II TYPE EQUATION OF A FUZZY MATRIX (II)

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ABSTRACT

In this paper we preliminarily discussed the conditions that a II type equation of a fuzzy matrix has a solution.

Keyword: II type equation of a fuzzy matrix.

3. THE CONDITIONS THAT A II TYPE EQUATION OF A FUZZY MATRIX HAS A SOLUTION

Let $B = (b_{ij})_{n\times n}$. In B let b_{nn}, b_{in}, and b_{ni} do not change, but change b_{ii} for b_{in}. And change other elements for zero, we get a fuzzy matrix B_i. i.e.

$$B_i = \begin{pmatrix} 0 & \cdots & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & & \vdots & \vdots & & & & \vdots \\ \vdots & & \vdots & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & & \vdots & b_{in} & 0 & \cdots & 0 & b_{in} \\ \vdots & & \vdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & & \vdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & b_{ni} & 0 & \cdots & 0 & b_{nn} \end{pmatrix} \quad (i=1,\dots,n-1) \qquad (3.1)$$

Let

$$B_o = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1\,n-1} & 0 \\ b_{21} & b_{22} & \cdots & b_{2\,n-1} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n-1\,1} & b_{n-1\,2} & \cdots & b_{n-1\,n-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \qquad (3.2)$$

Definition 3.1 Let $B = (b_{ij})_{n\times n}$. Both (3.1) are called the principal submatrices of B.

Theorem 3.1 Let $B=(b_{ij})_{n\times n}$. If $b_{in}=b_{ni}\leqslant b_{nn}$, the II type equation of every principal submatrix of B has a solution when the index $t = 1$.

Theorem 3.2 The II type equation of fuzzy square matrix $B = (b_{ij})_{n\times n}$ has solution if and only if

(1) $\forall i,j$, $b_{ij}=b_{ji}$; and

(2) $\forall i,j$, $b_{ii}\geqslant b_{ij}$.

Proof. $\Rightarrow$ By Theorem 2.10, (1) holds. Let $A=(a_{ij})_{n\times t}$ is a solution of II type equation of B when the index is t.

Then $B = A A'$. Therefore

$$b_{ij} = \bigvee_{k=1}^{t} (a_{ik} \wedge a_{jk}) \leq \bigvee_{k=1}^{t} (a_{ik}) = \bigvee_{k=1}^{t} (a_{ik} \wedge a_{ik}) = b_{ii}$$

$\Leftarrow$ For $B_2 \in \mathcal{M}_{2 \times 2}$, then II type equation of B_2 has solution.

We suppose $B_{n-1} \in \mathcal{M}_{(n-1) \times (n-1)}$, and B_{n-1} satisfi $\forall$ i,j, $b_{ij}=b_{ji}$, and $b_{ii} \geq b_{ij}$, and the II type equation of B has solution.

For $B_n \in \mathcal{M}_{n \times n}$. We have

$$B_n = B_0 + B_1 + B_2 + \dots + B_{n-1} \qquad (3.3)$$

By Theorem 3.1, the II type equation of B_1, ... , of B_{n-1} has solution, respectively.

We say that the II type equation of B_0 also has solution. Because let

$$B^* = \begin{pmatrix} b_{11} & \cdots & b_{1n-1} \\ \cdots & \cdots & \cdots \\ b_{n-1\,1} & \cdots & b_{n-1\,n-1} \end{pmatrix},$$

The elements of B^* satisfi $\forall$i,j, $b_{ij}=b_{ji}$, and $b_{ii} \geq b_{ij}$. Then by inductive assumption, the II type equation of B^*has a solution. Now

$$B_0 = \begin{pmatrix} b_{11} & \cdots & b_{1\,n-1} & 0 \\ \cdots & \cdots & \cdots & \cdots \\ b_{n-1\,1} & \cdots & b_{n-1\,n-1} & 0 \\ 0 & \cdots & 0 & 0 \end{pmatrix},$$

By Theorem 2.1, the II type equation of B_0 also has a solution. Therefore by Theorem 2.2 we gave the II type equation of (3.3) has a solution . i.e. the II type equation of $B_{n \times n}$ has a solution.

Theorem 3.3 The II type equation of $B = (b_{ij})_{n \times n}$ has a solution if and only if

(1) $\forall$ i,j, $b_{ij}=b_{ji}$; and

(2) $\forall$ i,j, $b_{ij} \leq b_{ii}$; and

(3) $\forall$ i,j, $b_{ij} \leq b_{jj}$; and

(4) $\forall$ i,j, $b_{ij} \leq b_{ii} \wedge b_{jj}$.

Theorem 3.4 Let $B = (b_{ij})_{n \times n}$, and $\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{pmatrix}$ is a permutation. And let $B^* = (b^*_{ij})_{n \times n}$, where $b_{ij} = b_{\sigma(i)\sigma(j)}$, $(i,j=1,2,\dots,n)$. Then

(1) If the II type equation of B has a solution, the II type equation of B^* also has a solution.

(2) If $A = (a_{ij})_{n \times m}$ is a solution of the II type eq-

uation of B, let $A^* = (a^*_{ij})_{n\times m}$, where $a^*_{ij} = a_{\sigma(i)\,j}$ $(1 \leq i \leq n;\ 1 \leq j \leq m)$, A^* also is a solution of the II type equation of B^*.

Now we consider the necessary and sufficient condition which II type equation of a fuzzy matrix has a solution when the index t = 1.

Theorem 3.5 The II type equation of a fuzzy matrix $B_{n\times n}$ has a solution when the index t = 1 if and only if a solution matrix has form $A = (a_1, \dots, a_n)'$.

In the decument [8] has definition:

If a row (or column) of a fuzzy matrix A has identical elements, then this row (or column) is called the same element row (or column) of A. If the minimal element of a fuzzy matrix A forms a same element row (or column) of A, then we are say that the row(or column) can be crossed out, and other rows (column) form a new matrix, which is called a submatrix of A.

Theorem 3.6 The II type equation of a fuzzy symmetrical matrix $B_{n\times n}$ has a solution when index t=1 if and only if for B and its submatrices can be crossed out rows and columns step by step up to a 1 × 1 submatrix. And a solution matrix is the column vector to contain maximal element of B.

Theorem 3.7 Let II type equation of $A \in \mathcal{M}_{n\ n}$ has a solution. $\beta(A) = 1$ if and only if fuzzy relation N-D equation of A has a solution and $\theta(A) = 1$.

Theorem 3.8 Let the II type equation of $B = (b_{ij})_{n\times n}$ has a solution, and $A = (a_{ij})_{n\times m}$ is a solution of the II type equation of B . Then a_{ij} have properties as following:

(1) $\forall$ i, let $a_{i\,k_i} = \max\{a_{i1}, \dots, a_{im}\}$, then $a_{ik_i} = b_{ii}$;

(2) To correspond every element b_{ij} of B, the element a_{lk} (l=i or j) in A is exist so that $a_{lk} = b_{ij}$;

(3) If both a_{ik_i} and a_{jk_j} are situated in same column, i.e. when $k_i = k_j$, then $b_{ij} = b_{ii} \wedge b_{jj}$.

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