A CRITICAL APPRECIATION ON FUZZY LOGIC CONTROLLER

Part II

Swapan Raha
Department of Mathematics
Visva-Bharati University
Santiniketan Bolpur 731235
INDIA

AND

Kumar S. Ray
Electronics & Communication
Sciences Unit
Indian Statistical Institute
203 Barrackpore Trunk Road
Calcutta 700 035, INIDA.

Abstract

In this paper, without denying the merits and novelty of the technique of approximate ressoning as presented by Zadeh [1] , we are proposing an alternative method for inexact ressoning that is simple to understand and implement. The proposed method desires an elementary knowledge in curve-fitting, a well-posed problem. It has been established that the proposed technique gives at least the same, if not better in all cases, results as found from the application of Zadeh's compositional rule of inference. The proposed method of inexact reasoning is first described and then illustrated with examples. The new results are compared with those obtained by the application of the existing rules of approximate reasoning. Ultimately we indicate the uselessness of the application of approximate reasoning in the design of fuzzy logic controller.

III. Numerical Examples

In this section our aim is to demonstrate through several numerical examples that the application of approximate reasoning for controller design can be more meaningfully replaced by the proposed technique.

Example 1 :

Let's consider the following premises

p <=> X is 'medium'

q   <=>   if   X   is   'low'   then   Y   is   'high'

in which X and Y range over the sets U & V respectively given by $U=V=1+2+3+4$ and the inexact concepts 'low', 'medium', 'high' are defined by

$$\text{'low'} = 1/_1 + .75/_2 + .50/_3 + .25/_4 = F$$

$$\text{'medium'} = .5/_1 + .75/_2 + .75/_3 + .5/_4 = H$$

$$\text{'high'} = .25/_1 + .50/_2 + .75/_3 + 1/_4 = G .$$

In terms of these definitions we thus have

$$p <=> X \text{ is 'medium'} -> \Pi_X = (.5\ .75\ .75\ .5) = H$$

and

$$q <=> \text{if X is 'low' then Y is 'high'} -> \Pi_{(X,Y)} = \text{'low'} X \text{'high'}$$

X \ Y	1	2	3	4
1	.25	.50	.75	1
2	.25	.50	.75	.75
3	.25	.50	.50	.50
4	.25	.25	.25	.25

$$= R .$$

Again

$$\Pi_Y = \Pi_X \circ \Pi_{(X,Y)} = H \circ R$$

$$= (.5 \quad .75 \quad .75 \quad .5) \circ \begin{pmatrix} .25 & .50 & .75 & 1 \\ .25 & .50 & .75 & .75 \\ .25 & .50 & .50 & .50 \\ .25 & .25 & .25 & .25 \end{pmatrix}$$

$$= (.25 \quad .50 \quad .75 \quad .75) \; .$$

Thus from the premises p and q, using compositional rule of inference, we obtain

$$r \leftarrow Y \text{ is } (.25 \quad .50 \quad .75 \quad .75) \; .$$

If instead, we take

$$q <=> \text{ if X is 'low' then Y is 'high'} \rightarrow \Pi_{(X,Y)} = \bar{F}' \oplus \bar{G}$$

we would have a relation R' in place of R as

$$\bar{F}' \oplus \bar{G} =$$

X \ Y	1	2	3	4
1	.25	.50	.75	1
2	.50	.75	1	1
3	.75	1	1	1
4	1	1	1	1

$$= R'$$

and

$$\Pi_Y = \Pi_X \circ \Pi_{(X,Y)} = H \circ R' = (.75 \quad .75 \quad .75 \quad .75) \quad \text{i.e. from p}$$

and q we obtain

$$r \leftarrow Y \text{ is } (.75 \quad .75 \quad .75 \quad .75) \; .$$

From the relation R we obtain the following obvious results :

i) if X is (1 o o o) then Y is (.25 .50 .75 1)

ii) if X is (o 1 o o) then Y is (.25 .50 .75 .75)

iii) if X is (o o 1 o) then Y is (.25 .50 .50 .50)

iv) if X is (o o o 1) then Y is (.25 .25 .25 .25)

which after defuzzification yields

i') if x = 1 then y = 4

ii') if x = 2 then y = 3.5

iii') if x = 3 then y = 3

iv') if x = 4 then y = 2.5 .

Here x and y are defined over U' and V' respectively. It can be easily seen that R' also gives the same collection of pairs. Using the above informations we find

$$y = 4.5 - .5x \qquad (1)$$

as the corresponding interpolation polynomial .

Now the defuzzified value of X = (.5 .75 .75 .5), obtained by taking the arithmetic mean of the generic values of X for which the membership values are maximum [3,5] in the set, will be 2.5 and from

$$y = 4.5 - .5x$$

for x = 2.5 we have y = 3.25.

Example 2 :

Let us consider the following problem in which we are given three fuzzy propositions of the form

if x is A then Y is B

where X and Y are two linguistic variables defined over U and V respectively and another fuzzy premise of the form

X   is A'

where A,A' are fuzzy subsets of U and B is a fuzzy subset of V. Let

$$U = 1+2+3+4+5 \quad , \quad V = 6+7+8 \quad .$$

Let the three compound assertions be

q_1 <=>   if X   is   'low' then Y is 'low'

q_2 <=>   if X   is 'medium' then Y is 'medium'

q_3 <=>   if X   is 'high' then Y is 'high'

and in terms of possibility distritution

q_1 <=> if X is (1/1 + .75/2 + .5/3 +.25/4 + 0/5) then Y is (1/6+.6/7+.3/8)

q_2 <=> if X is (.5/1 + .75/2 : 1/3 + .75/4 + .5/5)then Y is (.6/6+1/7+.6/8)

q_3 <=> if X is (0/1 +.25/2 + .5/3+.75/4+1/5)then Y is (.3/6+.6/7+1/8).

Thecorresponding relational matrix (using Zadeh's arithmetic rule) will
be

$R_1 =$

X\Y	6	7	8
1	1	.6	.3
2	1	.85	.55
3	1	1	.8
4	1	1	1
5	1	1	1

;

$R_2 =$

X\Y	6	7	8
1	1	1	1
2	.85	1	.85
3	6	1	.6
4	.85	1	.85
5	1	1	1

:

$R_3 =$

X\Y	6	7	8
1	1	1	1
2	1	1	1
3	.8	1	1
4	.55	.85	1
5	.3	.6	1

:

setting

$$R = R_1 \cap R_2 \cap R_3$$

and letting $\mu_R (x,y) = \min \left\{ \mu_{R1} (x,y), \mu_{R2} (x,y), \mu_{R3}(x,y) \right\}$
we find

X \ Y	6	7	8
1	1	.6	.3
2	.85	.85	.55
3	.6	1	.6
4	.55	.85	.85
5	.3	.6	.1

$$R =$$

Let the fuzzy proposition

$$X \text{ is } A'$$

induces a possibility distribution

$$(.25/1 + .50/2 + .75/3 + 1/4 + .75/5) \ .$$

Then compositional rule of inference gives the induced possibility distribution of Y as $(.6/6 + .85/7 + .85/8)$.

Whereas defuzzification of relations R_1, R_2, R_3 gives

R_1 -> if x is 1 then y is 6

R_2 -> if x is 3 then y is 7

R_3 -> if x is 5 then y is 8

x and y are defined over U' and V' respectively. Letting them as points in a two-dimensional plane we find that the corresponding interpolation polynomial will be

$$y = 5.5 + 0.5x \qquad\qquad (2)$$

Now the defuzzy value of the proposition

$$X \text{ is } A' \ <=> \ \Pi_x \ = \ (.25/1 + .50/2 + .75/3 + \ '1/4 +$$

$.75/5)$ is x=4 and from the relation (2) for x=4 we find y = 7.5, which is the same as the defuzzy value of "Y is $(.6/6 + .85/7 + .85/8)$".

IV. Critical Observation

In this section we view the entire method of approximate reasoning as follows :

Let us consider the proposition

p <=> X is F

where F is a fuzzy subset of U, the universe of discourse of X. Then consider a set of rectangular axes in the plane to represent u, the generic values of U and μ. Also let $U = u_1 + u_2 + u_3$; $u_1 = 1$, $u_2 = 2$, $u_3 = 3$ and $F = .6/1 + 1/2 + .6/3$. Now, let us plot the membership values against each u that are in F. We can view them as a series of trees of different heights standing on the axis of u. Let $V = v_1 + v_2 + v_3 + v_4$; $v_1 = 1$, $v_2 = 2$, $v_3 = 3$, $v_4 = 4$ and consider another axis through the origin and perpendicular to the plane of the existing set of axes to represent v, the generic values of V.

Then by cylindrical extension of the fuzzy set F over V we mean a rectangular forest in which trees are standing only at the junctions of the straight lines $u = u_i$: $i = 1,2,3$ and $v = v_j$; $j = 1,2,3,4$. They are standing in such a way that all trees in the series which are parallel to the axis of v are of the same height as that of the corresponding tree at u in F. Then defuzzification [3,5] of the cylindrical extension of F simply gives us those trees in a row the heights of which are maximum in the above mentioned forest. We take the corresponding straight line through them as the representation of the proposition p in our proposed technique. Obviously, this line passes through the point which corresponds to the defuzzy value of the fuzzy set F.

Now let us consider the proposition

$$q : \text{ if } X \text{ is } G \text{ then } Y \text{ is } H$$

where the universe of the linguistic variable Y is V and G, H are two fuzzy subsets defined on U and V respectively.
Let

$$G = 1/1 + .6/2 + .3/3 :$$

$$H = .25/1 + .50/2 + .75/3 + 1/4$$

and

$$R = \bar{G} \cap \bar{H} = G \times H$$

$$= \begin{pmatrix} .25 & .50 & .75 & 1 \\ .25 & .50 & .60 & .60 \\ .25 & .30 & .30 & .30 \end{pmatrix}$$

with reference to the frame uvu we may view R as a rectangular forest where trees are standing at the junctions of the straight lines

$$u = u_i \; ; \; i = 1,2,3,$$

$$v = v_j \; ; \; j = 1,2,3,4,$$

and the height of the tree at the junction $u = u_i$ and $v = v_j$ will be equal to $\mu(u_i, v_j)$ in R .

Let's now search every row of the forest along the axis of v for those trees for which the heights are maximum and collect the co-ordinates of the corresponding bases. In the process, we take only one such point in each row and if there be more than one such point, we take the mid-point of the line segment joining them. With these collection of points in the uv - plane we fit an interpolating polynomial which will represent a curve in the two-dimensional uv-plane. In our

proposed technique, this curve can be viewed as an extension to the defuzzification of the relational matrix R . Here we assume that all trees standing on the curve have the same height (we are not considering the respective heights in fitting the interpolation polynomial). Thus, in our proposed technique, the intersection of the curve with the straight line in the uv-plane can be viewed as an extension to the defuzzification of the conjunction/particularization of the cylindrical extension of F with that of $\overline{G} \cap \overline{H}$. In our proposed technique we thus have a collection of points and they can be viewed as the roots of the inference trees in the forest. They are then projected on the axis of v and the average of them is the corresponding inference. Again, we know that the defuzzy value of the cylindrical extension of the proposition p is a straight line which would stab the trees in the forest formed by the relational matrix R on the uv-plane. These stabbed-trees when projected on the axis of v gives us the required inference

$$r \leftarrow Y \text{ is } \overline{F} \text{ o } (\overline{G} \cap \overline{H}).$$

Looking at the defuzzy value of the inference r we can safely conclude that ultimately are not loosing any information due to the proposed technique.

V. Conclusion

The aim of this paper is to demonstrate that if the final control action "y" reduces to a single nonfuzzy value through some assumptions or approximation [3,5] there is no need to consider the concept of approximate reasoning. The same, if not better, result is

achieved through some conventional relation of the form $y = f(x)$. Specially, in case of process control where input and output variables are measurable, there is no need to consider fuzzy logic to interpret the vagueness of the linguistic statements given by an expert operator. An expert orerator is always aware of the operating range of the variables (input / output) of the plant and the linguistic variables (e.g. pressure is big, temperature is low etc.) stated by the operator varies within that operating range. Hence, when an operator says "pressure is big", he has got a definite nonfuzzy quantification about that bigness in his mind. At the time of knowledge acquisition, through proper question - answering system, we can easily collect that nonfuzzy value of "big pressure". Thus instead of defuzzifying the output we can easily defuzzify the vagueness of the linguistic statements at the begining of the fuzzy logic controller and store the experience of an operator through the conventional relation of the form $y=f(x)$. Hence, in our opinion, the concept of approximate reasoning can more appropriately be applied in medical consultancy, management decision - making, logic programming etc. rather than in controller design. At last we like to warn (very frankly) the fuzzy logic controller communities to re-evaluate the whole issue before they invest any further money for the research and development of fuzzy logic controller.