FUZZY TRANSCENDENTAL FIELD EXTENSIONS

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Abstract:

In this paper, we introduce the concept of fuzzy transcendental field extensions. We fuzzify the concept of algebraic field extension in a different manner than as previously done. We show that every fuzzy field extension has a fuzzy transcendence basis. We introduce neutral fuzzy field extensions.

Key words: fuzzy field extension, fuzzy algebraical, fuzzy algebraically independent, fuzzy transcendence basis, fuzzy subfield, fuzzy subring.

INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [8]. Rosenfeld [7] inspired the development of fuzzy algebraic structures. The development of fuzzy group theory and, to a much lesser extent, fuzzy ring theory has been rapid. However, little has been done in the development of fuzzy field theory. The purpose of this paper is to continue the work which has taken place in fuzzy field theory [1, 3 - 6]. We fuzzyify the notion of an algebraic field extension in a different manner than in [4]. We note the strengths and weaknesses of each fuzzification. We introduce the concept of fuzzy transcendental field extensions. We show that every fuzzy field extension has a fuzzy transcendence basis, Theorem 3.12. We discover a concept for fuzzy field extensions which ordinary field extensions do not possess. We say that a fuzzy field extension with this property is neutral. It appears that the notion of neutrality for fuzzy field extensions corresponds to that of equality for ordinary field extensions.

Let R denote a commutative ring with identity. A **fuzzy subset** X of R is a function from R into the closed interval [0, 1]. Let X and Y be fuzzy subsets of R. Then addition and multiplication of X and Y is given by for all $z \in R$, $(X + Y)(z) = \sup\{\min\{X(x), Y(y)\} \mid z = x + y\}$ and $(XY)(z) = \sup\{\min\{X(x), Y(y)\} \mid z = xy\}$. If $c \in R$ and $t \in [0, 1]$, we let c_t denote the fuzzy subset of R defined by $c_t(z) = t$ if $z = c$ and $c_t(z) = 0$ if $z \neq c$. c_t is called a **fuzzy singleton**. We say that $X \subseteq Y$ if and only if $X(z) \leq Y(z)$ for all $z \in R$. We say that $X \subset Y$ if $X \subseteq Y$ and $\exists\, z \in R$ such that $X(z) < Y(z)$. If $\mathcal{S}$ is a collection of fuzzy subsets of R, then we define the fuzzy subsets $\bigcap_{X \in \mathcal{S}}$ and $\bigcup_{X \in \mathcal{S}}$ of R by for all $z \in R$, $(\bigcap_{X \in \mathcal{S}})(z) = \inf\{X(z) \mid X \in \mathcal{S}\}$ and $(\bigcup_{X \in \mathcal{S}})(z) = \sup\{X(z) \mid X \in \mathcal{S}\}$. If $t \in [0, 1]$, we let $X_t = \{x \in R \mid X(x) \geq t\}$ and $X^* = \{x \in R \mid X(x) > 0\}$. For S a subset of R, we let δ_S denote the characteristic function of S, i. e., $\delta_S(z) = 1$ if $z \in S$ and $\delta_S(z) = 0$ otherwise. A fuzzy subset A of R is called a **fuzzy subring** of R if and only if $A(1) = 1$ and for all $x, y \in R$, $A(x - y) \geq \min\{A(x), A(y)\}$ and $A(xy) \geq \min\{A(x), A(y)\}$. We let $\mathfrak{F}(R)$ denote the set of all fuzzy subrings of R. Let F be a field. A fuzzy subset A of F is called a **fuzzy subfield** of F if and only if A is a fuzzy subring of F such that for all $x \in F$, $x \neq 0$, $A(x^{-1}) = A(x)$. We let $\mathfrak{F}(F)$ denote the set of all fuzzy subfields of F. If $A, B \in \mathfrak{F}(F)$ and $B \subseteq A$, we write A/B and call A/B a **fuzzy field extension**. We let $\mathfrak{F}(A)$ denote the set of all fuzzy subsets X of F such that $X \subseteq A$ and $\mathfrak{F}(A/B)$ the set of all fuzzy subfields C of F such that $B \subseteq C \subseteq A$. If $A \in \mathfrak{F}(F)$ and $t \in [0, 1]$, then A_t and A^* are subfields of F. We let N denote the set of positive integers. A fuzzy subset X of F is said to have the sup property if and only if every nonempty subset of Im(X), the image of X, has a maximal element.

1. FUZZY GENERATORS

Proposition 1.1. Let X and Y be fuzzy subsets of R. Let $A \in \mathcal{F}(R)$. If $X, Y \subseteq A$, then $X + Y \subseteq A$ and $XY \subseteq A$.

Proposition 1.2. Let X be a fuzzy subset of F. Let $A \in \mathcal{F}(F)$. Define the fuzzy subset X^{-1} of F by $\forall z \in F$, $z \neq 0$, $X^{-1}(z) = X(z^{-1})$ and $X^{-1}(0) = X(0)$. If $X \subseteq A$, then $X^{-1} \subseteq A$.

For any fuzzy singleton x_t of F, it follows easily that $(x_t)^{-1} = (x^{-1})_t$.

Definition 1.3. Let $A, B \in \mathcal{F}(R)$, $B \subseteq A$, and let X be a fuzzy subset of R such that $X \subseteq A$. Define B[X] to be the intersection of all $C \in \mathcal{F}(R)$ such that $B \cup X \subseteq C \subseteq A$.

Definition 1.4. Let $A, B \in \mathcal{F}(F)$, $B \subseteq A$, and let X be a fuzzy subset of F such that $X \subseteq A$. Define B(X) to be the intersection of all $C \in \mathcal{F}(F)$ such that $B \cup X \subseteq C \subseteq A$.

It follows easily that B[X] is a fuzzy subring of R in Definition 1.3 and that B(X) is a fuzzy subfield of F in Definition 1.4.

In the following results, we use the notation $\sum (b_i)_{u_i}(x^i)_t$ to denote $\sum_{i_1=0}^{m_1} \cdots \sum_{i_n=0}^{m_n} (b_{i_1 \ldots i_n})_{u_{i_1 \ldots i_n}} (x_1{}^{i_1})_{t_1} \cdots (x_n{}^{i_n})_{t_n}$ where $n \in N$; $i_j, m_j \in N \cup \{0\}$; $u_{i_1 \ldots i_n}, t_j \in [0, 1]$; $j = 1, \ldots, n$.

Theorem 1.5. Let $A, B \in \mathcal{F}(R)$ be such that $B \subseteq A$ and let X be a fuzzy subset of R such that $X \subseteq A$. Define the fuzzy subset S of R by $\forall z \in R$,

$S(z) = \sup\{(\sum (b_i)_{u_i}(x^i)_t)(z) \mid B(b_i) = u_i,\ X(x_j) = t_j \text{ if } i_j \neq 0 \text{ and } t_j \in \{X(x_j), 1\} \text{ if } i_j = 0;\ i_j = 0, 1, \ldots m_j;\ j = 1, \ldots, n;\ n \in N\}$.
Then $S = B[X]$.

Theorem 1.6. Let $A, B \in \mathcal{F}(F)$ be such that $B \subseteq A$ and let X be a fuzzy subset of F such that $X \subseteq A$. Define the fuzzy subset S of F by $\forall z \in F$,

$S(z) = \sup\{(\sum (b_i)_{u_i}(x^i)_t)(\sum (c_j)_{v_j}(y^j)_s)^{-1}(z) \mid B(b_i) = u_i,\ X(x_k) = t_k \text{ if } i_k \neq 0 \text{ and } t_k \in \{X(x_k), 1\} \text{ if } i_k = 0;\ i_k = 0, 1, \ldots, m_k;\ k = 0, 1, \ldots n;\ n \in N;\ B(c_j) = v_j,\ X(y_h) = s_h \text{ if } j_h \neq 0 \text{ and } s_h \in \{X(y_h), 1\} \text{ if } j_h = 0;\ j_h = 0, 1, \ldots, q_h;\ h = 1, \ldots, r;\ r \in N\}$.
Then $S = B(X)$.

2. FUZZY ALGEBRAIC INDEPENDENCE

If X is a fuzzy subset of F, we let $\mathcal{S}(X) = \{x_t \mid t = X(x) > 0\}$. If $\mathcal{S}$ is a set of fuzzy singletons such that $x_t, x_s \in \mathscr{S}$ implies $t = s$, then we let $X(\mathscr{S})$ denote the fuzzy subset of F defined by $X(\mathscr{S})(x) = t$ if $x_t \in \mathscr{S}$ and $X(\mathscr{S})(x) = 0$ if $x_t \notin \mathscr{S}$. Clearly, $X(\mathscr{S}(X)) = X$ and $\mathscr{S}(X(\mathscr{S})) = \mathscr{S}$. We let $\mathbf{i} = (i_1, ..., i_n)$.

In the remainder of the paper, A, B $\in \mathscr{F}(F)$ and $B \subseteq A$.

Definition 2.1. Let $X \in \mathscr{F}(A)$. Then X(or $\mathscr{S}(X)$) is said to be **fuzzy algebraically independent** over B if and only if $\forall\ (x_1)_{t_1}, ..., (x_n)_{t_n} \in \mathscr{S}(X)$, $\forall\ b_1, ..., b_n \in F$, $\forall\ s \in (0, 1]$, $\sum(b_{\mathbf{i}})_{u_{\mathbf{i}}}(x^{\mathbf{i}})_t = 0_s$ where $B(b_{\mathbf{i}}) \geq u_{\mathbf{i}}$ and $X(x_j) \geq t_j$ $(j = 1, ..., n)$ implies $b_{\mathbf{i}} = 0$ for all $\mathbf{i}$.

If $c_t \subseteq A$ is fuzzy algebraically independent over B, then c_t is also said to be **fuzzy transcendental** over B.

Proposition 2.2. Let $X \in \mathscr{F}(A)$. Then X is fuzzy algebraically independent over B if and only if $\forall\ s \in (0, 1]$, X_s is algebraically independent over B_s.

Proposition 2.3. Let $X \in \mathscr{F}(A)$. Then X is fuzzy algebraically independent over B if and only if X^* is algebraically independent over B^*.

Proposition 2.4. Let $X \in \mathscr{F}(A)$.

(i) $\forall\ t \in (0, 1]$, $B_t(X_t) \subseteq B(X)_t$;

(ii) If B(X) has the sup property, then $\forall\ t \in (0, 1]$ $B_t(X_t) = B(X)_t$.

Proposition 2.5. Let $X \in \mathscr{F}(A)$. Then $B(X)^* = B^*(X^*)$ and $B[X]^* = B^*[X^*]$.

Let $X \in \mathscr{F}(A)$. We say that X is maximally fuzzy algebraically independent over B if and only if X is fuzzy algebraically independent over B and there does not exist $Y \in \mathscr{F}(A)$ such that Y is fuzzy algebraically independent over B and $X \subset Y$.

Proposition 2.6. Let $X \in \mathscr{F}(A)$. Suppose $\forall\ x \in X^*$ that $X(x) = A(x)$. Then X is maximally fuzzy algebraically independent over B if and only if X^* is a transcendence basis of A^*/B^*.

The **cardinality** of a fuzzy subset X of F is defined to be the cardinality of X^*.

Corollary 2.7. A/B has maximal fuzzy algebraically independent fuzzy subsets of F and the cardinality of each is unique.

3. FUZZY ALGEBRAIC FIELD EXTENSIONS AND FUZZY TRANSCENDENCE BASES

Let $c_t \subseteq A$, $t > 0$. Suppose that c_t is not fuzzy algebraically independent over B. Then $\exists\ n \in N$, $\exists\ b_i \in B^*$, $\exists\ s, u_i \in (0, 1]$ such that $B(b_i) = u_i$ for $i = 0, 1, ..., n$ and such that $0_s = \sum_{i=1}^{n} (b_i)_{u_i}(c_t)^i$ with not all $b_i = 0$. If the only such s that exist for which such an equation holds are strictly less than t, then c_t is not fuzzy algebraic over B according to [4, Definition 1.1]. There are other complications that arise with [4, Definition 1.1] which we point out in this section. Hence it would seem that c_t should be called fuzzy algebraic over B if c_t is not fuzzy transcendental over B. However there are also complications with this definition which we point out in this section and the next. In order to compare the two ideas of fuzzy algebraic, we define some new terminology. This terminology should be considered temporary.

Definition 3.1. Let $c_t \subseteq A$ with $t > 0$. Then c_t is called **fuzzy algebraical** over B if and only if c_t is not fuzzy transcendental over B. If every such c_t is fuzzy algebraical over B, then A/B is called **fuzzy algebraical**; otherwise A/B is called **fuzzy transcendental.**

Definition 3.2. Let $c_t \subseteq A$ with $t > 0$. If $\exists\, s \in (0, t]$ such that $c_s \subseteq B$, then c_t is called **neutral** over B. If every such c_t is neutral over B, then A/B is called **neutral.**

Clearly, A/B is neutral if and only if $A^* = B^*$. Suppose that for $c \in F$, $t = A(c) > B(c) = s > 0$. Then $c_t + (-c_s) = 0_s$ so that c_t is fuzzy algebraical over B. In fact, we can think of c_t as being a root of a first degree polynomial in x, $x + (-c_s)$, with $c_s \subseteq B$.

Example 3.3. Let $F = P(t)(c)$ where P is a field, t is transcendental over P, and c is a root of the polynomial $f(x) = x^2 + tx + t$ over P(t). By Eisenstein's criterion, f(x) is irreducible over P(t). Let $A = \delta_F$. Define the fuzzy subset B of F by $B(y) = 1$ if $y \in P$, $B(y) = 3/4$ if $y \in P(t) - P$, and $B(y) = 1/2$ if $y \in F - P(t)$. Then B is a fuzzy subfield of F and $B \subsetneq A$. Now $c_{3/4} + (-c_{1/2}) = 0_{1/2}$ and so $c_{3/4}$ is neutral over B. In fact, A/B is neutral. Also, $(c_{3/4})^2 + t_{3/4}c_{3/4} + t_{3/4} = 0_{3/4}$. Thus $c_{3/4}$ is fuzzy algebraic over B. Of course, f(x) is not irreducible over B^*. $\forall\, t \in (3/4, 1]$, c_t is neutral over B, but not fuzzy algebraic over B for if $\sum_{i=0}^{n} (b_i)(c_t)^i = 0_t$, then $B(b_i) \geq u_i \geq t$ which is impossible since b_i would be in P, but c is not algebraic over P.

One of the strengths of the concept of a fuzzy algebraic field extension is its characterization in terms of level subfields [4, Theorem 2.1]. We now show that the concept of a fuzzy algebraical field extension A/B is characterized in terms of A^*/B^*.

Proposition 3.4. A/B is fuzzy algebraical if and only if A^*/B^* is algebraic.

Corollary 3.5. Let $X \in \mathcal{F}(A)$. Then $B(X) = B[X]$ if and only if B(X)/B is fuzzy algebraical.

Corollary 3.6. Let $C \in \mathcal{F}(A/B)$. Then A/B is fuzzy algebraical if and only if A/C and C/B are fuzzy algebraical.

Definition 3.7. Let $c_t \subseteq A$ with $t > 0$. Suppose that c is a root of a polynomial $p(x) = k_n x^n + \dots + k_1 x + k_0$ over B^*. We say that c_t is **fuzzy algebraical** with respect to p(x) over B^* if and only if $0_s = \sum_{i=0}^{n} (k_i)_{u_i}(c_t)^i$ for some $u_i \in (0, 1]$ where $B(k_i) \geq u_i$ for $i = 0, 1, \dots, n$ and $s \leq t$. For $s = t$, we say that c_t is **fuzzy algebraic** with respect to p(x) over B^*.

Lemma 3.8. [2] Let k, b $\in B^*$, $k \neq 0$. If $B(k) \neq B(b)$, then $B(kb) = \min\{B(k), B(b)\}$.

Proposition 3.9. Let $c_t \subseteq A$ with $t > 0$. Suppose that c is algebraic over B^*. Let p(x) be the minimal polynomial of c over B^*. If c_t is fuzzy algebraical (not fuzzy algebraic) with respect to p(x), then c_t is fuzzy algebraical (not fuzzy algebraic) with respect to every irreducible polynomial over B^* which has c as a root.

Example 3.10. Let F, c, and A be defined as in Example 3.3 where P is a perfect field of characteristic $p > 0$. Define the fuzzy subset B of F by $B(y) = 1$ if $y \in P$, $B(y) = 3/4$ if $y \in P(t^p) - P$, $B(y) = 1/2$ if $y \in P(t) - P(t^p)$, and $B(y) = 0$ if $y \in F - P(t)$. Then B is a fuzzy subfield of F and $B \subseteq A$. Let $p(x) = x^2 + tx + t$ and $q(x) = x^{2p} + t^p x^p + t^p$. Then $c_{3/4}$ is fuzzy algebraic with respect to q(x) over B^* since $(c_{3/4})^{2p} + (t_{3/4})^p(c_{3/4})^p + (t_{3/4})^p = 0_{3/4}$ and $c_{3/4}$ is fuzzy algebraical (not fuzzy algebraic) with respect to p(x) over B^* since $(c_{3/4})^2 + t_{1/2}c_{3/4} + t_{1/2} = 0_{1/2}$.

Proposition 3.11. Define the fuzzy subset $B^{(n)}$ of F by $B^{(n)}(x) = A(x)$ if $x \in B^*$ and $B^{(n)}(x) = 0$ if $x \notin B^*$. Then $B^{(n)} \in \mathcal{F}(A/B)$. Suppose that $\inf\{A(x) \mid x \in B^*\} \geq \sup\{A(x) \mid x \notin B^*\}$. If A/B is fuzzy algebraical, then $A/B^{(n)}$ is fuzzy algebraic.

$B^{(n)}$ is called the **neutral closure** of B in A. Let $c_t \subseteq A$ with $t > 0$. If $c_t + (-c)_s = 0_s$ with $s > 0$ and $(-c)_s \subseteq B^{(n)}$, then $B^{(n)}(c) \geq s > 0$ and so $c \in B^{(n)*}$. Hence $c_t \subseteq B^{(n)}$. That is, if c_t is neutral over $B^{(n)}$, then $c_t \subseteq B^{(n)}$.

We now turn our attention to fuzzy transcendence bases.

Definition 3.12. Let $X \in \mathcal{F}(A)$. Then X is called a **fuzzy transcendence basis** of A/B if and only if X is fuzzy algebraically independent over B and A/B(X) is fuzzy algebraical.

Theorem 3.13. A/B has a fuzzy transcendence basis and the cardinality of a fuzzy transcendence basis is unique. In fact, X is a fuzzy transcendence basis of A/B if and only if X^* is a transcendence basis of A^*/B^*.

It is clear that not every fuzzy field extension A/B has a fuzzy transcendence basis X in the sense that A/B(X) is fuzzy algebraic. For example, let A/B be a nontrivial neutral fuzzy field extension.

4. FUZZY ALGEBRAICAL CLOSURES

Throughout this section we assume that F has characteristic $p > 0$. If the concept of algebraic field extension is fuzzified by way of fuzzy algebraical field extensions, then the concepts of fuzzy purely inseparable [4] and fuzzy separable algebraic [4] field extensions should be modified accordingly. We introduce new terminology here also.

Definition 4.1. Suppose that $c_t \subseteq A$ with $t > 0$. Then c_t is said to be **fuzzy pure inseparable** over B if and only if $\exists\ e \in N \cup \{0\}$, $\exists\ b_0, b_1 \in B^*$, $\exists\ s, u_0, u_1 \in (0, 1]$, $B(b_i) = u_i$ for $i = 0, 1$ such that $(k_1)_{u_1}(c_t)^{p^e} + (k_0)_{u_0} = 0_s$. A/B is called **fuzzy pure inseparable** if and only if every $c_t \subseteq A$ with $t > 0$ is fuzzy pure inseparable over B.

Definition 4.2. Suppose that $c_t \subseteq A$ with $t > 0$. Then c_t is said to be **fuzzy separable algebraical** over B if and only if $\exists\ n \in N$, $\exists\ b_i \in B^*$, $\exists\ s, u_i \in (0, 1]$, $B(b_i) = u_i$ for $i = 0, 1, \ldots, n$ such that $0_s = \sum_{i=0}^{n} (k_i)_{u_i}(c_t)^i$ and the polynomial $\sum_{i=0}^{n} k_i x^i$ (in x) is separable over B^*. A/B is called **fuzzy separable algebraical** if and only if every $c_t \subseteq A$ with $t > 0$ is fuzzy separable algebraical over B.

Now $c_t \subseteq A$ $(t > 0)$ is neutral over B if and only if c_t is fuzzy pure inseparable and fuzzy separable algebraical over B, yet in either event it is not necessarily the case that $c_t \subseteq B$. If c_t is fuzzy purely inseparable and fuzzy separable algebraic over B, then $c_t \subseteq B$, [3]. Another complication or difference from the fuzzy purely inseparable case is that c_t may be fuzzy pure inseparable over B without $(c_t)^{p^e} \subseteq B$ for some $e \in N$.

Proposition 4.3. (i) A/B is fuzzy pure inseparable if and only if A^*/B^* is purely inseparable.

(ii) A/B is fuzzy separable algebraical if and only if A^*/B^* is separable algebraic.

Corollary 4.4. Let $C \in \mathcal{F}(A/B)$.

(i) A/B is fuzzy pure inseparable if and only if A/C and C/B are fuzzy pure inseparable.

(ii) A/B is fuzzy separable algebraical if and only if A/C and C/B are fuzzy separable algebraical.

Proposition 4.5. Let $c_t \subseteq A$ with $t > 0$. If c_t is fuzzy algebraical (pure inseparable or separable algebraical) over B, then $B(c_t)/B$ is fuzzy algebraical (pure inseparable or separable algebraical).

Definition 4.6 Let $X \in \mathcal{F}(A)$. Then X is called a **fuzzy separating transcendence basis** of A/B if and only if X is fuzzy algebraically independent over B and A/B(X) is fuzzy separable algebraical.

Proposition 4.7. A/B has a fuzzy separating transcedence basis if and only if A^*/B^* has a separating transcendence basis. In fact, X is a fuzzy separating transcendence basis of A/B if and only if X^* is a separating transcendence basis of A^*/B^*.

Let $C \in \mathcal{F}(A/B)$. We say that C is the **fuzzy algebraical closure** of B in A if and only if C/B is fuzzy algebraical and $\forall\ c_t \subseteq A$ with $t > 0$, c_t fuzzy algebraical over B implies $c_t \subseteq C$. Fuzzy pure inseparable and fuzzy separable algebraical closures of B in A are defined similarly.

Proposition 4.8. (i) B has a fuzzy algebraical (pure inseparable, separable algebraical) closure in A. In fact, $B^{(c)}$ $(B^{(i)}, B^{(s)})$ is the fuzzy algebraical (pure inseparable, separable algebraical) closure of B in A if and only if $B^{(c)*}$ ($B^{(i)*}$, $B^{(s)*}$) is the algebraic (purely iseparable, separable algebraic) closure of B^* in A^*, i. e., $B^{(c)*} = B^{*(c)}$ ($B^{(i)*} = B^{*(i)}$, $B^{(s)*} = B^{*(s)}$).

(ii) $B^{(c)} \supseteq B^{(i)}, B^{(s)}$ and $B^{(c)}/B^{(s)}$ is fuzzy pure inseparable.

We now show that B does not necessarily have a fuzzy algebraic closure in A.

Example 4.9. Let $F = P(x)$ where P is a perfect field of characteristic $p > 0$ and x is transcendental over P. Let $A = \delta_F$. Define the fuzzy subset B of F by $B(y) = 1$ if $y \in P$ and $B(y) = i/(i+1)$ if $y \in P(x^{p^i}) - P(x^{p^{i+1}})$ for $i = 0, 1, \ldots$. Then B is a fuzzy subfield of F and $B \subseteq A$. Define the fuzzy subset C_i of F by $C_i(y) = 1$ if $y \in P$, $C_i(y) = i/(i+1)$ if $y \in F - P(x^{p^{i+1}})$, and $C_i(y) = B(y)$ if $y \in P(x^{p^{i+1}})$ for $i = 1, 2, \ldots$. Then $C_i \in \mathfrak{F}(A/B)$ for $i = 1, 2, \ldots$. Also $A = \bigcup_{i=1}^{\infty} C_i$. Let $c_t \subseteq C_i$. Suppose that $t = 1$. Then $c \in P = (C_i)_t = B_t$ and c is algebraic over B_t. Suppose that $0 < t < 1$. Then $B_t = P(x^{p^j})$ for some j. Hence c is algebraic (purely inseparable) over B_t. Thus c_t is fuzzy algebraic over B [4, Theorem 2.1]. Hence C_i/B is fuzzy algebraic for $i = 1, 2, \ldots$. Now A/B is not fuzzy algebraic since for $c \in F - P$, $c_1 \subseteq A$, but c is not algebraic over $B_1 = P$, [4, Theorem 2.1]. That is, A is a union of an ascending sequence of fuzzy subfields of F each fuzzy algebraic over B, but A is not fuzzy algebraic over B. Thus B does not have a fuzzy algebraic closure in A. However A/B is fuzzy algebraical so A is the fuzzy algebraical closure of B in A.

REFERENCES

1. D. S. Malik and J. N. Mordeson, Fuzzy subfields, Fuzzy Sets and Systems, to appear.
2. D. S. Malik and J. N. Mordeson, Fuzzy subgroups of abelian groups, preprint.
3. J. N. Mordeson, Fuzzy field extensions, Fuzzy Sets and Systems, to appear.
4. J. N. Mordeson, Fuzzy algebraic field extensions, Fuzzy Sets and Systems, to appear.
5. S. Nanda, Fuzzy fields and linear spaces, Fuzzy Sets and Systems, 19 (1986) 89 - 94.
6. M. T. Abu Osman, On t-fuzzy subfields and t-fuzzy vector subspaces, Fuzzy Sets and Systems, 33 (1989) 111 - 117.
7. A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512 - 517.
8. L. A. Zadeh, Fuzzy Sets, Inform. and Control 8 (1965) 338 - 353.