Fuzzy Connected Sets In Fuzzy Topological Spaces.

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Abstract : The interrelationship between different notions of connectedness of fuzzy sets introduced by Ming and Ming[3], Zheng Chong You[5], Saha[4],Azmal and Kohli[1] is investigated. A new concept of connectedness called C_δ - connctedness of a fuzzy set is introduced and the validity of the standard results for the new concept of connectedness are examined.

Let (X,T) be a fuzzy topological Space.

If $A \in I^X$ be connected in the sense of Ming and Ming[3](resp.Saha[4]), then we say that A is C_M -connected (resp. C_S -connected.)

The following results are obtained:-

Theorem 1: If $D \in I^X$ be C_1 -connected, then D is C_S - connected.

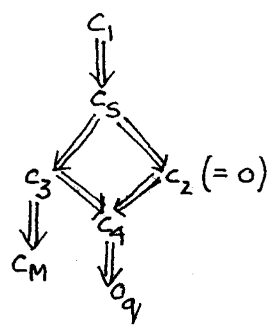
Theorem 2: If $D \in I^X$ be C_S-connected, then D is C_2 - connected.

Theorem 3: If $D \in I^X$ be $C_{\bar{S}}$-connected, then it is C_3 - connected.

~~Theorem 4: $D \in I^X$ is C_2-connected iff it is θ-connected.~~

Theorem 5: A fuzzy C_4-connected set is O_q-connected.

Remark 6: In a fuzzy topological space (X,T), the interrelationship between the classes of O-connected, O_q-connected and C_i-connected ($i=1,2,3,4,S,M$) fuzzy sets can be described by the following diagram.



Examples have been given to show that the reverse implications do not hold.

A new concept of connectedness of a fuzzy set.

In a topological space (X,T) a subset E of X is connected iff there does not exist any non-empty proper clopen subset in (E,T_E) where T_E is the reletivization of T in E. The open (closed) subset of (E,T_E) are the intersection of open (closed) subsets of (X,T) with E.

Let (X,T) be a fuzzy topological space and $E \in I^X$. We define $T_E = \{B \wedge N;\ N \in T\}$ and $C(T_E) = \{B \wedge P;\ P \in C(T)\}$. We call the members of $T_E(C(T_E))$ as fuzzy open (or closed) sets in D.

Definition 7: $D \in I^X$ is said to be C_5-connected if there does not exist any non-zero proper fuzzy clopen set in E.

Examples have been given to show that C_5-connectedness is totally different from all other types of connectedness mentioned earlier.

Theorem 8: Let f:(X,T)---(Y,S) be bijective and fuzzy continuous. If $D \in I^X$ be C_5-connected, then f(D) is also C_5 -connected in Y.

Theorem 9: Let $\{A_i ; i \in J\}$ be a collection of C_5 -connected fuzzy sets, any two of which are intersecting. Then $A = \bigvee_{i \in J} A$ is C_5-connected.

Remark 10: The fuzzy closure of C_5 -connected fuzzy set is not necesserily C_5-connected.

C_5 -component is defined and the validity of some well known results are investigated.

Referencess

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