Fuzzy Reliability of Complex System

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Abstract : This paper is one of continuation of "Fuzzy Rliability" . It developsgeneral reliability into fuzzy reliability in complex system by means of basicconcepts and principles of fuzzy mathematics. In this paper , author has adiscussion on method establishing mathematics model of fuzzy reliability forcomplex system.

Key Words : fuzzy reliability of complex system.

1. Introduction

We still employ the classificatory method of a system in general reliabilitytheory. A complex system is defined as the system consisting of severalelements are complex functional relations.

Two systems as shown in Figure 1 and Figure 2 are two complex system.

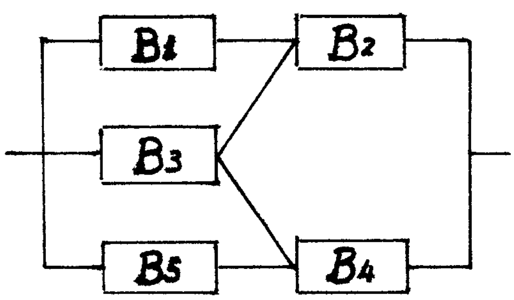

Figure 1

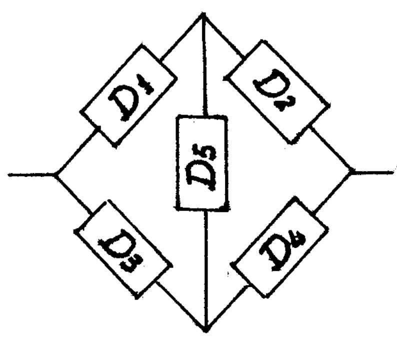

Figure 2

2. Indexes of Fuzzy Reliability

Now we only discuss the complex system as shown in Figure 1 . It may beassumed that failure of any element would occur independently of the operationother components.

We employ the denotations as follows:

S — the system is successful .

B_j — a element B_j is successful, $j = 1,2,3,4,5$.

$\overline{B_j}$ — a element B_j is failure, $j = 1,2,3,4,5$.

R_s — general reliability of the complex system .

$\underset{\sim}{R}_s$ — fuzzy reliability of the complex system.

R_j — general reliability of a element B_j .

$\underset{\sim}{R}_j$ — fuzzy reliability of a element B_j .

$\underset{\sim}{A}_i$ — discussing one of performance subsets .

$\mu_{\underset{\sim}{A}_i}(R_s)$ — degree of membership of R_s in $\underset{\sim}{A}_i$.

$\mu_{\underset{\sim}{A}_i}(R_j)$ — degree of membership of R_j in $\underset{\sim}{A}_i$.

By means of a definition of fuzzy conditional probability, we have

$$P\ (\ B_j \ \dot{\wedge}\ \underset{\sim}{A}_i\ ) = P\ (\ \underset{\sim}{A}_i | B_j\ )\ P\ (\ B_j\ )\ , \tag{1}$$

$$P\ (\ S\ \dot{\wedge}\ \underset{\sim}{A}_i\ ) = P\ (\ \underset{\sim}{A}_i | S\ )\ P\ (\ S\ )\ , \tag{2}$$

where the sign $\dot{\wedge}$ denotes algebraic product.

By means of general reliability theory and fuzzy reliability theory, we have

$$\underset{\sim}{R}_j = P\ (\ B_j\ \dot{\wedge}\ \underset{\sim}{A}_i\ )\ , \tag{3}$$

$$\underset{\sim}{R}_s = P\ (\ S\ \dot{\wedge}\ \underset{\sim}{A}_i\ )\ , \tag{4}$$

$$P\ (\ B_j\ ) = R_j\ , \tag{5}$$

$$P\ (\ S\ ) = R_s\ , \tag{6}$$

$$P\ (\ \underset{\sim}{A}_i | B_j\ ) = \mu_{\underset{\sim}{A}_i}(\ R_j)\ , \tag{7}$$

$$P\ (\ \underset{\sim}{A}_i | S\ ) = \mu_{\underset{\sim}{A}_i}(\ R_s)\ . \tag{8}$$

Substituting Eq.s (3)、(5)、(7) into Eq.(1) , then

$$\underset{\sim}{R}_j = \mu_{\underset{\sim}{A}_i}(\ R_j) \cdot R_j\ . \tag{9}$$

Substituting Eq.s (4)、(6)、(8) into Eq.(2) , then

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_i}(\ R_s) \cdot R_s\ . \tag{10}$$

If employ a analysing method , then the complex system may be dividedinto two case . At first , the element B_3 is successful , just then thecomplex system may be simplified to that as shown in Figure 3 . Next , theelement B_3 is failure , just then the complex system may be simplified to thatas shown in Figure 4 .

According to the formula for prior probability of probability theory, asuccessful probability of the complex system is expressed

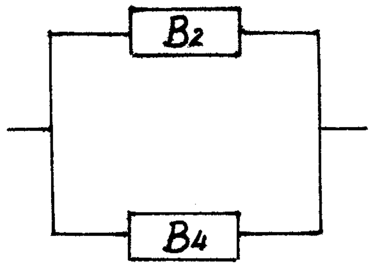

Figure 3

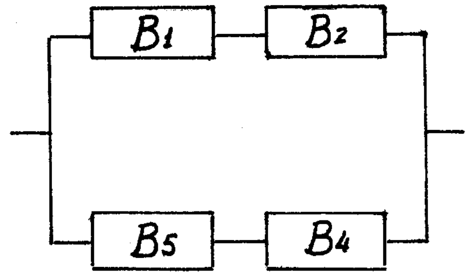

Figure 4

$$P(S) = P(S|B_3)\,P(B_3) + P(S|\overline{B_3})\,P(\overline{B_3})\,. \qquad (11)$$

Evidently ,

$$P(S) = R_s \,,$$
$$P(B_3) = R_3 \,,$$
$$P(\overline{B_3}) = 1 - R_3 \,,$$
$$P(S|B_3) = 1 - (1-R_2)(1-R_4) \,,$$
$$P(S|\overline{B_3}) = 1 - (1-R_1R_2)(1-R_4R_5) \,,$$

Substituting these exprassions into Eq. (11) , we obtain

$$\begin{aligned} R_s &= [1-(1-R_2)(1-R_4)]\,R_3 + [1-(1-R_1R_2)(1-R_4R_5)](1-R_3) \\ &= R_1R_2 + R_2R_3 + R_3R_4 + R_4R_5 - R_1R_2R_3 - R_2R_3R_4 \\ &\quad - R_3R_4R_5 - R_1R_2R_3R_4 + R_1R_2R_3R_4R_5 \\ &= \sum_{\substack{j=1 \\ k=j+1}}^{4} R_jR_k - \sum_{\substack{j=1 \\ k=j+1 \\ l=j+2}}^{3} R_jR_kR_l - \prod_{\substack{j=1 \\ j\neq 3}}^{5} R_j + \prod_{j=1}^{5} R_j \,. \end{aligned} \qquad (12)$$

Substituting Eq. (12) into Eq. (10) , we obtain fuzzy reliability for thecomplex system

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_l}(R_s) \cdot \Big[\sum_{\substack{j=1 \\ k=j+1}}^{4} R_jR_k - \sum_{\substack{j=1 \\ k=j+1 \\ l=j+2}}^{3} R_jR_kR_l - \prod_{\substack{j=1 \\ j\neq 3}}^{5} R_j + \prod_{j=1}^{5} R_j \Big] \,. \qquad (13)$$

Substituting Eq. (9) into Eq. (13) , then

$$\underset{\sim}{R}_s = \mu_{\underset{\sim}{A}_t}(R_s) \cdot \Big[\sum_{\substack{j=1\\k=j+1}}^{4} \frac{\underset{\sim}{R}_j \underset{\sim}{R}_k}{\mu_{\underset{\sim}{A}_t}(R_j)\mu_{\underset{\sim}{A}_t}(R_k)} - \sum_{\substack{j=1\\k=j+1\\l=j+2}}^{3} \frac{\underset{\sim}{R}_j \underset{\sim}{R}_k \underset{\sim}{R}_l}{\mu_{\underset{\sim}{A}_t}(R_j)\,\mu_{\underset{\sim}{A}_t}(R_k)\,\mu_{\underset{\sim}{A}_t}(R_l)} - \prod_{\substack{j=1\\j\neq 3}}^{5} \frac{\underset{\sim}{R}_j}{\mu_{\underset{\sim}{A}_t}(R_j)} + \prod_{j=1}^{5} \frac{\underset{\sim}{R}_j}{\mu_{\underset{\sim}{A}_t}(R_j)} \Big] . \tag{14}$$

Eq. (13) is the general expression which express the relation between thefuzzy reliability for the complex system and the general reliability forthe element . Eq. (14) is the general expression which express the relationbetween the fuzzy reliability for the complex system and fuzzy reliabilityfor the elements .

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