# FUZZY OSCILLATORS

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Abstract: The concepts of fuzzy oscillator and fuzzy controlled oscillator are introduced as a counterpart of the crisp oscillator and (voltage) controlled oscillator. The block diagrams of such devices are presented and computer simulations are discussed.

## 1. Introduction

As well known, a crisp oscillator is a systems that autonomously generates a periodic signal at its output. If the frequency of the output signal (and possibly its harmonic content) can be controlled by a signal (generally a voltage or a d.c. current) presented to its input, then one gets a controlled oscillator (generally known as VCO -- voltage controlled oscillator). Such systems find an extensive use in communication technology (e.g. in the so called PLLs -- phase locked loops) and in industrial electronics.

With the advent of "fuzzy PLLs" /1/, /2/,/3/, the question was asked if fuzzy oscillators can be built up. this paper gives an answer and tries to explain how to build such a fuzzy system. In this purpose, the concept of fuzzy feedback is defined, (section 2), then the principles of fuzzy oscillators are discussed (section 3) and finally simulation results are presented (section 4).

## 2. Fuzzy feedback

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This concept was previously introduced in /4/, /5/. The simplest way the feedback can be realised for a discrete time fuzzy set is indicated in Fig. 1 a. In this figure, R denotes the rules describing the fuzzy system (i.e. the mapping of the fuzzy set $y_{n-1}$ into the set $y_n$ ), and denotes the delay (i.e. a system delaing the output with respect to the input by time $\tau$ ). A two-input system accepts two feedback paths derived from the case already discussed. Such a double feedback loop is sketched in fig. 1.b. Multiple inputs fuzzy systems can accept a multiple-loops feedback, as sketched in fig. 1 c.

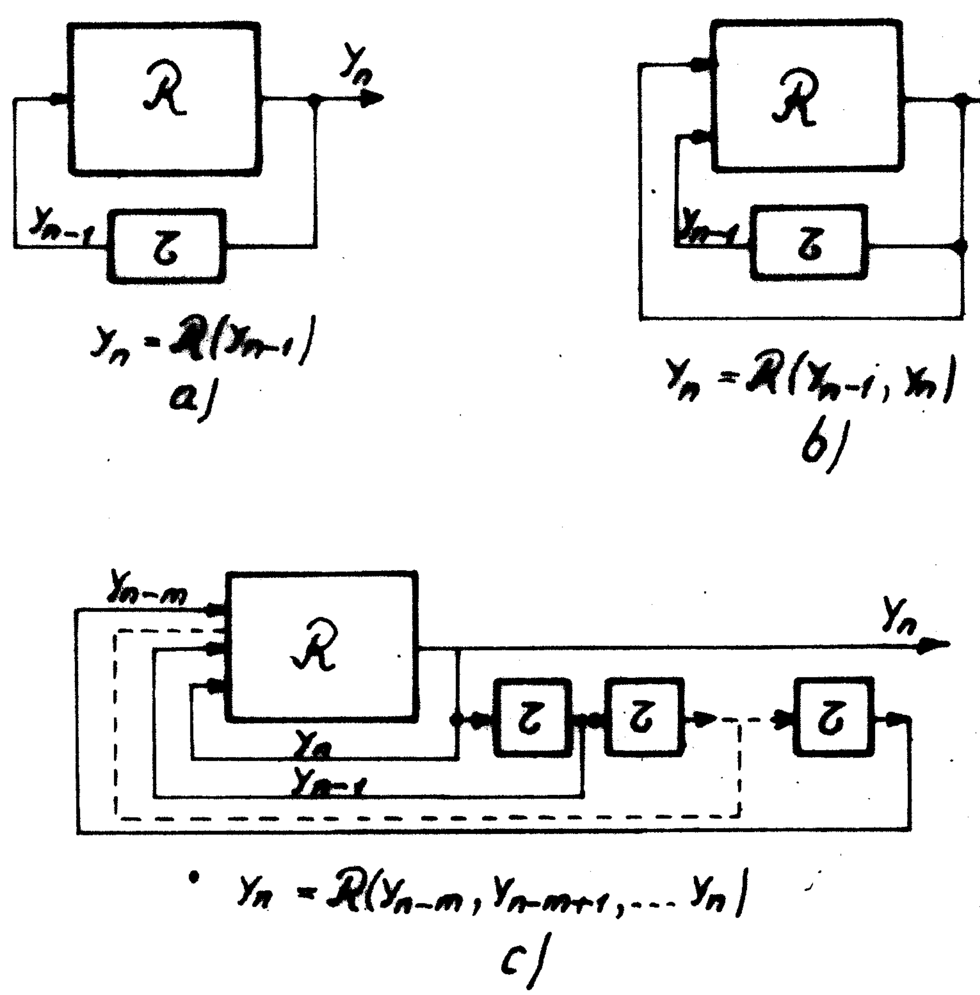



Fig. 1. Simple and multiple feedback loops

Note that a two-input fuzzy system can use only one of the inputs to realize the feedback loop, while the other input can be used to control the behaviour of the system; see Fig. 2.

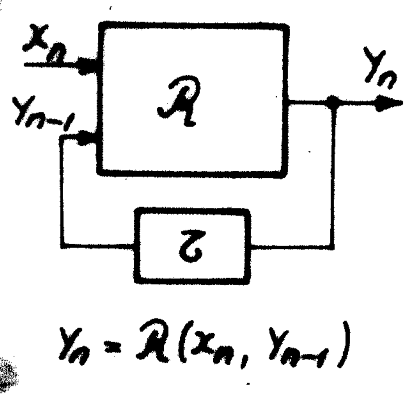



Fig. 2. A two-input, one feedback loop fuzzy system

However, the above discussed feedback fuzzy systems are not the true counterparts of the crisp systems with feedback because of the lack of summing nodes. It is obvious that a crisp summing node can not serve as a fuzzy node. Thus, we need a specific concept of node for fuzzy systems. (For the sampling node, no difference occurs between the fuzzy and the crisp case).

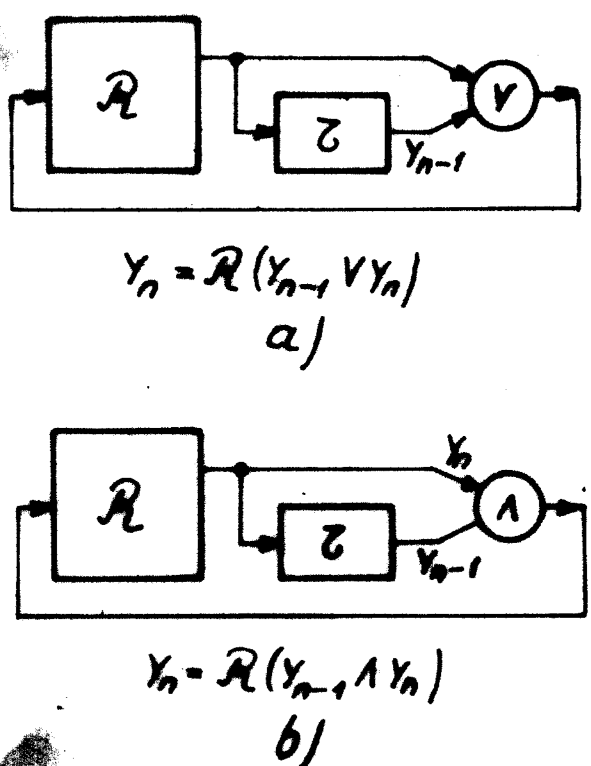
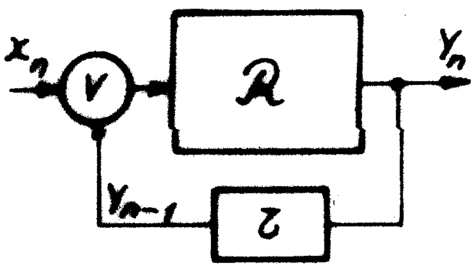



Fig. 3. Feedback fuzzy systems using fuzzy nodes

By 'fuzzy node' will be denoted an elementary two-input, one-output fuzzy system performing either the meet (meet node), or the join operation. Fuzzy systems with feedback, using fuzzy nodes, are sketched in Fig. 3 for two elementary cases.

The extension of the feedback fuzzy system presented in fig. 2, using a fuzzy node, is presented in Fig. 4. In the system sketched in fig. 4, the input can also be used to control the behaviour of the output, thus allowing for the design of controlled fuzzy oscillators (as explained in the next sections).




$$Y_n = \mathcal{R}(x_n \vee Y_n)$$

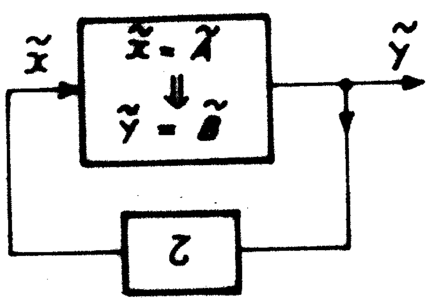
Fig. 4. Feedback system based on fuzzy node

### 3. Basical fuzzy oscillators

#### 3.1. Fixed frequency fuzzy oscillators

It is well known that a crisp oscillator is in fact a crisp unstable system exhibiting a periodical unstability (in contrast to quasi-periodical and to chaotic unstabilities). Technically, a crisp oscillator is realised by providing suitable positive feedback to a stable system.

Following the above idea, let us consider the simplest feedback configuration sketched in fig. 5. For the analogy



$$(\tilde{Y}_0 = 0)$$

Fig. 5. The basical fuzzy oscillator

$$\tau: \begin{array}{l} \tilde{y}(t<t_0)=A_1 \\ \tilde{y}(t>t_0)=A_2 \end{array} \Rightarrow \begin{cases} \tilde{x}(t<t_0+\tau)=B_1 \\ \tilde{x}(t>t_0+\tau)=B_2 \end{cases}$$

with the crisp case is complete, let us suppose that the fuzzy system, described by the set of rules:

$$R_k : \text{IF } x \text{ is } A_k \text{ THEN } y \text{ is } B_k \qquad (1)$$

is the fuzzy equivalent of an amplifier. Let us detail what this means. A crisp amplifier has two main features: i) monotonic (and even linear) behaviour -- at least in the so called 'active', or 'linear' region of the input--, and ii) its characteristic function is increasing. Let us translate this in a linguistic description. Suppose the input is described by the linguistic degrees A1, A2, ... ,An (ordered increasingly), and the output of the amplifier is described by the linguistic degrees B1, B2, ... , Bn (also increasingly ordered). The above properties require that:

i) if x is Ak and y is Bh, then if x is $A_{k+q}$ implies y is $B_r$, by necessity $B_r$ is higher than $B_h$;

ii) the linearity of the characteristic function ask -- at least if the linguistic description is good enough-- that : (If x is $A_k$ Then y is $B_{k+q}$) THEN (If x is $A_{k+1}$ then y is $B_{k+q+1}$).

Note that the value of q will determine the 'linearity region' of the system: it will include n-q linguistic degrees.

Also note that such a system can be conveniently described by introducing a fuzzy operator which we shall name the 'linguistic shift operator', or simply 'shift operator. Let denote it by $\mathbb{D}$. Then, the 'fuzzy increasing system', i.e. the equivalent of the crisp amplifier, is briefly described by the system of rules:

IF x is A1 THEN y is Bq (initial condition) (2)

$$\text{If } x \text{ is } \mathbb{D}(A_k) = \mathbb{D}^{k-1}(A_1) \text{ THEN } y \text{ is } \mathbb{D}^{k-1}(B_q) \qquad (3)$$

In eq. (3), $\mathbb{D}^k_{(Ai)} := \mathbb{D}(\ \mathbb{D}(\ldots(\ \mathbb{D}(A_i)\ldots)$ (4)

Let us also remark that the inverse of the shift operator, (in the sense of monotonicity) can be defined in a similar manner and can be used to introduce the counterpart of the 'inversing (crisp) amplifier'. In Fig. 6 are sketched the two basical fuzzy systems defined by using the direct (right), and respectively the inverse (left) shift operators. To simplify notations, in fig. 6 one considers that the input and the output are characterized by the same class of linguistic degrees.

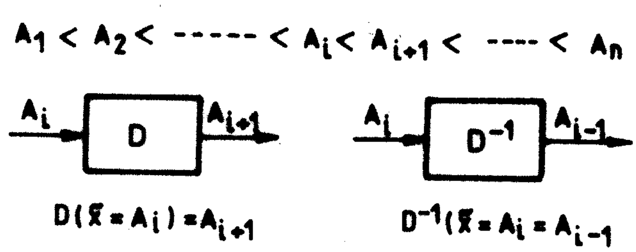



Fig. 6. Fuzzy systems defined by means of the shift operators.

Further discussion of the fuzzy shifting operators, including bi-dimensional shift operators, will be given in another paper.

Comming back to the system in fig. 5, let us check the first moments after starting up behaviour. Obviously, the conditions already asked for do not describe this behaviour. Thus, initial conditions have to be described, as well as first moments behaviour. Consider the system is started at time to. The output of the system in the off state will be imposed as:

$$y(t < t0) = A1 \ (=B1) \qquad (5)$$

Thus,

$$x(t \geq to + \tau) = A1$$

(As the delay does not change the nature of its input, the input and the output to the fuzzy system have to be described by the same linguistic degrees).

Let us impose that at the start, the output jumps to the value :

$$y(t_o \leq t < t_o + \tau) = A2 \qquad (6)$$

Then, the input will follow the output with a delay $\tau$ :

$$x(t_o + \tau \leq t < t_o + 2\tau) = A2$$

Now, the system depassed the initial (starting) regime and its behaviour is described by eq. (1) and eq. (2) and (3).

The system in fig. 5 and which behaviour is above described is, however, not an oscillator. It will behave exactly as a crisp amplifier provided with positive feedback: it will 'saturate'. Indeed, once the output reaches the maximal linguistic degree, the above description does not predict the next step. The saturation process can be then introduced by the condition:

IF x is An THEN y is An (7)

To get the desired oscillator, the saturation should be avoided. By analogy to the crisp case -- more exactly,to crisp flip-flops realised by using operational amplifiers -- let us provide a 'reset' condition, as bellow:

IF x is An THEN y is A1 (8)

Then we get:

Proposition 1. The fuzzy system described by eqs. (1), (2),(with q=2), (3), (with k = 1, ..., n-1), (5), (6) and (8) is an unstable system with periodical unstability.

The proof follows directly from the construction.

Concluding, we got a fuzzy oscillator, with a discrete output.

A simple example of fuzzy oscillator can be built using only five linguistic degrees to describe the input and the output. The 'linguistic waveform' at the output of this oscillator is plotted in fig. 7.

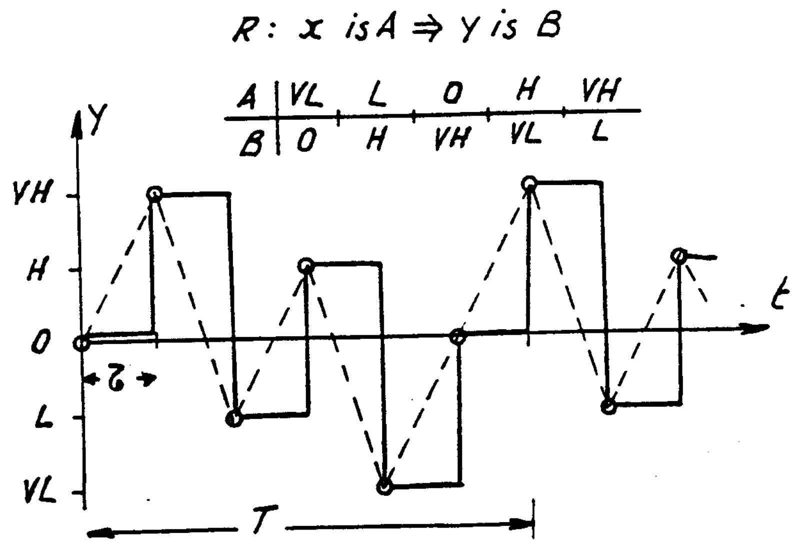



Fig. 7. The output of the basical fuzzy oscillator with 5 linguistic degrees

The question could be asked how to build up a fuzzy oscillator using a nonlinear fuzzy system with feedback. Such a question is important in some applications, for example when a specific 'linguistic waveform' is needed. To answer this question and to determine an algorithm to solve such a problem, let us first turn back to the simple oscillator already described. First let draw the graph of the outputs (in fact, of the states of the system) -- see fig. 8, left side. By re-drawing the graph, one gets a cyclic graph. The following is obvious:

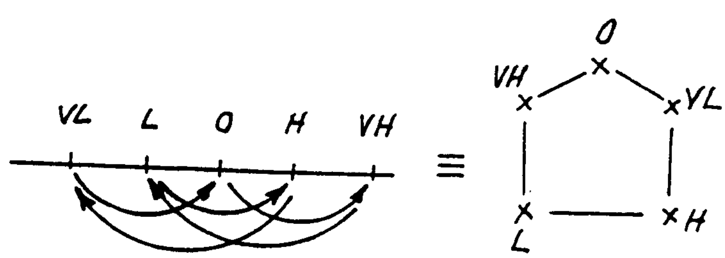



Fig. 8

Lemma: For a fuzzy system is an oscillator, the graph of its state transitions should be cyclic.

Now, the algorithm for building a fuzzy oscillator with given output waveform is transparent:

Algorithm

Step 1- Using the output waveform, draw the output graph in a cyclic form.

Step 2- Apply the $\mathbb{D}$ operator to the graph obtained in step 1 to determine the corresponding inputs; thus, the rules describing the system can be layed down (except for the starting rules).

Step 3- Fix the off-condition output and then write the starting up rule(s).

With the configuration as in fig. 5, the system is already described.

### 3.2. Controlled fuzzy oscillators

The controlled fuzzy oscillator can be built either as a two-inputs, no node feedback system, or as an one-input, one-node feedback system. A detailed discussion of the controlled oscillator will be presented in another paper. Note that in the case of the two-input, no-node case, the discussion in Section 3.1. directly applies in developping the controlled oscillator.

## 4. Computer simulation : brief comments

The behaviour of the no-node oscillators is easy to check even by hand-computation. This is not the case of controlled,or uncontrolled oscillators involving nodes in the feedback loop(s). Indeed, in this case, the output of the system is by no means restricted to a finite set of fuzzy values. To check that the system outputs an oscillatory, periodic signal computer simulation is needed.

Various oscillators were simulated (PASCAL programs) to determine their behaviour (period, waveform after defuzzification, locking time). It was found that fuzzy oscillators can be used to generate a large variety of waveform. Chaotic behaviour of some feedback fuzzy systems was also proved.

It is believed that fuzzy oscillators can be succesfuly used in many applications in communications engineering.