FUNDAMENTAL g-t-NORMS

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Let g be a normalized additive generator, i.e. a strictly increasing continuous mapping $g: [0,\infty] \to [0,\infty]$ with $g(0) = 0$ and $g(1) = 1$. g induces an order-reversing involution c on $[0,1]$, $c(a) = g^{-1}(1 - g(a))$ for $a \in [0,1]$. Thus we are able to define a fuzzy complementation A'^g for a fuzzy subset A of an universum X via $A'^g(x) = c(A(x))$ for any $x \in X$.

On the other hand, g induces a pseudo-addition $\oplus$ on $[0,\infty]$ via $a \oplus b = \tilde{g}^{-1}(g(a) \oplus g(b))$, $a,b \in [0,\infty]$. Let T be a triangular norm. Its g-dual S^g is defined via

$$S^g(a,b) = c(T(c(a),c(b))) \quad , \ a,b \in [0,1] .$$

Obviously S^g is a t-conorm. Note that $\tilde{g}^{-1}(u) = g^{-1}(\min(u, g(\infty)))$.

<u>Definition 1</u>. T is a fundamental g-t-norm iff

i) for any $a \in [0,1]$: $T(a,a) = a$ or for any $a \in]0,1[$: $T(a,a) < a$

ii) for any $a,b \in [0,1]$: $T(a,b) \oplus S^g(a,b) = a \oplus b$.

If $g(a) = a$ (it is enough to take $g(a) = a$ on $[0,2]$), i.e. if g is identity, then g can be omitted and we get Frank's fundamental t-norm [1] .

<u>Theorem 1</u>. Let g, h be two normalized additive generators and let T be a fundamental g-t-norm. Then

i) k defined via $k(a) = g(h(a))$ for $a \in [0,\infty]$ is a normalized additive generator

ii) T_h defined via $T_h(a,b) = h^{-1}(T(h(a),h(b)))$ for $a,b \in [0,1]$ is a fundamental k-t-norm

iii) corresponding k-t-conorm S_h^k satisfies $S_h^k(a,b) = h^{-1}(S^g(h(a),h(b)))$ for $a,b \in [0,1]$.

__Corollary 1.__ T is a fundamental g-t-norm iff $T_{g^{-1}h}$ is a fundamental h-t-norm.

__Corollary 2.__ The system $\mathcal{F}_g$ of all fundamental g-t-norms is induced by the Frank's family of fundamental t-norms $\mathcal{F} = \{T_s,\ s \in [0,\infty]\}$, see [1] , i.e. $\mathcal{F}_g = \{T_{s,g} = (T_s)_g,\ s \in [0,\infty]\}$,

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\begin{aligned}
T_{s,g}(a,b) &= \min(a,b) = T_0(a,b) && \text{for } s = 0 \\
&= g^{-1}(g(a).g(b)) && s = 1 \\
&= \max(a \oplus b, 1) && s = \\
&= g^{-1}\left(\log_s\left(1 + \frac{(s^{g(a)}-1).(s^{g(b)}-1)}{s-1}\right)\right) && s \in]0,1[\cup]1,\infty[
\end{aligned}
$$

for $a,b \in [0,1]$.

The properties of Frank's family $\mathcal{F}$ are preserved for the g-case, too. E.g. any g-t-norm satisfying ii) of Definition 1 is either a fundamental g-t-norm or an ordinal sum of fundamental g-t-norms. The continuity of the family $(T_{s,g})$ with respect to s holds, too. For $0 < s < r < \infty$ we have

$$T_0 \leq T_{s,g} \leq T_{r,g} \leq T_{\infty,g} .$$

__References__

[1] Frank, M.J.: On the simultaneous associativity of F(x,y) and x + y - F(x,y). Aequationes Math. 19 (1979), 194-226.