THE SOLUTION OF THE II TYPE EQUATION OF A FUZZY MATRIX AND THE COMPUTATION OF THE CONTENT FOR REALIZABLE FUZZY MATRICES

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ABSTRACT

In this paper, we studied the solution of the II type equation of a fuzzy matrix. And we got that the computational method of the content for realizable matrices on lattice [0,1].

1. THE SOLUTION OF THE II TYPE EQUATION OF A FUZZY MATRIX

For a fuzzy matrix $B = (b_{ij})_{n \times n}$ we consider the solution of the II type equation of B.

First by Theorem 3.2 in [9] we verify B whether satisfis $b_{ij} = b_{ji} \le b_{ii}$, $\forall i,j$. If B satisfied this condition, II type equation of B has a solution. Otherwise it has not any solution.

Second, we study the solution of the II type equation of the B.

1.1 The solution when the index t = 1

This time we need solve the equation $B = X_{n \times 1} X'$.

Theorem 1.1 The II type equation of a symmetric fuzzy matrix $B = (b_{ij})_{n \times n}$ has a solution when the index t = 1 if and only if $(b_{11}, \dots, b_{nn})'$ is its solution and it has only this solution.

1.2 The solution when the index t = 2

This time we need solve the equation $B = X_{n \times 2} X'$.

When solveing this equation, we need following basic equations:

Basic equation 1 $x + y = b$

Its solutions are : $x = b$, $y = [0,b]$ or $x = [0,b], y=b$.

Basic equation 2 $xy + zw = b$

Its solutions see the table 1.

Example 1.1 Let

$$B = \begin{bmatrix} 0.8 & 0.7 & 0.6 \\ 0.7 & 0.8 & 0.5 \\ 0.6 & 0.5 & 0.6 \end{bmatrix}$$

Table 1

x	[b,1]	[b,1]	b	b	[0,b]	[0,b]	[0,1)	[0,1)
y	b	b	[b,1)	[b,1]	[0,1)	[0,1)	[0,b]	[0,b)
z	[0,1]	[0,b]	[0,b]	[0,1)	b	[b,1)	[b,1)	b
w	[0,b]	[0,1)	[0,1]	[0,b)	[b,1]	b	b	[b,1)

First. By Theorem 3.2 in [9] we see the II type equation of B has a solution.

Second. $(b_{11}, b_{22}, b_{33})' = (0.8 \;\; 0.8 \;\; 0.6)'$ and

$$\begin{pmatrix} b_{11} \\ b_{22} \\ b_{33} \end{pmatrix} (b_{11} \; b_{22} \; b_{33}) = \begin{pmatrix} 0.8 & 0.8 & 0.6 \\ 0.8 & 0.8 & 0.6 \\ 0.6 & 0.6 & 0.6 \end{pmatrix} \neq B.$$

Therefore by Theorem 1.1 we know the II type equation of B has not any solution when the index $t = 1$.

Third. We solve all solutions of the II type equation of B when the index $t = 2$. Let $B = X_{n \times 2} X'$. It equivalent to the system of equations (1.5)

$$\begin{cases} x_{11} + x_{12} = 0.8 & (1) \\ x_{11}\, x_{21} + x_{12}\, x_{22} = 0.7 & (2) \\ x_{11}\, x_{31} + x_{12}\, x_{32} = 0.6 & (3) \\ x_{21} + x_{22} = 0.8 & (4) \\ x_{21}\, x_{31} + x_{22}\, x_{32} = 0.5 & (5) \\ x_{31} + x_{32} = 0.6 & (6) \end{cases} \qquad (1.5)$$

we obtained that

$$X_1 = \begin{pmatrix} 0.7 & 0.8 \\ 0.8 & [0,0.5] \\ 0.5 & 0.6 \end{pmatrix} \qquad X_3 = \begin{pmatrix} 0.7 & 0.8 \\ 0.8 & 0.5 \\ [0,0.5] & 0.6 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 0.8 & 0.7 \\ [0,0.5] & 0.8 \\ 0.6 & 0.5 \end{pmatrix} \qquad X_4 = \begin{pmatrix} 0.8 & 0.7 \\ 0.5 & 0.8 \\ 0.6 & [0,0.5] \end{pmatrix}$$

1.3 The solution when the index $t \leq n$

This time the II type equation of $B_{n \times n}$ is $B = X_{n \times t} X'$. i.e.

$$\begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \cdots & \cdots & \cdots \\ b_{1n} & \cdots & b_{nn} \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1t} \\ \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nt} \end{pmatrix} \begin{pmatrix} x_{11} & \cdots & x_{n1} \\ \cdots & \cdots & \cdots \\ x_{1t} & \cdots & x_{nt} \end{pmatrix} \qquad (1.6)$$

It equivalent to the system of equations (1.7).

$$\begin{cases} (1\times 1) \\ \cdots\cdots \\ (i\times j) \\ \cdots\cdots \\ (n\times n) \end{cases} \qquad (1.7)$$

Where $(i\times j)$ -th equation is $(i,j = 1, \ldots ,n;\ i \le j)$:

$$b_{ij} = x_{i1}x_{j1} + \cdots + x_{it}x_{jt}$$

It equivalent to the system of equations (1.8).

$$\left\{\begin{array}{l} \begin{cases} x_{i1}x_{j1} = b_{ij} \\ x_{i2}x_{j2} \le b_{ij} \\ \cdots\cdots \\ x_{it}x_{jt} \le b_{ij} \end{cases} \\ \text{or} \begin{cases} x_{i1}x_{j1} \le b_{ij} \\ x_{i2}x_{j2} = b_{ij} \\ x_{i3}x_{j3} \le b_{ij} \\ \cdots\cdots \\ x_{it}x_{jt} \le b_{ij} \end{cases} \\ \cdots\cdots \\ \text{or} \begin{cases} x_{i1}x_{j1} \le b_{ij} \\ \cdots\cdots \\ x_{i\,t-1}\,x_{j\,t-1} \le b_{ij} \\ x_{it}\,x_{jt} = b_{ij} \end{cases} \end{array}\right. \qquad (1.8)$$

When solveing system of equations (1.8), we need following basic equations:

Basic equation 3 $\quad xy = b$

Its solution is $x = b,\ y = [0,b]$; and $x = [0,b]$, $y = b$.

Basic equation 4 $\quad xy \le b$

Its solution is $x = [0,b]$, $y = [0,1]$; and $x = [0,1], y = [0,b]$

We can find all solutions of (1.8) with both basic equation 3 and 4, thus we can find all solutions of (1.7). Therefore all solutions of (1.6) are

$$X = \begin{pmatrix} \mathfrak{X}_{11} & \cdots & \mathfrak{X}_{1t} \\ \cdots & \cdots & \cdots \\ \mathfrak{X}_{n1} & \cdots & \mathfrak{X}_{nt} \end{pmatrix},$$

where $\mathfrak{X}_{ij} = \bigcap x_{ij}$, $(i=1,\ldots,n;\ j=1,\ldots,t)$. If $\mathfrak{X}_{ij} = \phi$.

for an ij, then the II type equation of B have not any solution when the index is t.

Example 1.2 Let

$$B = \begin{bmatrix} 0.5 & 0.3 & 0.3 \\ 0.3 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.8 \end{bmatrix}.$$

Find the solutions of the II type equation of B when the index t = 3.

Solution. Let $B = X_{3\times 3} X'$, where $X_{3\times 3} = (x_{ij})_{3\times 3}$. It equivalent to the system of equation (1.9).

$$\begin{cases} x_{11} + x_{12} + x_{13} = 0.5 \\ x_{11} x_{21} + x_{12} x_{22} + x_{13} x_{23} = 0.3 \\ x_{11} x_{31} + x_{12} x_{32} + x_{13} x_{33} = 0.3 \\ x_{21} + x_{22} + x_{23} = 0.9 \\ x_{21} x_{31} + x_{22} x_{32} + x_{23} x_{33} = 0.6 \\ x_{31} + x_{32} + x_{33} = 0.8 \end{cases} \quad (1.9)$$

By Theorem 2.9 in [9], we may let

$$x_{ii} = \max\{x_{i1}, x_{i2}, x_{i3}\} \quad (i=1,2,3).$$

By Theorem 3.8 in [9], then $x_{ii} = b_{ii}$. Thus (1.9) may write to

$$\begin{cases} x_{11} = 0.5 & (1) \\ 0.5\, x_{21} + 0.9\, x_{12} + x_{13} x_{23} = 0.3 & (2) \\ 0.5\, x_{31} + x_{12} x_{32} + 0.8\, x_{13} = 0.3 & (3) \\ x_{22} = 0.9 & (4) \\ x_{21} x_{31} + 0.9\, x_{32} + 0.8\, x_{23} = 0.6 & (5) \\ x_{33} = 0.6 & (6) \end{cases} \quad (1.10)$$

We obtained the table 2.

Every column in the table 2 (where x_{11}=0.5, x_{22}=0.9, x_{33} =0.8) is a solution of the II type equation of B when the index t=3. By Theorem 2.9 in [9],to exchange every column in this solution, we may obtain all solutions the II type equation of B when the index t=3. They have sixty.

Table 2

x_{11}										
x_{12}	0.3	[0,.3]	0.3	[0,.3]	0.3	0.3	[0,.3]	[0,.3]	0.3	[0,.3]
x_{13}	[0,.3]	0.3	[0,.3]	0.3	0.3	0.3	0.3	[0,.3]	[0,.3]	[0,.3]
x_{21}	[0,.3]	[0,.3]	[0,.3]	0.3	[0,.3]	[0,.3]	0.3	0.3	[0,.3]	0.3
x_{22}										
x_{23}	0.6	[.3,.6]	[0,.6]	[0,.6]	[0,.6]	0.6	0.6	[0,.6]	0.6	0.6
x_{31}	[0,.3]	[0,.3]	[0,.3]	[0,.3]	[0,.3]	[0,.3]	[0,.3]	0.3	0.3	0.3
x_{32}	[.3,.6]	0.6	0.6	0.6	0.6	[0,.6]	[0,.6]	0.6	[0,.6]	[0,.3]
x_{33}										
or.	3,9 17	5,11 15	3,9 15	1,11 15	3,11 15	3,11 17	1,12 17	1,7 15	3,7 17	1,8 15

1.4 The solution when the index $t > n$.

This time the II type equation of B is $B = X_{n \times t} X'$. i.e.

$$\begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \cdots & \cdots & \cdots \\ b_{1n} & \cdots & b_{nn} \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1n} & \cdots & x_{1t} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nn} & \cdots & x_{nt} \end{pmatrix} X' \quad (1.11)$$

The solution is similar for solving the equation (1.6), we do not go into details.

2. THE COMPUTATION OF CONTENT FOR REALIZABLE FUZZY MATRICES

In 1982 Liu Wangjin [1] first introduced the concepts of the realizable fuzzy symmtric matrix and its content. And many estimated formulas of the content of a fuzzy symmtric matrix are gave cf. [1—8]. But so far, people can not directly compute it. In this section we may compute directly the content of any fuzzy matrix with the II type equation of the fuzzy matrix when the content of the fuzzy matrix is exist.

Definition 2.1 Let $B \in \mathcal{M}_{n \times n}$. If there exist $A \in \mathcal{M}_{n \times m}$ such that $B = A A'$, then B is called realizable, and A is called an realization matrix of B.

Definition 2.2 Let $B \in \mathcal{M}_{n \times n}$ is realizable. Note

$$\gamma(B) = \min\{ m \mid \exists A \in \mathcal{M}_{n \times m} \text{ such that } A A' = B \},$$

$\gamma(B)$ is called the content of B.

By section 1 we have that

Theorem 2.1 Let $B \in \mathcal{M}_{n \times n}$ is a fuzzy symmtric matrix. B is realizable if and only if the II type equation of B has a solution, and $r(B) = \beta(B)$.

Example 2.1 Let B is same for the example 1.1, find $\Upsilon(B)$.
Solution. By the example 1.1, $\Upsilon(B) = 2$. And this time all realization matrices are gaved in the example 1.1.

Example 2.2 Let B is same for the example 1.2, find $\Upsilon(B)$.
Solution. By Theorem 3.2 in [9], We easy see $\Upsilon(B)>1$. Let

$$B = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix} \begin{pmatrix} x_{11} & x_{21} & x_{31} \\ x_{12} & x_{22} & x_{32} \end{pmatrix} \qquad (2.1)$$

We easy verify the equation (2.1) has not any solution. Then $\Upsilon(B) > 2$. In the example 1.2 we found that the II type equation of B has a solution when the index t = 3. Therefore $\Upsilon(B) = 3$. And this time all realization matrices are gave in the example 1.2.

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