LIMIT CALCULATION THEOREM
OF QUOTIENT OF THE TWO GREY FUNCTIONS

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From the paper (1) and (2) we know: Let x, x_0 be two rational grey number, $f(x)$, $g(x)$ be two grey functions and both $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$ be finite. Then

$$\lim_{x\to x_0}[f(x)\pm g(x)] = \lim_{x\to x_0} f(x) \pm \lim_{x\to x_0} g(x)$$

$$\lim_{x\to x_0}[f(x)\cdot g(x)] = \lim_{x\to x_0} f(x) \cdot \lim_{x\to x_0} g(x)$$

On the basis, Discussing limit calculation of grey function.

Let the following x, x_0 be two rational grey numbers, $f(x)$, $g(x)$ be two grey functions and both $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$ be finite.

Lemma: If $\lim_{x\to x_0} g(x) = B$ and if $0 \bar{\in} [p(B), Q(B)]$. Then

$$\lim_{x\to x_0} 1/g(x) = 1/B.$$

Proof: According to definition of rational grey number quotient as $0 \bar{\in} [a, b]$, then $1/[a, b] = [1/b, 1/a]$, $1/(\underset{\sim}{a}, b] = [1/b, \underset{\sim}{1/a})$

From the paper (1) we know,

$\lim_{x\to x_0} 1/g(x) = 1/B$ if and only if

$$\lim_{x\to x_0} p[1/g(x)] = p[1/B] = 1/Q[B]$$

$$\lim_{x\to x_0} Q[1/g(x)] = Q[1/B] = 1/p[B]$$

$$\lim_{x\to x_0}(\inf 1/g(x)) = \inf 1/B = \inf B$$

Hence To show that lemmma is trues, it is enough to show that three equalitys above is trues.

At first proving: If there esists $\delta>0$, such that $d[x, x_0] < \delta$, there must be $0 \bar{\in} [p[g(x)], Q[g(x)]]$. Since $0 \bar{\in} [p[B], Q[B]]$, there are and only be following two circumstances:

1, As $p[B]>0$

since $\lim\limits_{x\to x_0} g(x) = B$ then $\lim\limits_{x\to x_0} p[g(x)] = p[B]$

for $\varepsilon = p[B]$ if there exists $\delta>0$ such that $d[x, x_0]<\delta$ there must be:

$$|p[g(x)] - p[B]| < p[B]$$

$0< P[g(x)] < 2p[B]$. Thus as $d[x, x_0] < \delta$ there must be $0 \bar{\in} [p[g(x)], Q[g(x)]]$.

2, As $Q[B]<0$

Choose: $\varepsilon = -Q[B]>0$. In the same manner we nust have

$$0 \bar{\in} [p[g(x)], Q[g(x)]].$$

Now. Under the circumstance of $d[x, x_0]<\delta$, we will prove the three equalitys above trues.

First proving: $p[1/g(x)] = p[1/B] = 1/Q[B]$

According to definition of rational grey number quotient we know:

$$\lim_{x\to x_0} p[1/g(x)] = \lim_{x\to x_0} 1/Q[g(x)]$$

$$\lim_{x\to x_0} 1/Q[g(x)] = 1/\lim_{x\to x_0} Q[g(x)]$$

since $\lim\limits_{x\to x_0} g(x) = B$, Acoording to equivalence of grey space and Euclidean three-space, then

$$\lim_{x\to x_0} Q[1/g(x)] = 1/Q[B] = p[1/B]$$

In the same manner proving :

$$\lim_{x\to x_0} Q[1/g(x)] = 1/p[B] = Q[1/B]$$

proving again: $\lim\limits_{x\to x_0}(\inf 1/g(x)) = \inf 1/B = \inf B.$

Since $\lim_{x \to x_0} g(x) = B$

then for $\varepsilon = 1 > 0$ if exists $\delta_1 > 0$, $\delta > \delta_1$, such that $d[x, x_0] < \delta_1$

We have :

$$d[g(x), B] = \max\{|p[g(x)]-p[B]|, |Q[g(x)]-Q[B]|, |\inf g(x) - \inf B|\} < 1.$$

Then $|\inf g(x) - \inf B| < 1$.

Next since $\inf g(x) \in \{0, 1\}$, $\inf B \in \{0, 1\}$.

Then $\inf g(x) = \inf B$. $\lim_{x \to x_0} (\inf g(x)) = \inf B$.

Thus $$\lim_{x \to x_0} (\inf 1/g(x)) = \inf 1/B = \inf B.$$

The lemma follows.

Theorem: If $\lim_{x \to x_0} f(x) = A$ $\lim_{x \to x_0} g(x) = B$ and $0 \bar{\in} [p[B], Q[B]]$. Then

$$\lim_{x \to x_0} f(x)./g(x) = A/B.$$

Proof : According to lemma and multiplication rule of grey limit.

It can be know $\lim_{x \to x_0} f(x)/g(x) = \lim_{x \to x_0} f(x) \cdot 1/g(x)$

$$= \lim_{x \to x_0} f(x) \cdot \lim_{x \to x_0} 1/g(x) = A \cdot 1/B = A/B.$$

Then $\lim f(x)/g(x) = A/B$.

The theorem follows.

References

(1) , Wu Heqin and Yue Changan: Introduction on Grey Mathematics (in Chinese) Hebei people' s Education Publiching House, 1989.

(2) . Wu Heqin and Yue Changan: <Grey Limit>, (in China) Heilongjiang Water Conservancy Training school.

(3) . Yue Changan. The complex fuzzy metric space-The grey metric space, n° 38 issue 89 of BUSEFAL.