ON THE DEFINITION OF A FUZZY SUBGROUP

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Abstract: Using the idea of quasi-coincidence of a fuzzy point with a fuzzy set, some new concepts of a fuzzy subgroup are introduced and their acceptibility are investigated. Some fundamental properties of one such viable fuzzy subgroup are obtained.

Unless otherwise mentioned G will denote a group with e as the identity element. If a fuzzy point x_t belongs to (resp. be quasi-coincident with) $A \in I^G$, then we write $x_t \in A$ (resp. $x_t\, q\, A$). If $x_t \in A$ and (resp. or) $x_t\, q\, A$, then we write $x_t \in\wedge q\, A$ (resp. $x_t \in\vee q\, A$).

α, β will denote any one of (i) $\in$, (ii) q, (iii) $\in\vee q$ (iv) $\in\wedge q$. $x_t \bar{\alpha}\, A$ will mean that $x_t \alpha\, A$ does not hold.

Definition 1: A fuzzy subset A of G is said to be a (α, β)-fuzzy subgroup of G ($\alpha \neq \in\wedge q$) if $\forall\, x, y \in G$ and $t_1, t_2 \in I-\{0\}$

(i) $x_{t_1} \alpha\, A,\ y_{t_2} \alpha\, A \implies (xy)_{M(t_1,t_2)} \beta\, A$

(ii) $A(x) = A(x^{-1})$.

Remark 2: The fuzzy subgroup, defined by Rosenfeld [5], is simply a $(\in, \in)$-fuzzy subgroup.

Remark 3: The case of $\alpha = \in\wedge q$ is omitted since there exist fuzzy subsets A s.t. $\{x_t\ ;\ x_t \in\wedge q\, A\}$ is empty. Infact, if $A(x) \leqslant .5 \ \forall x \in G$, then A is such a fuzzy subset.

Theorem 4: Let A be a non-zero (α,β)-fuzzy subgroup of G. Then

(i) $A(e) > 0$;

(ii) $A_0 = \{x \in G;\ A(x) > 0\}$ is a subgroup of G.

Theorem 5: Let A be a non-zero (α,β)-fuzzy subgroup of G where (α,β) = (i) $(\in,q)$, (ii) $(\in,\in\wedge q)$, (iii) $(q,\in)$, (iv) $(q,\in\wedge q)$ (v) $(\in\vee q,q)$, (vi) $(\in\vee q,\in\wedge q)$, (vii) $(\in\vee q,\in)$.
Then $A = \chi_{A_0}$, the characteristic function of A_0.

Theorem 6: Let A be a non-zero (q,q)-fuzzy subgroup of G. Then A is constant on A_0.

Theorem 7: Let H be any subgroup of G. Let $A: G \longrightarrow I$ be s.t. $A(x) = 0 \ \forall x \in G-H$. Then A is a $(q,\in\vee q)$-fuzzy subgroup of G if any one of the following holds.

(i) A is a non-zero constant on H.

(ii) $A(x) \geqslant .5$ and $A(x) = A(x^{-1}) \ \forall x \in H$.

Theorem 8: Let A be a $(q,\in\vee q)$-fuzzy subgroup of G s.t. A is not constant on A_0. Then $A(x) \geqslant .5 \ \forall x \in A_0$.

Remark 9: A is a $(\in\vee q,\in\vee q)$-fuzzy subgroup or $(\in,\in)$-fuzzy subgroup of G implies that A is a $(\in,\in\vee q)$ -fuzzy subgroup of G.

Example is given to show that the converse is not true.

Remark 10: A necessary condition for A to be a $(\in,\in\vee q)$-fuzzy subgroup of G is $x_{t_1}, y_{t_2} \in A \Longrightarrow (xy^{-1})_{M(t_1,t_2)} \in\vee q\ A$.
However example can be found to show that the condition is not a sufficient condition.

Remark 11: The only non-trivial generalisation of a fuzzy subgroup defined by Rosenfeld obtained in this manner is the cconcept of a $(\in, \in\vee q)$-fuzzy subgroup.

In what follows by a fuzzy subgroup we shall mean a $(\in, \in\vee q)$-fuzzy subgroup of G.

Theorem 12: For any subset H of G, χ_H is a fuzzy subgroup of G iff H is a subgroup of G.

Theorem 13: Let $\{A_i ; i \in J\}$ be any family of fuzzy subgroups of G. Let $A = \bigcap_{i\in J} A_i$. Then A is a fuzzy subgroup of G.

Theorem 14: Let G and G' be two groups and let $f : G \longrightarrow G'$ be a homomorphism. Let A,B be two fuzzy subgroups G and G' respectively. Then

(i) $f^{-1}(B)$ is a fuzzy subgroup of G;

(ii) If A satisfies the 'sup property', then f(A) is a fuzzy subgroup of f(G).

Remark 15: If A be a fuzzy subgroup of G, then $\forall t \in I-\{0\}$ $A_t = \{x \in G ; A(x) \geqslant t\}$ may not be a subgroup of G.

Definition 16: A fuzzy subgroup H of G is said to be a fuzzy normal subgroup of G if $A(xax^{-1}) \geqslant A(a)$ $\forall$ $x, a \in G$.

Remark 17: If A be a fuzzy normal subgroup of G in the sense of Mukherjee and Bhattacharya [4], then $A([x,y]) \geqslant A(x)$ $\forall$ $x, y \in G$ where $[x,y]$ denotes the commutator of x,y. But this is not necessarily true if A be a fuzzy normal subgroup of G in the above sense.

Definition 18: Let A be a fuzzy subgroup of G. $\forall x \in G$ $\hat{A}_x$ (resp. $\check{A}_x$) $: G \longrightarrow I$ defined by $\hat{A}_x(y) = A(yx^{-1})$ (resp. $\check{A}_x(y) = A(x^{-1}y)$) $\forall y \in G$. is called the fuzzy left (resp. right) coset of G determined by x and A.

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If A be a fuzzy normal subgroup of G, then $\hat{A}_x = \check{A}_x \; \forall x \in G$.

<u>Theorem 19:</u> Let A be a fuzzy normal subgroup of G. Let $\mathfrak{F}$ be the set of all fuzzy cosets of A. Then $\mathfrak{F}$ is a group if the composition be defined by $\hat{A}_x . \hat{A}_y = \hat{A}_{xy} \; \forall x, y \in G$

Let $\bar{A} : \mathfrak{F} \dashrightarrow I$ be defined by $\bar{A}(\hat{A}_x) = A(x) \; \forall x \in G$.

Then $\bar{A}$ is a fuzzy normal subgroup of .

The validity of some other results, analogus to those obtained by Mukherjee and Bhattacharya[5] in the case of (ϵ,ϵ)-fuzzy subgroups are examined.

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