FUZZY NUMBERS AND CLASSICAL INTEGRAL TRANSFORMATIONS

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The aim of the paper is to show some connetions between fuzzy numbers calculus based on convolution principle and some classical integral transformations, especially the Mellin one.

The analysis requires the notion of the modified fuzzy number published e.g. in [1]. In order to get better survey. let us re all its its meaning.

Let $\mathbb{R}$ be the set of the real numbers endoved with usual topology. Let us denote $\mathcal{F} = \left\{ f \in [0, \infty)^{\mathbb{R}};\ f$ - piecewise continuous, $\overline{\text{supp}\ (f)}$ - compact, $0 < \text{essup}\ (f) < \infty \right\}$, $\mathcal{F}^+ = \left\{ f \in \mathcal{F};\ \text{supp}\ (f) \subset \subset [0, +\infty) \right\}$. Let us define the equivalence relation on the set $\mathcal{F}$ in the following way: $f \text{ E } g \overset{\text{def.}}{\Leftrightarrow} \exists\ \alpha \in (0, \infty): f = \alpha\, g$.
The elements of the factor set $\Phi = \mathcal{F} / \text{E}$ resp. $\Phi^+ = \mathcal{F}^+ / \text{E}$ will be called the proper modified fuzzy numbers resp. nonnegative proper ones. Because every proper modified fuzzy number $\text{F} \in \Phi / \text{E}$ is uniquely determined by an arbitrary element $f \in \text{F}$, we can without fear of being confused, write only f instead of F. Instead of the notion "modified fuzzy number" we can use its shortened form "fuzzy number".In order to complete the set of the fuzzy numbers we must consider the class $\mathcal{D}$ of Dirac δ-functions $\delta_a(x)$ for $a \in \mathbb{R}$. The function δ_a can be regarded as a representative of the crisp number a to the contrary of the "proper" one. Ultimately we shall work with the set $\Phi \cup \mathcal{D}$ instead of Φ.

Let $\xi: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ ($\xi: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$) be the binar operation. How to define the generalized fuzzy valued binar operation $\tilde{\xi}: \Phi \times \Phi \to \Phi$ ($\tilde{\xi}: \Phi^+ \times \Phi^+ \to \Phi^+$) associated with the operation ξ ?

One of many various answers can be formulated as follows:

$$\tilde{\xi}[f, g] = \int_{\mathbb{R}\ (\mathbb{R}^+)} f(u)\ g\Big[\omega(x, u)\Big]\ \psi(x, u)\ du$$

where ω is the function which satisfies the condition $\xi[u, \omega(x, u)] = x$ for $u \in \mathbb{R}$ ($\mathbb{R}^+$).

The sugested binary operation $\tilde{\xi}$ does not depend on the choice of $\psi(x, u)$ from the point of limit properties in the following sense:

$$f_n \to \delta_a\ ,\ g_n \to \delta_b \ \Rightarrow\ \tilde{\xi}[f_n, g_n] \to \delta_{\xi(a,b)}.$$

In order to transfer the reasonable properties of ξ on the operation $\tilde{\xi}$ applied on the proper fuzzy numbers we must do special choice of the function $\psi(x, u)$. The two cases are analysed in [1].

A/ Let $\xi(x, y) = x + y$. The choice $\psi(x, y) \equiv 1$ leads to the well known integral

$$\tilde{\xi}(f, g)(x) = (f \tilde{+} g)(x) = \int_{\mathbb{R}} f(u)\ g(x - u)\ du$$

which does not requive the special discusion.

B/ Let $\xi(x, y) = xy$. In order to guarantee the commutativity property of the binar operation $\tilde{\xi}: \Phi^+ \times \Phi^+ \to \Phi^+$ the only choice $\psi(x, u) = 1 / u$ is possible. Then we have

$$\tilde{\xi}(f, g)(x) = (f \tilde{.} g)(x) = \int_{\mathbb{R}^+} f(u)\ g(x/u)\ du/u.$$

The practical computation of these integrals, especially of the second one, is not easy. It often leads to the nonelementary functions even if f and g are very simple. One of the power ful tools is to use the so called Mellin transform

$$\mathcal{M}: \mathcal{F}^+ \to \mathbb{K}^{\mathbb{K}},\ f \mapsto \mathcal{M}(f; s) = \int_0^{+\infty} f(x)\ x^{s-1} dx.$$

It is easy to prove

$$\mathcal{M}(f \tilde{.} g) = \mathcal{M}(f)\ \mathcal{M}(g)$$

and then

$$(f \tilde{.} g) = \mathcal{M}^{-1}\Big[\mathcal{M}(f)\ \mathcal{M}(g)\Big].$$

It is useful to recall the simple relation between Fourier, Laplace and Mellin transform

$$\mathcal{M}(f;\ s) = \mathcal{F}\left\{f(e^{x});\ is\right\} = \mathcal{L}\left\{f(e^{t});\ -s\right\} + \mathcal{L}\left\{f(e^{-t});\ s\right\}$$

which extends considerably the possibilities of successful computation. Very suitable tables of integral transformations are e.g. [2].

REFERENCES

[1] Harman,B.: Sum and product of the modified real fuzzy numbers. In: International symposium on fuzzy approach to reasoning and decision-making, Czecho - Slovakia, Bechyně 25.-29. 6. 1990.

[2] Bateman, H., Erdélyi, A.: Tables of integral transforms, New York 1954.