APPLYING FUZZY RELATIONS TO MODEL APPROXIMATE REASONING

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Fuzzy relations have become recently the main tool to model approximate reasoning in at least two new approaches. On one hand, fuzzy equivalence relations -i.e.fuzzy relations which are reflexive, symmetric and transitive with respect some t-norm T- play a central role in the work carried out by E.H. Ruspini [3] on the semantics of fuzzy logic. On the other hand, it has been shown in [1] that the duality between preorders and Tarski's consequence operators also holds in the fuzzy case.

In fact, in [2] it is shown that the Compositional Rule of Inference -when the so-called forward inference functions are used- defines a closure operator on the set of fuzzy-truth values, i.e. when defined by means of:

$$M_I(h)(y)=\sup_x m_I(h(x),I(x,y)),$$

where m_I is a Modus Ponens generating function for the implication function I [4]. M_I satisfies, among others, the following properties:

i) $M_I(h)\geq h$ for any h in $[0,1]^{[0,1]}$.
ii) If $h\geq h'$ then $M_I(h)\geq M_I(h')$.
iii) $M_I \circ M_I=M_I$

One of the main advances in [1] consists in the fact that the above result remain when in the Compositional Rule of Inference,

a) the implication function I used is replaced by an arbitrary T-fuzzy preorder R (i.e. a symmetric and transitive -with respect to the t-norm T- fuzzy relation), and

b) the Modus Ponens generating function m_I is replaced by the t-norm T.

In other words, given such a T-preorder R, the function

$$C_R(h)(y)=\sup_x T(h(x),R(x,y))$$

defines a consequence operator in the sense of Tarski, i.e. C_R satisfies the above properties (i) to (iii).

From this standpoint, the representation theorem for fuzzy transitive relations [5] allows us to address from a new perspective some interesting questions concerning the use of fuzzy tools to model approximate reasoning. Thus, for instance, in [2] it has been pointed out that the relationship between the inference rule and the way in which

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conditionals statements are represented -through suitable implication connectives, fuzzy preorders or whatever means- should be reversible. The above definition of the function C_R makes clear that it is possible to fix first the way in which inferences are made and then, the conditional statements can be defined according to some convenient criteria. To explain this, let it be noticed that if, for a given t-norm T and a fuzzy subset of the unit interval h, if R_h stands for the preorder generated by h through the representation theorem [5], i.e.

$$R_h(x,y)=T^\wedge(h(x)|h(y))$$

then it turns out that R is the biggest preorder for which $C_R(h)=h$. In particular, when h=j, the identity on the unit interval, R_h is precisely the R-implication associated with the t-norm T. In other words, each R-implication function, I, is the biggest fuzzy preorder for which the associated consequence operator C_I satisfies

$$C_I(j)=j$$

More generally, the representation theorem allows us to fix the preorder provided that some initial conditions are given. If such conditions are given in the form of a family of functions $\{h_j\}_{j\in J}$ for which $C_R(h_j)=h_j$ -as it is done in [2]- then

$$R(x,y)=\mathrm{Inf}_{j\in J}T^\wedge(h_j(x)|h_j(y))$$

is the biggest preorder for which the initial conditions are fulfilled. These, and related facts, will be exploited in order to give better solutions to the selection of suitable sets of fuzzy truth values, as formulated in [2].

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