \}.$$

Analogically they are defined also over interval valued IFS [6].

Modal operators "necessity" and "possibility" from intuitionistic fuzzy sets theory are defined as follows:

$$A = \{ <x, \mu_A(x), 1-\mu_A(x)> \mid x \in E \},$$

$$A = \{ <x, 1-\tau_A(x), \tau_A(x)> \mid x \in E \},$$

for every IFS A and for every interval valued IFS A:

$$A = \{ <x, M_A(x), [\inf N_A(x), 1-\sup M_A(x)]> \mid x \in E \},$$

$$A = \{ <x, [\inf M_A(x), 1-\sup N_A(x)], N_A(x)> \mid x \in E \},$$

Now there exist 7 extensions to the above operators but all they can be expressed by the use of the operators $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$.

To each proposition (in the classical meaning) one can assign its truth value: Truth - 1, or False - 0. In the case of fuzzy logics this truth value is a real number in the interval [0,1] and can be called "truth degree" of a particular proposition. IFS theory adds one more value - "false degree" - which will be in the interval [0,1] too. Thus we assign to the proposition p two real numbers $\mu(p)$ and $\tau(p)$, for which the following constraint is valid:

$$0 \le \mu(p) + \tau(p) \le 1.$$

Let this be done by the evaluation function V defined so that $V(p) = <\mu(p), \tau(p)>$. Hence the function $V: S \to [0,1] \times [0,1]$, where x denotes cartesian product, gives the truth and false degrees from the class of all propositions. The negation p of the proposition p will be defined through $V(p) = <\tau(p), \mu(p)>$. When the values V(p) of the propositions p and q are known, the evaluation function V for operations AND and OR are defined in the following way:

$$V(p \text{ AND } q) = <\min(\mu(p), \mu(q)), \max(\tau(p), \tau(q)>,$$

$$V(p \text{ OR } q) = <\max(\mu(p), \mu(q)), \min(\tau(p), \tau(q)>,$$

3.Intuitionistic Fuzzy Structures

Man's knowledge consists of statements which cannot be guarantied to be true. These statements can be either facts or rules and InF-structures to express the degree of truth and the degree of false are used. An InF-structure can be:

(i) an interval valued InF-object <M, N>, where $M = [\mu_i, \mu_S]$ and $N = [\tau_i, \tau_S]$ are subsets of the interval [0,1] , and following constraint is valid:

$$\mu_i + \tau_S = \sup M + \sup N \leq 1;$$

(ii) an INF-threshold pair $<t_\mu, t_\tau>$ constrained to :

$$0 \leq t_\mu + t_\tau \leq 1;$$

(iii) an INF-pair $<\mu,\tau>$, where

$$0 \leq \mu + \tau \leq 1;$$

4. Representation of the clauses

An InF-Prolog program consists of finite number of rules. A rule is of the form(the numeration is as above)

$$<H_\mu, H_\tau> \quad H_0 :- B_1, B_2, \ldots, B_m. \quad <B_\mu, B_\tau>.$$

where H_0 is called head of the clause, B_i, $1 \leq i \leq m$ are atoms(or some IFS operators expressed as built-in predicates) and $m \geq 1$. This should be considered as a Prolog clause with addition of the InF-structure. It can be given the following interpretation. For each assignment of each variable occurring in the clause, if $B_1, B_2, \ldots, B_m$ are all true with some degrees satisfying constraints B_μ and B_τ the H_0 is true with some degrees corrected by H_μ and H_τ. In next sections we describe the way degrees of an conclusion can be obtained.

Facts are in the form of:

$$<F_\mu, F_\tau> \quad F_0. \;,$$

where F_μ and F_τ are InF-Structures from case (ii).

An goal is in the form of:

$$?- A_1, A_2, \ldots, A_m. \; <B_\mu, B_\tau>,$$

where A_i, $1 \leq i \leq m$ are atoms and B_μ, B_τ are InF-objects from case (i) or (ii).

When the InF-structure is missing, we assume that the values of the head are equal to the body's values $<\mu_H, \tau_H> = <\mu_B, \tau_B>$ to preserve the degrees of uncertainty during the inference.

5. Operational Model

Here we wish to highlight the operational differences of InF-Prolog with traditional logic programming.

In the frames of the InF-PROLOG program the rules are activated after the calculation of the truth-value degrees of the rule's body.

5.1 Calculating values of the clause body

Let R be an InF-Clause denoted by:

R :- B_0, B_1, ... m R :- B , B , ... , B .

If μ_{Bi} and τ_{Bi} are truth and false degrees of the atom B_i, then for values $<\mu_B, \tau_B>$ of the body we get:

$$<\mu_B, \tau_B> = <\mathrm{Min}_{1\le i\le m}\mu_{Bi}, \mathrm{Max}_{1\le i\le m}\tau_{Bi}>,$$

which corresponds to the operation AND (conjunction denoted by ",") from IFL and in the case of the operation disjunction denoted by ";" for values $<\mu_B, \tau_B>$ we get:

$$<\mu_B, \tau_B> = <\mathrm{Max}_{1\le i\le m}\mu_{Bi}, \mathrm{Min}_{1\le i\le m}\tau_{Bi}>.$$

Except the operations conjunction and disjunction in the body of an InF-Prolog clause an unordered collection from operations and operators defined over every IFS may appear. Most of them are implemented as built-in predicates which handle uncertainty of InF-Prolog goals in different manners. These predicates are called **certainty predicates** because they remove or modify the uncertainty degrees of the goals derived. Certainty predicates has an strong theoretical background and correspond to the operators from IFS theory. Some of definitions are the following:

not(G) - the goal G is proved with some truth and false degrees $<\mu_G, \tau_G>$ and for the resulting degrees μ and τ we get $<\mu, \tau> = <\tau_G, \mu_G>$.

ness(G) - the goal G is proved with some truth and false degrees $<\mu_G, \tau_G>$ and for the resulting degrees μ and τ we get $<\mu, \tau> = <\mu_G, 1-\mu_G>$. This predicate corresponds to the modal operator "necessity" from IFS and removes the uncertainty (which is $CF = 1 - \mu_G - \tau_G$) adding it to the false degree.

poss(G) - the same as ness(G) but this predicate corresponds to the modal operator "possibility" from IFS and adds the CF to the truth degree,hence $<\mu, \tau> = <1-\tau_G, \tau_G>$.

5.2 Calculating uncertainty degrees of goals

The calculated values of the body $<\mu_B, \tau_B>$ are basis for calculation of the values of the head $<\mu_H, \tau_H>$. There are several ways for this, which correspond to various combination from different InF-Structures and operators from IFL Theory.

1. Interval rule

case (a)

$$\frac{[\mu_{Hi}, \mu_{HS}]\ [\tau_{Hi}, \tau_{HS}]\quad H :- B \quad [\mu_{Bi}, \mu_{BS}]\ [\tau_{Bi}, \tau_{BS}] \qquad [\mu_B]\ [\tau_B]\quad B}{[\mu_H, \tau_H]\quad H}$$

$$\mu_H = \mu^H_{Hi} + \alpha_\mu.(\mu_{HS} - \mu_{Hi}^H),$$
$$\tau_H = \tau^H_i + \alpha_\tau.(\tau_{HS} - \tau^H_i),$$ where:

$$\alpha_\mu = \frac{\mu_B - \mu_{Bi}}{\mu_{BS} - \mu_{Bi}}, \text{if } \mu^{BS} > \mu^B_i \text{ and } \alpha_\mu = {}^1/_2 \text{ in otherwise,}$$

$$\alpha_\tau = \frac{\tau_B - \tau_{Bi}}{\tau_{BS} - \tau_{Bi}}, \text{if } \tau^{BS} > \tau^B_i \text{ and } \alpha_\tau = {}^1/_2 \text{ in otherwise;}$$

case (b)

$$\frac{[\mu_{Hi}, \mu_{HS}]\ [\tau_{Hi}, \tau_{HS}]\quad H :- B \quad [b_\mu]\ [b_\tau] \qquad [\mu_B]\ [\tau_B]\quad B.}{[\mu_H, \tau_H]\quad H}$$

$[\mu_{Hi}, \mu_{HS}]$ and $[\tau_{Hi}, \tau_{HS}]$ are InF-Structures from case (i), $[b_\mu]$ $[b_\tau]$ are thresholds and then the values $<\mu_H, \tau_H>$ are obtained from case (A) with :

$$\alpha_\mu = \frac{\mu_B - b_\mu}{1 - b_\mu}, \text{ if } b_\mu < 1 \text{ and } \alpha_\mu = {}^1/_2 \text{ in otherwise,}$$

$$\alpha_\tau = \frac{\tau_B}{b_\tau}, \text{ if } b_\tau > 0 \text{ and } \alpha_\tau = {}^1/_2 \text{ in otherwise;}$$

2. F rule

$$\frac{[\alpha, \beta]\quad H :- B \quad <B_\mu, B_\tau> \qquad [\mu_B]\ [\tau_B]\quad B}{[\mu_H, \tau_H]\quad H}$$

where $\alpha, \beta \in [0,1]$ and $0 \leq \alpha + \beta \leq 1$) and B_μ, B_τ(may be absent) are

InFStructures from case (i) or B_μ, B_τ are thresholds and $0 \le B_\mu + B_\tau \le 1$ then:

$$<\mu_H, \tau_H> = F_{\alpha,\beta} (< \mu_B, \tau_B>),$$

3.G rule

$[\alpha]$ $[\beta]$ H :- B $<B_\mu, B_\tau>$
$[\mu_B]$ $[\tau_B]$ B

$[\mu_H, \tau_H]$ H

where $\alpha, \beta \in [0,1]$ and B_μ, B_τ(may be absent) are InF - Structures from case (i) or B_μ, B_τ are thresholds and $0 \le B_\mu + B_\tau \le 1$ then:

$$<\mu_H, \tau_H> = G_{\alpha,\beta} (< \mu_B, \tau_B>),$$

4. D rule

$[\alpha]$ H :- B $<B\mu, B\tau>$
$[\mu B]$ $[\tau B]$ B

$[\mu H, \tau H]$ H

Where $\alpha \in [0,1]$ and B_μ, B_τ(may be absent) are InF-Structures from case (i) or B_μ, B_τ are thresholds and $0 \le B_\mu + B_\tau \le 1$ then:

$$<\mu_H, \tau_H> = D_\alpha (< \mu_B, \tau_B>);$$

6.Derivation of Goals from Programs

Let P denote a InF-Prolog program. Let G_1 be a goal ,we say that there is a **derivation step** from G_1 to goal G_2 if G_1 takes the form:

$$?\text{-}\ A_1, A_2, \ldots, A_m \quad <\mu_{BG1}, \tau_{BG1}>.$$

where $m \ge 1$, and P contains a rule of the form:

$$B_0 :\text{-}\ D_1, D_2, \ldots, D_k.$$

where $k \ge 0$, and values $<\mu_{HB0}, \tau_{HB0}>$ of the above rule are already calculated, then G_2 is in form of:

$$?\text{-}\ A_1, \ldots A_{i-1}, A_i = B_0, A_{i+1}, \ldots, A_m \quad <\mu_{BG1}, \tau_{BG1}>.$$

where $1 \le i \le m$, if the following constraints are valid:

for $\mu_{BG1} = [\mu_i, \mu_s]$ and $\tau_{BG1} = [\tau_i, \tau_s]$,

$$\mu_i \le \mu_{HB0} \le \mu_s,$$

$$\tau_i \le \tau_{HB0} \le \tau_s.$$

if μ_{BG1} and τ_{BG1} are thresholds

$$\mu_{BG1} \leq \mu_{HB0},$$

$$\tau_{BG1} \geq \tau_{HB0}.$$

A **derivation** sequence is possibly infinite sequence of goals [13] wherein is a derivation step to each goal from the preceding goal. A derivation sequence is intuitionistic *successful* if it is finite and its last goal is empty and for answer degrees $<\mu_H, \tau_H>$ of the question $\mu_H \geq \tau_H$ is valid. If $\mu_H < \tau_H$ the sequence is **intuitionistic failed.** Finally, an **unknown** sequence is a finite sequence where no derivation step, using the selected atom, is possible from the last goal in the sequence. Hence, the answer degrees of truth and false are $\mu_H = \tau_H = 0$.

Thus an InF-Prolog program and goal are executed by a process of continuously reducing any remaining atoms in the subgoal. There are two non-deterministic aspects in obtaining a derivation sequence. An atom selection rule must determine which atom should be reduced next in a goal. In case of more than one proof paths for the selected goal a search strategy on the other hand determines which rule is to be used in the reduction of a given atom in the subgoal. In present implementation InF-Prolog uses the atom selection strategy as in Prolog, but the search strategy is much more extended. It can be based on the values of the already proved goals. Such a techniques can be divided into the groups:

(a) search for rule with max μ and min τ, or v.v.;
(b) rule fires only if $\mu \geq \tau$;
(c) combination of degrees from different proof paths(for more details see Dubois and Prade [10,11]).

Finally we shall show three examples.

Example 1

This example is based on Interval rule case (a) and shows how the inference is directed by the interval constraints on the end of the rule:

```
[0.6,0.8] [0.1, 0.2]  d(X):- p(X), l(X)   [0.4,0.7 ] [0,0.2].
[0.4,0.7] [0.1, 0.2]  d(X):- c(X)         [0.5,0.8 ] [0,0.1].
[0.5,0.8] [0.15,0.2]  p(X):- e(X), r(X)   [0.3,0.75] [0,0.2].

[0.7] [0.2]  r(a).        [0.8] [0.1]  r(b).
[0.6] [0.2]  l(a).        [0.9] [0.0]  l(b).
```

```
[0.7] [0.2]  e(a).        [0.9] [0.1]  e(b).
[0.6] [0.3]  c(a).        [0.3] [0.6]  c(b).
```

The answer to a question on d(X) will look as:

```
?- d(X).
     SUCCESS  0.780 : 0.200
     X = a
```

Example 2

The same as example 1 but using thresholds as a constraint instead of intervals(facts are the same as above):

```
[0.6,0.8] [0.1, 0.2]  d(X):- p(X), l(X)   [0.4] [0.2].
[0.4,0.7] [0.1, 0.2]  d(X):- c(X)         [0.5] [0.1].
[0.5,0.8] [0.15,0.2]  p(X):- e(X), r(X)   [0.3] [0.2].
```

There exist two answers to the question:

```
?- d(X).
     SUCCESS 0.676 0.200
     X = a
     More/y,n/ ;
     SUCCESS 0.705 0.187
     X = b
```

Example 3:

Let the truth-values of p(X), q(X) and r(X) are already calculated as <0.4, 0.3>, <0.7, 0.2> and <0.5, 0.0>, respectively. The truth-value of the goal

```
?- a(X).
```

for clause

```
a(X) :- ( p(X); poss(q(X)) ), ( ness(p(X)); r(X) ).
```

is <0.7, 0.2>. If:

(a) [0.4] h(X) :- a(X) [0.6,0.8] [0.1,0.2].
then the rule will be activated, because $0.7 \in [0.6\ 0.8]$ and $0.2 \in [0.1\ \ 0.2]$ and using rule (D) the truth-value of h(X) will be calculated as:

$$D_{0.4}(<0.7,\ 0.2) = <0.74, 0.26>;$$

(b) [0.4][0.5] h(X) :- a(X) [0.75] [0.25].,
then the rule will be activated, because $0.7 < 0.75$ and $0.2 < 0.25$ and from rule (F) we obtain:

$$F_{0.4,0.5}(<0.7,\ 0.2>) = <0.74,\ 0.25>.$$

(c) [0.3, 0.6] h(X) :- a(X) [0.6,0.8] [0.1,0.2], then the rule will be activated as in the case (a) and the truth-value of h(X) will be calculated as:

$$G_{0.3,0.6}(<0.7,0.2>) = <0.21, 0.12>.$$

7. Conclusion

In this paper we have discussed a logic programming system which uses intuitionistic fuzzy set theory to model various forms of uncertainty associated with the facts and rules in the knowledge base. The theory uses ideas from Atanassov's intuitionistic fuzzy set theory specialized to the particular form of knowledge representation and inference mechanism of the InF-Prolog.

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