CONSONANT CREDIBILITY STRUCTURES WITHIN MULTI-EXPERT SYSTEMS BELIEF SPACES

Venera TURCU DE REYES*
University of Bucuresti
Faculty of Mathematics
14, Academiei
70109 Bucuresti 1 Romania

Rolanda PREDESCU**
Institute of Research
for Informatics
8-10, Averescu
71316 Bucuresti 1 Romania

Abstract

A particular class of belief spaces is introduced, as an illustration of uncertain knowledge processing within multi-expert systems.

Under the basic assumption that the context generating a belief space is heterogeneous, containing both evidences with known probability and hypotheses, with known credibility and plausability, three main consequences are analysed: (a) the splitting of the belief space into three categories of facts, according to their heterogeneous derivational support; (b) the construction of adequate probability and mass distribution functions; (c) the construction of specific credibility and plausability functions that "reveal" the consonant credibility structure, inner to the belief space.

1. Introduction

A special approach to credibility functions concerns their property of consonance, namely the possibility to determine nested levels of evolving belief within a universe of discourse, considered as a frame of discernment.

The consonance principles were introduced by Shafer /7/ and represent a specific aspect within the Dempster-Shafer belief theory/1,7/. They were largely debated in /2,3/.

A credibility structure is called consonant if its focal elements are nested, namely they can be indexed such that $An \supset \ldots \supset A1$, for $m(A1) \geq \ldots \geq m(An)$ where m(Ai) is the mass attached to Ai. An important property /7/ states that a belief function derives from a consonant credibility structure iff

$$Pl(A \cup B) = \max(Pl(A), Pl(B)) \qquad (1.1)$$

or, equivqlently,

$$Cr(A \cap B) = \min(Cr(A), Cr(B)) \qquad (1.2)$$

We recall that in the framework of belief spaces theory /5/ a basic concept is that of context, seen as a set [illegible]$= \{H1, \ldots, Hn\}$ of hypotheses.

As well, a belief space is actually a context, along with all the facts derivable from it.

*) visiting doctorand

**) mail should be addressed to this author

The elements used to derive a fact are called the origin set(OS) of that fact. A fact is called supported if $OS \in \mathcal{P}(C)$.

2. Consonant credibility within heterogeneous belief spaces

In the present approach we use the basic assumptions of /6/. The knowledge that an expert system deals with is assimilated to a belief space HB, where the context is heterogeneous. Namely $C = E \cup H$, where $e \in E$ are evidences with known probability p(e), while $h \in H$ are hypotheses with a given credibility Cr(h) and plausability Pl(h). The sets of rules are expressed by a multivalued mapping HE: $HB \longrightarrow \mathcal{P}(HB)$. Thus, the same conclusion $F \in HB$ may be supported by several origin sets(sets of premises) $OS = \{A1,..,Am\}, Ai \subset HB$.

A fact is called: (i) completely supported, when $HE(F) \subset \mathcal{P}(C)$; (ii) partially hypothetical, when $HE(F) \subset \mathcal{P}(HB)$; (iii) completely hypothetical, when $HE(F) = \emptyset$. The three classes will be denoted by CS, PH, and CH, respectively.

Two probability type distributions are defined on C:

- a lower one, cred: $C \longrightarrow [0,1]$, with $cred(e) = \begin{cases} p(e), & e \in E \\ Cr(\{e\}), & e \in H \end{cases}$ (2.1)

- a higher one, plaus: $C \longrightarrow [0,1]$, with $plaus(e) = \begin{cases} p(e), & e \in E \\ Pl(\{e\}), & e \in H \end{cases}$ (2.2)

Two mass type distributions are further associated to $F \in CS$:

- lm: $2^{HB} \longrightarrow [0,1]$, $lm(F) = \min_{OS} \sum_{e \in OS} cred(e)$ (2.3)

- hm: 2^{HB}--- 0,1 , $hm(F) = \max_{OS} \sum_{e \in OS} plaus(e)$ (2.4)

Credibility and plausability are defined, for facts of CS and PH, at the level of a single origin set OS, by:

$$CR\text{-}S(F) = \left[\sum_{Ai \in OS} cr(Ai) \times dc\text{-}i \times N(F|Ai)\right] \times s$$

$$PL\text{-}S(F) = \left[\sum_{Ai \in OS} pl(Ai) \times dc\text{-}i \times \Pi(F|Ai)\right] \times s$$

where dc-i is the degree of contribution of Ai in concluding F, s is the certainty degree attached to F, due to OS, and:

$$cr(Ai) = \begin{cases} lm(Ai), & Ai \in CS \\ CR\text{-}C(Ai), & Ai \in PH \\ CR(Ai), & Ai \in CH \end{cases} \qquad pl(Ai) = \begin{cases} hm(Ai), & Ai \in CS \\ PL\text{-}C(Ai), & Ai \in PH \\ PL(Ai), & Ai \in CH \end{cases}$$

At the level of the whole belief space we define:

$$CR\text{-}C(F) = \min_{OSi \in HE(F)} CR\text{-}Si(F) \; ; \; PL\text{-}C(F) = \max_{OSi \in HE(F)} PL\text{-}Si(F)$$

CR and PL for facts of CH are taken from those facts of HB that are most similar to them, and also credible and plausible to a highest degree. The procedure used to detect these"reliable" similarities is a specific adaptation of some fuzzy pattern-matching and fuzzy decision-making methods described in /4/ and /8/. The concept of multi-expert system is supported by the acceptance of several HBi, with $\{OSj\}_j$, located in $\bigcup_i \mathcal{P}(HBi)$. Under the above circumstances, the following propositions hold.

Proposition 2.1. The function lm in (2.3) is a mass distribution.

Proof. We have to show that:

$lm(\emptyset) = 0$ (2.1.1.) and $\sum_{A \subset HB} lm(A) = 1$ (2.1.2.)

Indeed, $lm(\emptyset) = \begin{cases} p(\emptyset) = 0, & E = \emptyset \\ cred(\emptyset) = Cr(\emptyset) = 0, & H = \emptyset \end{cases}$ Thus, (2.1.1.) is satisfied.

For (2.1.2.), we have:

$$\sum_{A \subset HB} lm(A) = \begin{cases} \sum_{A \subset HB} \min_{OS \in HB(A)} \sum_{ei \in OS} p(ei), \; OS \subseteq E & (2a) \\ \sum_{A \subset HB} \min_{OS \in HB(A)} \sum_{ei \in OS} Cr(\{ei\}), OS \subseteq H & (2b) \end{cases}$$

In the case (2a), we have:

$$\sum_{A \subset HB} \min_{OS \in HB(A)} \sum_{ei \in OS} p(ei) = \min_{OS \in \mathcal{P}(HB)} \sum_{ei \in OS} p(ei)$$

Since p is a probability, in the case $ci \in OS = E$ we have $\sum_{ei \in E} p(ei) = 1$.

Therefore, $\min_{OS \in \mathcal{P}(HB), OS \subseteq E} \sum_{ei \in OS} p(ei) = 1$.

In the case (2b), we have:

$$\sum_{A \subset HB} \min_{OS \in HB(A)} \sum_{ei \in OS} Cr(\{ei\}) = \min_{OS \in \mathcal{P}(HB)} \sum_{ei \in OS} Cr(\{ei\})$$

Since Cr is a credibility function, $Cr(\{ei\}) = m(\{ei\})$. Hence, for $ci \in OS = HB$, we have $\sum_{\{ei\} \subset HB} Cr(\{ei\}) = \sum_{\{ei\} \subset HB} m(\{ei\}) = 1$.

Thus, $\min \sum_{\{ei\} \subset HB} Cr(\{ei\}) = 1$ and (2.1.2.) is satisfied.

Similarly, it can be shown that hm in (2.4) is a mass distribution.

In this case, for OS = HB, we have:

$$\sum_{\{ei\} \subset HB} Pl(ei) = \sum_{\{ei\} \subset HB} \sum_{j | ei \in Aj} m(Aj) = \sum_j m(Aj) = 1.$$

Proposition 2.2. The plausability PL-C derives from the consonant credibility structure of HB.

Proof. We have to show that for any $F, G \subset HB$,

$$PL\text{-}C(F \cup G) = \max (PL\text{-}C(F), PL\text{-}C(G))$$

Indeed, using the adequate definition-based substitution, we obtain:

$$\max_{OSi \in HB(F \cup G)} PL\text{-}Si(F \cup G) =$$
$$= \max \Big[\max_{OSi \in HB(F)} \sum_{Aj \in OSi} pl(Aj) \times dc\text{-}j \times \Pi(F|Aj) \times e ,$$
$$\max_{OSi \in HB(G)} \sum_{Aj \in OSi} pl(Aj) \times dc\text{-}j \times \Pi(G|Aj) \times e \Big] .$$

Hence, according to (1.1), HB underlies a consonant credibility structure, with respect to hm. The credibility consonance within HB, with respect to lm, results in a similar manner from (1.2), satisfied by CR-C.

3. Conclusions

An appropriate set of credibility functions was constructed, for a heterogeneous space of uncertain facts, in order to illustrate the convergence of belief levels towards a consonant credibility structure.

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