On Fuzzy Semi-Connectedness

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Abstract: The concept of fuzzy semi-open and semi-closed sets have been utilised to define four types of semi-seperation of fuzzy sets corrosponding to the notions of seperation[5], Q-seperation[3], Weak seperation[4], Strong-seperation[5] and eight types of semi-connectedness viz. S_iC, SC_i-connectedness for i=1,2,3,4 corrosponding to the concept of O-connectedness[5], connectedness[3], connectedness[4], O_q-connectedness[5] and C_i-connectedness[1] of a fuzzy set. Interrelationship between these notions of semi-connectedness of a fuzzy set and their properties have been discussed.

S_i-seperated fuzzy sets (i=1,2,3,4)

Let (X,T) be a fuzzy topological space.

Definition 1: Two non-zero fuzzy sets A and B in (X,T) are said to be S_1 (resp. S_2)-seperated if there exist two fuzzy semi-open (resp. semi-closed) sets G,H such that $G \geqslant A, H \geqslant B$, $G \wedge B = 0, H \wedge A = 0$.

If A,B be S_1-seperated and if in addition $G(x)+H(x) > 1 \; \forall x$ in A_o and $H(x)+B(x) > 1 \; \forall x$ in B_o, then A,B are said to be S_4-seperated.

Remark 2: $A,B \in I^X$ are S_2-seperated iff $A \wedge \overline{B}_S = B \wedge \overline{A}_S = 0$ where $\overline{A}_S$ denote the fuzzy semi-closure of A.

Definition 3: Two non-zero fuzzy sets A and B are said to be S_3-seperated if $A \bar{q} \bar{B}_s$, and $B \bar{q} \bar{A}_s$.

Remark 4: $A, B \in I^X$ are said S_3-seperated iff there exist fuzzy semi-open sets G,H such that $A \leqslant G$, $B \leqslant H$, $A \bar{q} H$, $B \bar{q} G$

Remark 5: Seperation[5], Q-seperation[3], Weak seperation [4] and strong-seperation[5] of fuzzy sets imply respectively their S_1, S_2, S_3, S_4-seperations. But examples are given to show that the converse is not true.

Remark 6: Let $A, B \in I^X$ be S_i-seperated for i=1,2,3,4. If i=2 and if $A \vee B$ be fuzzy semi-closed, then A,B are fuzzy semi-closed. If $A \vee B$ be fuzzy semi-open (resp.semi-closed when $i \neq 2$), then A,B are not necessarily fuzzy semi-open(resp. semi-closed).

Theorem 7: Let $A, B \in I^X$ be such that $A_o \wedge B_o = 0$, $(\bar{A}_S)_{A_o \vee B_o} = (\bar{A}_S)_{A_o}$ and $(\bar{B}_S)_{A_o \vee B_o} = (\bar{B}_S)_{B_o}$. Then A,B are S_2-seperated.

Remark 8: Example is given to show that the conditions of the above theorem are not necessary.

Theorem 9: $A, B \in I^X$ are S_2-seperated if $(\bar{A}_S)_{A_o \vee B_o}$ and $(\bar{B}_S)_{A_o \vee B_o}$ are S_2-seperated.

Remark 10: Example is given to show that the converse of the above theorem is not true.

Semi-connected fuzzy sets

Definition 11: $D \in I^X$ is said to be S_iC-connected if D can not be expressed as the join of two non-zero S_i -seperated fuzzy sets A and B for i=1,2,3,4.

Definition 12: $D \in I^X$ is said to be SC_i -connected (i=1,2,3,4) if there do not exist fuzzy semi-open sets A,B such that respectively

SC_1 : $D \leqslant A \vee B$, $A \wedge B \leqslant 1-D$, $D \wedge A \neq 0$, $D \wedge B \neq 0$

SC_2 : $D \leqslant A \vee B$, $D \wedge A \wedge B = 0$, $D \wedge A \neq 0$, $D \wedge B \neq 0$

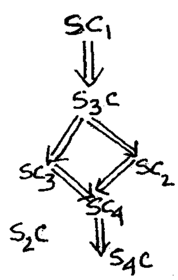
SC_3 : $D \leqslant A \vee B$, $A \wedge B \leqslant 1-D$, $A \nleqslant 1-D$, $B \nleqslant 1-D$.

SC_4 : $D \leqslant A \vee B$, $D \wedge A \wedge B = 0$, $A \nleqslant 1-D$, $B \nleqslant 1-D$.

<u>Remark 13</u> : S_1 C-connected===>O-connected[5], S_2 C-connected===>connected[3], S_3C-connected===>connected[4], S_4 C-connected===> O_q -connected[5], SC_i -connected===>C_i -connected[1] for i=1,2,3,4. But the reverse implications are not true.

<u>Remark 14</u>: Examples are constructed to show that that SC_2 ==≠=>SC_1 , SC_3 ==≠=>SC_1 , SC_4 ==≠=>SC_3 , SC_4==≠=>SC_2 , SC_3==≠=>S_2C, S_3C==≠=>SC_1 , S_2C==≠=>SC_3 , SC_2 ==≠=>S_3C, S_4C==≠=>SC_4 , S_4C==≠=>S_1C, S_2C==≠=>S_4C, S_4C==≠=>S_2C.

<u>Remark 15</u> Interrelationship between these notion of semi-connectedness of a fuzzy set in a fuzzy topological space is described by the following diagram.



Examples have been given to show that the reverse implications do not hold.

<u>Theorem 16</u>: Let $D \in I^X$ be such that for any two fuzzy points x_λ ,$y_\mu \in D$, there exists a $S_iC(SC_j)$-connected fuzzy subset A such that $x_\lambda, y_\mu \in A \leqslant D$. Then D is $S_iC(SC_j)$-connected

for i =1,2,3 and j =1,2,3,4. The result is not true for i =4.

Theorem 17: The join of any collection of $S_i C(SC_j)$ -connected fuzzy sets any two of which are intersecting(overlapping in case j =3,4) is $S_i C(SC_j)$-connected.

Remark 18: (i) The join of two overlapping S_4C-connected fuzzy sets is not necessarily S_4C-connected.

(ii) The conditions "intersecting" or "overlapping" in the above theorem are necessary.

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