On Fuzzy Strongly Irresolute Mappings

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Abstract : The concept of fuzzy strongly irresolute(f.s.i.) mappings has been defined using the notions of fuzzy semi-open and semi-closed sets and some properties of such mappings are obtained.

Let (X,T) and (Y,T') be two fuzzy topological spaces. f will denote a mapping of (X,T) into (Y,T'). The family of all fuzzy semi-open (resp.semi-closed) sets of (X,T) will be denoted by $SO(T)$ (resp.$SC(T)$).

The fuzzy semi-closure(resp.semi-interior) of $A \in I^X$ denoted by $\bar{A}_s$ (resp. A_s°) is defined by

$\bar{A}_s = \wedge\{A_i \in SC(T),\ A_i \geqslant A\}$ (resp. $A_s^\circ = \vee\{A_i \in SO(T),\ A_i \leqslant A\}$.

Definition 1 : f is said to be f.s.i. if $\forall\, A \in I^X$ $f(\bar{A}_s) \leqslant f(A)$.

Remark 2 : f is f.s.i. iff $f(A_s^d) \leqslant f(A)$ $\forall\, A \in I^X$, where A_s^d denotes the set of all semi-accumulation points of A.

Theorem 3 : f is f.s.i. iff $f^{-1}(B) \in SC(T)$ $\forall\, B \in I^Y$.

Corollary 4 : f is f.s.i. iff $f^{-1}(B) \in SO(T)$ $\forall\, B \in I^Y$.

Remark 5 : Every f.s.i. mapping is fuzzy irresolute (f.i.) but the converse is not true.

Theorem 6 : If f be f.i. and $g : (Y,T') \longrightarrow (Z,T'')$ be f.s.i., then $g.f : (X,T) \longrightarrow (Z,T'')$ is f.s.i.

Theorem 7 : Let $f_i : (X_i, T_i) \longrightarrow (Y_i, S_i)$ be f.s.i. for $i = 1,2$ and let $X = X_1 \times X_2$, $Y = Y_1 \times Y_2$ where X_1 is product related to X_2. Then $f : (X, T_1 \times T_2) \longrightarrow (Y, S_1 \times S_2)$ defined by

$f(x_1, x_2) = (f_1(x_1), f_2(x_2))\ \forall\ x_1 \in X_1,\ x_2 \in X_2$ is f.s.i.

Theorem 8 : Let f be f.s.i. and let $g : (Y,T') \longrightarrow (Z,T'')$ be any mapping. Then $g.f : (X,T) \longrightarrow (Z,T'')$ is f.s.i.

Theorem 9 : Let $f : X \longrightarrow \prod X_i$ be f.s.i. and let $\forall\ i \in J$ $f_i : X \longrightarrow X_i$ be defined by $f_i(x)$ = ith cordinate of $f(x)\ \forall\ x \in X$ Then f_i is f.s.i.

Definition 10 : $D \in I^X$ is said to be S_2C-connected if D cannot be expressed as $D = B \vee C$ where $\bar{B}_S \wedge C = B \wedge \bar{C}_S = 0$.

Theorem 11 : Let f be f.s.i. and injective and let $A \in I^X$ be S_2C-connected. Then $(f(A))_0$ consists of only one point.

Theorem 12 : A semi-closed crisp subset of a fuzzy semi-compact space (X,T) is fuzzy semi-compact.

Definition 13 : f is said to be fuzzy semi-compact if the inverse image of every fuzzy semi-compact crisp subset in Y is fuzzy semi-compact in X.

Theorem 14 : Let f be f.s.i. where (X,T) is a fuzzy semi-compact space. Then f is fuzzy semi-compact.

Theorem 15 : Let f be f.s.i. If $A \in I^X$ be fuzzy semi-compact, then $(f(A))_0$ is finite.

Theorem 16 : Let f be f.s.i. and surjective. If X is fuzzy nearly semi-compact or fuzzy almost semi-compact, then Y is fuzzy semi-compact.

Definition 17: f is said to be a fuzzy completely irresolute mapping if the inverse image of every fuzzy semi-open set in Y is fuzzy regularly semi-open in X.

Theorem 18 : Let f be surjective and completely irresolute mapping. If (X,T) be fuzzy nearly semi-compact, then (Y,T') is fuzzy semi-compact.

Theorem 19 : If $f: (X,T) \longrightarrow (Y,T')$ be a fuzzy completely irresolute mapping and g: $(Y,T')\dashrightarrow(Z,T'')$ be fuzzy irresolute mapping, then g.f: $(X,T)\dashrightarrow(Z,T'')$ is fuzzy completely irresolute.

Remark 20 : Every f.s.i. mapping is fuzzy completely irresolute but the converse is not true.

References

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